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Robust model predictive control for uncertain singular systems with state delay

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Abstract: In this paper, we propose a model predictive control (MPC) law for a discrete time uncertain singular system with state delay and input constraints. The model uncertainty is assumed to be polytopic, and the delay is assumed to be unknown, but with a known upper bound. Using zero equation with the free variable matrix, we derive a sufficient condition for cost monotonicity in terms of LMI, which can be easily solved by an efficient convex optimization algorithm. The MPC problem is formulated to minimize the upper bound of infinite horizon cost that satisfies the sufficient conditions. A numerical example is included to illustrate the effectiveness of the proposed method.

Keywords: Robust MPC, Singular system, Time delay, LMI framework

1. INTRODUCTION

Model predictive control (MPC), also called as moving or receding horizon control, is a effective control method that allows to handle multi-variable linear systems with constraints systematically. Moreover, other advantages are known that MPC can easily handle time-varying system and provide good tracking performance. The drawbacks of MPC are the difficulty of incorporating plant model uncertainties and time-delays. In most practical control systems, uncertainties and time-delays have studied and often adversely affect performance and stability. Recently, many researchers have focused on the robust MPC of uncertain time delay systems but only MPC algorithms have not been developed to explicitly handle time-delay singular systems.

In [1], Kothare, et al, (1996) proposed a way to synthesize an MPC law that robustly stabilizes a closed loop system with model uncertainty, the authors assumed that the delay index was known. In [2], W.H. Kwon, et al, (2003) suggested the simple receding horizon control for continuous time systems with state delay, where the reduction technique was adopted so that an optimal problem for state delayed systems could be transformed into an optimal problem for delay free ordinary systems. The suggested control method can't guarantee the stability. In [3], Jeong and Park (2005) proposed a constrained MPC for an uncertain time varying system with an unknown bounded state delay using state feedback. In order to implement MPC for an unknown delay index, the authors defined the optimization problem for a known upper bound of delay index and proved the equivalence of the two optimization problems between a known upper bound of the delay index and an unknown delay index. They then, proved feasibilities and closed loop stabilities using the equivalence property. On the other hand, singular systems have attracted a lot researchers from mathematics and control communities due to the fact that singular systems can better describe the behavior of some physical systems than state-space ones [4], [5]. Singular systems are also referred to as descriptor systems, implicit systems, generalized state-space systems or semi-state systems, and a great number of fundamental notions and results based on the theory of regular systems have been extended to the area of singular systems [4]-[6]. Recently, the problems of stability analysis and stabilization for singular systems with time delay have been studied [7]. However, the results didn't use model predictive control schemes.

In this paper, we propose a model predictive control(MPC) algorithm for uncertain singular systems with unknown bounded delay. The proposed method is derived from the minimization problem of infinite horizon. Furthermore, we use zero equations with free variables in the terminal inequality condition to derive for delay independent LMI condition for cost monotonicity. These design technique using zero equations also guaranteed the system performance by reducing the upper bound of worst-case cost functions with respect to uncertainties. We consider an equivalent optimization problem to design MPC for discrete time uncertain singular systems with unknown bounded delay, and then solve the original optimization problem using the equivalence property. Finally, a numerical example shows the effectiveness of the proposed method.

Notation. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ is the set of $m \times n$ real matrix. $|\cdot|$ represents the absolute value. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. $diag\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix. I denotes an identity matrix with appropriate dimension. $\|x\|_{\mathcal{Q}}$ denotes $x^T\mathcal{Q}x$.

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2. PROBLEM STATEMENTS

Consider the following discrete-time uncertain singular systems with state delay

$$Ex(k+1) = A(k)x(k) + \bar{A}(k)x(k-d) + B(k)u(k),$$
$$x(k) = \Xi(k), \quad k \in [-d^*, 0] \tag{1}$$

subject to input constraints

$$-\bar{u} \le u(k) = K(k)x(k) \le \bar{u},\ \bar{u} > 0,\ \text{for all } k \in [0, \infty) \tag{2}$$

where $x(k) \in \mathbb{R}^n$ is the state, $u(k) \in \mathbb{R}^m$ is the control input, and $\Xi(k) \in \mathbb{R}^n$ is the initial condition. d is an unknown constant integer representing the number of delay index in the state, but being assumed $0 \le d \le d^*$ with the bounded value d^*. The system matrix $[A(k), \bar{A}(k), B(k)]$ is uncertain but belongs to a polytope Ω at all instants. That is

$$[A(k), \bar{A}(k), B(k)] \in \Omega$$
$$= Co\{[A_1, \bar{A}_1, B_1], [A_2, \bar{A}_2, B_2], \dots [A_p, \bar{A}_p, B_p]\} \tag{3}$$

where Co denotes the convex hull and $[A_i, \bar{A}_i, B_i]$ are vertices of the convex hull, The matrix $E \in \mathbb{R}^{n\times n}$ may be singular and it is assumed that rank $E = r \le n$. It is assumed that the system (1) is stabilizable by the state feedback control, and that the state $x(k)$ is available at every sampling instant k.we define the following infinite horizon quadratic cost function [1].

$$J_\infty(k) \triangleq \sum_{j=0}^{\infty} \{||x(k+j|k)||_{\mathscr{Q}} + ||u(k+j|k)||_{\mathscr{R}}\} \tag{4}$$

where $\mathscr{Q}, \mathscr{R}$ are given positive definite symmetric matrices. $x(k+j|k)$ and $u(k+j|k)$ are the predicted state and input, respectively, with $x(k|k) = x(k)$, $x(k-i|k) = x(k-i)$ for $i \ge 1$.
Consider the min-max optimization problem

$$\min_{u(k+j|k), j\ge 0} \max_{[A(k+j), \bar{A}(k+j), B(k+j)]\in\Omega} J_\infty(k) \tag{5}$$

subject to

$$-\bar{u} \le K(k)x(k+j|k) \le \bar{u} \quad j \in [0, \infty) \tag{6}$$

Then the state feedback controller with input constraints (6) yields less conservative sufficient conditions in terms of LMIs so that it allows us to design a more robust model predictive control law. In order to impose an upper bound of the cost (4), we define the following quadratic function as

$$V(x(k+j|k), P(k), P_d(k), d)$$
$$\triangleq \|x(k+j|k)\|_{E^T P(k) E} + \sum_{i=1}^{d} \|(x(k+j-i|k)\|_{P_d(k)} \tag{7}$$

where $P(k),\ P_d(k)$ and assume any positive definite symmetric matrices $P(k) > 0,\ P_d(k) > 0$ satisfy the following inequality for given $[A(k+j), \bar{A}(k+j),\ B(k+j)] \in \Omega, j \ge 0$ and d:

$$\Delta V(j,k) = V(x(k+j+1|k), P(k), P_d(k), d)$$
$$- V(x(k+j|k), P(k), P_d(k), d)$$
$$\le -(\|x(k+j|k)\|_{\mathscr{Q}} + \|u(k+j|k)\|_{\mathscr{R}}). \tag{8}$$

Then, the cost monotonicity is guaranteed for an uncertain system with state delay (1). For the cost in (4) to be finite, we must have $x(\infty|k) = 0$ or $V(x(\infty|k), P(k), P_d(k), d) = 0$. Summing both sides of inequality (8) from $j = 0$ to ∞, we get an upper bound of the worst cost (4)

$$\max_{[A(k+j), \bar{A}(k+j), B(k+j)]\in\Omega} J_\infty(k)$$
$$\le V(x(k|k), P(k), P_d(k), d). \tag{9}$$

Then, the min-max problem (5) is relaxed to the following minimization problem which minimizes the upper bound of the worst cost as

$$\min_{P(k), P_d(k)} V(x(k|k), P(k), P_d(k), d)$$
$$\text{subject to} \quad (6) \text{ and } (8). \tag{10}$$

In order to convert the minimization problem (10) into an LMI optimization problem that can be efficiently solved by a convex optimization algorithm [16], we define the upper bound of the object function in (10) as

$$V(x(k|k), P(k), P_d(k), d) \le \gamma(k) \tag{11}$$

where $\gamma(k)$ is the nonnegative upper bound of V to be minimized.
Then, the minimization of $V(x(k|k), P(k), P_d(k), d)$ is equivalent to

$$\min_{\gamma(k), K(k), P(k), P_d(k)} \gamma(k)$$
$$\text{subject to} \quad (6), (8) \text{ and } (11). \tag{12}$$

3. MAIN RESULTS

In this section, a novel LMI condition for cost monotonicity is presented before giving our main result for MPC design.

3.1 LMI condition for cost monotonicity

Let us define

$$x = x(k), \ \ \delta x \triangleq x(k+1) - x(k), \ \ x_d = x(k-d) \tag{13}$$

Then, for state feedback controller $u(k) = K(k)x(k)$, the state equation of the system (1) is written as

$$\begin{bmatrix} \Sigma(k) - E & \bar{A}(k) & -I \end{bmatrix} \begin{bmatrix} x \\ x_d \\ \delta x \end{bmatrix} = 0. \tag{14}$$

where $\Sigma(k) = A(k) + B(k)K(k)$

For nonzero free variable matrices $N_1(k) \in \mathbb{R}^{n\times n}$, $N_2(k) \in \mathbb{R}^{n_p\times n}$ and $N_3(k) \in \mathbb{R}^{n_p\times n}$, the following equality is always satisfied

$$\begin{bmatrix} x \\ x_d \\ \delta x \end{bmatrix}^T \begin{bmatrix} N_1(k) \\ N_2(k) \\ N_3(k) \end{bmatrix} \begin{bmatrix} \Sigma(k) - I & \bar{A} & -I \end{bmatrix} \begin{bmatrix} x \\ x_d \\ \delta x \end{bmatrix} = 0. \tag{15}$$

By using Eq. (15), we derive a new sufficient condition satisfying the terminal inequality (8) for any uncertainty in the following theorem.

In this subsection, a novel LMI condition for cost monotonicity is presented before giving our main result for MPC design.

Theorem 1 The terminal inequality (8) is satisfied for any uncertainty, if there exist $X(k)$, $Y(k)$, $Z(k)$, H, $Q(k) = Q^T(k) > 0$ and $Q_d(k) = Q_d^T(k) > 0$ satisfying the following LMI:

$$\begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & \Phi_{14} & 0 & 0 & 0 \\ \star & \Phi_{22} & \Phi_{23} & \Phi_{24} & 0 & 0 & 0 \\ \star & \star & \Phi_{33} & \Phi_{34} & \Phi_{35} & \Phi_{36} & \Phi_{37} \\ \star & \star & \star & \Phi_{44} & 0 & 0 & 0 \\ \star & \star & \star & \star & \Phi_{55} & 0 & 0 \\ \star & \star & \star & \star & \star & \Phi_{66} & 0 \\ \star & \star & \star & \star & \star & \star & \Phi_{77} \end{bmatrix} \geq 0 \tag{16}$$

where

$$\begin{aligned} &\Phi_{11} = -X(k) - X^T(k),\ \ \Phi_{12} = \bar{A}(k)Q_d(k) - Y(k), \\ &\Phi_{13} = (A(k) - E(k))Q(k) + B(k)H(k) - Z(k) + X^T(k), \\ &\Phi_{14} = X^T(k),\ \Phi_{22} = -Q_d(k),\ \Phi_{23} = \Phi_{24} = Y^T(k), \\ &\Phi_{33} = Z(k) + Z^T(k),\ \Phi_{34} = Z^T(k),\ \Phi_{35} = Q(k), \\ &\Phi_{36} = Q^T(k)\mathscr{Q}^{T/2},\ \Phi_{37} = H^T(k)\mathscr{R}^{T/2},\ \Phi_{44} = -Q(k), \\ &\Phi_{55} = Q_d(k),\ \ \Phi_{66} = \Phi_{77} = \gamma(k),\ \ H(k) \triangleq K(k)Q(k), \\ &Q(k) \triangleq \gamma^{-1}(k)P^{-1}(k)\ \ Q_d(k) \triangleq \gamma^{-1}(k)P_d^{-1}(k), \\ &l = 1, 2, \cdots p. \end{aligned} \tag{17}$$

Proof:

$$\begin{aligned} \Delta V(j,k) &= ||x(k+j+1|k)||_{E^TP(k)E} \\ &\quad - ||x(k+j|k)||_{E^TP(k)E} \\ &\quad + \sum_{i=1}^{d} ||x(k+j+1-i)||_{P_d(k)} - \sum_{i=1}^{d} ||x(k+j-i)||_{P_d(k)} \\ &= ||x(k+j+1|k)||_{P(k)} - ||x(k+j|k)||_{P(k)} \\ &\quad - ||x(k+j-d|k)||_{P_d(k)} + ||x(k+j|k)||_{P_d(k)} \\ &= \zeta^T \begin{bmatrix} P_d(k) & 0 & P(k) \\ 0 & -P_d(k) & 0 \\ P(k) & 0 & P(k) \end{bmatrix} \zeta \\ &< -\zeta^T diag\{\mathscr{Q} + K^T(k)\mathscr{R}K(k), 0, 0\}\zeta \end{aligned} \tag{18}$$

where

$$\begin{aligned} &\zeta = [\bar{x}^T \ \ \bar{x}_d^T \ \ \delta\bar{x}^T]^T, \\ &\bar{x} = x(k+j|k),\ \ \delta\bar{x} = x(k+j+1|k) - x(k+j|k), \\ &\bar{x}_d = x(k+j-d|k). \end{aligned} \tag{19}$$

After adding the left hand side of the equation (15) to Eq. (18), then we have

$$\begin{aligned} &\begin{bmatrix} P_d(k) & 0 & P(k) \\ 0 & -P_d(k) & 0 \\ P(k) & 0 & P(k) \end{bmatrix} \\ &\quad + \begin{bmatrix} N_1(k) \\ N_2(k) \\ N_3(k) \end{bmatrix} \begin{bmatrix} A_{cl}(k) & \bar{A}(k) & -I \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\ &+ \begin{bmatrix} A_{cl}(k) & \bar{A}(k) & -I \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}^T \begin{bmatrix} N_1(k) \\ N_2(k) \\ N_3(k) \end{bmatrix}^T \\ &< -\begin{bmatrix} \mathscr{Q} + K^T(k)\mathscr{R}K(k) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \end{aligned} \tag{20}$$

where $A_{cl} = A(k) + B(k)K(k) - I$. Inequality (20) can be rewritten as

$$\begin{aligned} &\begin{bmatrix} \mathcal{Q}(k) + P_d(k) + K^T\mathcal{R}K & 0 & 0 \\ 0 & -P_d(k) & 0 \\ 0 & 0 & P(k) \end{bmatrix} \\ &\quad + \Gamma(k)\Psi(k) + \Psi^T(k)\Gamma^T(k) < 0 \end{aligned} \tag{21}$$

where

$$\begin{aligned} \Gamma(k) &= \begin{bmatrix} N_1(k) & 0 & P(k) \\ N_2(k) & P_d(k) & 0 \\ N_3(k) & 0 & 0 \end{bmatrix}, \\ \Psi(k) &= \begin{bmatrix} A_{cl}(k) & \bar{A}(k) & -I \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}. \end{aligned} \tag{22}$$

For the simplicity, we define

$$\begin{aligned} M(k) &\triangleq \begin{bmatrix} 0 & 0 & Q(k) \\ 0 & Q_d(k) & 0 \\ X(k) & Y(k) & Z(k) \end{bmatrix} \\ &= \begin{bmatrix} N_1(k) & 0 & P(k) \\ N_2(k) & P_d(k) & 0 \\ N_3(k) & 0 & 0 \end{bmatrix}^{-T}. \end{aligned} \tag{23}$$

Pre- and post multiplying $M^T(k)$ and $M(k)$ in the left-hand side of inequality (21), respectively, then we obtain

that

$$\begin{bmatrix} X^T \\ Y^T \\ Z^T \end{bmatrix} P(k) \begin{bmatrix} X & Y & Z \end{bmatrix} + \begin{bmatrix} -X - X^T & \bar{A}Q_d(k) - Y & A_{cl}QY + X^TY - Z \\ \star & -Q_d(k) & Y^T \\ \star & \star & Z + Z^T \end{bmatrix} + \begin{bmatrix} Q(k)^TQQ(k) + Q(k)^TQ_dQ(k) + K^T\mathcal{R}K & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} < 0. \quad (24)$$

By the Schur complements [16], we can derive the following as

$$\begin{bmatrix} g_{11} & g_{12} & g_{13} & g_{14} & 0 & 0 & 0 \\ \star & g_{22} & g_{23} & g_{24} & 0 & 0 & 0 \\ \star & \star & g_{33} & g_{34} & g_{35} & g_{36} & g_{37} \\ \star & \star & \star & g_{44} & 0 & 0 & 0 \\ \star & \star & \star & \star & g_{55} & 0 & 0 \\ \star & \star & \star & \star & \star & g_{66} & 0 \\ \star & \star & \star & \star & \star & \star & g_{77} \end{bmatrix} \geq 0 \quad (25)$$

where

$$\begin{aligned} &g_{11} = -X(k) - X^T(k),\ \ g_{12} = \bar{A}(k)Q_d(k) - Y(k) \\ &g_{13} = (A(k) - E(k))Q(k) + B(k)H(k) - Z(k) + X^T(k) \\ &g_{14} = X^T(k),\ g_{22} = -Q_d(k),\ g_{23} = g_{24} = Y^T(k) \\ &g_{33} = Z(k) + Z^T(k),\ g_{34} = Z^T(k),\ g_{35} = Q(k), \\ &g_{36} = Q^T(k)\mathscr{Q}^{T/2},\ g_{37} = H^T(k)\mathscr{R}^{T/2},\ g_{44} = -Q(k), \\ &g_{55} = Q_d(k),\ \ g_{66} = g_{77} = \gamma(k),\ \ H(k) \triangleq K(k)Q(k), \\ &Q(k) \triangleq \gamma^{-1}(k)P^{-1}(k)\ \ Q_d(k) \triangleq \gamma^{-1}(k)P_d^{-1}(k), \\ &l = 1, 2, \cdots p. \end{aligned} \quad (26)$$

Since (28) is affine in $[A(k), \bar{A}(k), B(k)]$, it is satisfied for all

$$\begin{aligned} &[A(k), \bar{A}(k), B(k)] \in \Omega \\ &= Co\{[A_1, \bar{A}_1, B_1], [A_2, \bar{A}_2, B_2], \ldots [A_p, \bar{A}_p, B_p]\}. \end{aligned} \quad (27)$$

if and only if there exist $X(k)$, $Y(k)$, $Z(k)$, $Q(k)$, $Q_d(k)$ and γ satisfying (10). This completes the proof. Then, finally we have the inequality (16). This completes the proof. ■

3.2 Model predictive controller design

Next, we derive the LMI for the inequality (11). The inequality (11) is rewritten as

$$\|x(k|k)\|_{E^TP(k)E} + \sum_{i=1}^{d^*} \|(x(k-i|k)\|_{P_d(k)} \leq \gamma(k). \quad (28)$$

Choosing $P(k) = \gamma(k)Q^{-1}(k)$, $\bar{P}_d = \gamma(k)\bar{Q}_d^{-1}$, and using Schur complement [16], we have

$$\begin{bmatrix} 1 & x^T(k|k)E^T & \cdots & x^T(k-d^*|k) \\ Ex(k|k) & Q(k) & \cdots & 0 \\ x^T(k-1|k) & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ x(k-d^*|k) & 0 & \cdots & Q_d(k) \end{bmatrix} \geq 0. \quad (29)$$

Lastly, we convert the input constraint (6) into LMI. Let $E \triangleq \{z | z^TQ^{-1}(k)z \leq 1\}$ be invariant ellipsoids for the predicted states [1]. From (6), for $i = 1, 2, \cdots, m$,

$$\begin{aligned} \max_{j>1} |u_i(k+j|k)| &= \max_{j>1} |K_i(k)x(k+j|k)| \\ &= \max_{j>1} |H_i(k)Q^{-1}(k)x(k+j|k)| \\ &\leq \max_{j>1} |H_i(k)Q^{-1}(k)x(k+j|k)| \\ &\leq \max_{z_1 \in E_l} |(H_i(k)Q^{-1}(k)z_1)_i| \\ &\leq \|(H(k)Q^{-1/2}(k))_i\|_2^2 \\ &= (H(k)Q^{-1}(k)H^T(k))_{ii}. \end{aligned} \quad (30)$$

Assume that there exists a symmetric matrix $G(k)$ such that

$$(H(k)Q^{-1}(k)H^T(k))_{ii} \leq g_{ii}(k). \quad (31)$$

where $g_{ii}(k)$ is the i-th diagonal entry of $G(k)$. Using Schur complements [16], the input constraints (6) are satisfied, if for $i = 1, 2, \cdots, m$,

$$\begin{bmatrix} G(k) & H(k) \\ H^T(k) & Q(k) \end{bmatrix} \geq 0,\ \ g_{ii}(k) \leq \overline{u}_i^2 \quad (32)$$

where $\overline{u}_i$ is the i-th element of $\overline{u}$.
Now, the minimization problem (12) is converted to the following LMI optimization problem

$$\min_{\gamma(k), G(k), H(k), Q(k)} \gamma(k) \quad \text{subject to} \quad (16), (29) \text{and} \quad (32). \quad (33)$$

The feedback gain matrices $K(k) = H(k)Q^{-1}(k)$ are found from the solutions of the LMI optimization problem (33). However, since the delay index d is unknown, we cannot get the solutions for the minimization problem (12) directly. In order to overcome this difficulty, we consider a quadratic function with known d^* in place of d as

$$V(x(k|k), P(k), P_d(k), d^*) \leq \gamma(k) \quad (34)$$

under the assumption that $P_d(k)$ are fixed to any positive definite constant matrices $\bar{P}_d$ at all instants k. Then, we have the following optimization problem

$$\begin{aligned} \mathscr{P}(P(k), \bar{P}_d, d^*) &\min_{\gamma(k), K(k), P(k)} \gamma(k) \\ &\text{subject to } (16), (29), \text{and } (32). \end{aligned} \quad (35)$$

Jeong and Park [3] showed the equivalence of the two optimization problems (12) and (35) under the assumption that $P_d(k)$ are fixed to constant matrices $\bar{P}_d$ and have the same solutions. Therefore, we will solve the minimization problem (35) associated with known d^*. Based on the equivalence, we can prove the feasibility and the closed loop stability. The proof is omitted for simplicity.

4. NUMERICAL EXAMPLE

In this section, a robust model predictive controller $u(k) = Kx(k)$, will be established for the uncertain discrete singular system with time-delay (1).

We first consider the nominal discrete singular system (1) with time-delay described by

$$Ex(k+1) = A(k)x(k) + \bar{A}(k)x(k-d) + B(k)u(k)$$

where,

$$E = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A(k) = \begin{pmatrix} 1.509 & 0 & 0 \\ -0.0509 & 1 & 0 \\ 0.0509\alpha(k) + \vartheta\beta(k) & -0.4\alpha(k) + \varrho\beta(k) & 1 \end{pmatrix}$$

$$\bar{A}(k) = \begin{pmatrix} 0.0218 & 0 & 0 \\ -0.0218 & 0 & 0 \\ 0.0218\alpha(k) + \varpi\beta(k) & 0 & 0 \end{pmatrix}$$

$$B(k) = \begin{pmatrix} -0.1429 \\ 0 \\ 0 \end{pmatrix} \tag{36}$$

$\alpha(k)$ and $\beta(k)$ are time varying uncertain parameters belong to the following sets

$$\alpha(k) \in [0 \;\; 1], \;\; \beta(k) = 1 - \alpha(k),$$
$$\vartheta = 0.609, \; \varrho = -0.5, \; \varphi = 0.258 \tag{37}$$

Simulation parameters are as follows: the initial value of the state is $x(0) = [0.5\pi, 0.75\pi, -5]^T$; the state and the input weighting matrices are $Q = diag(10, 10, 10)$ and $R = 1$, $d = 4$, $d^* = 10$, $r_n = 10$ All optimization problems are solved by using matlab 7.2.0. We easily proved the closed loop stability through the monotonic decreasing property for all $k \geq 0$. However, the results may be conservative because we assume a constant matrix $\bar{P}_d$ at all times k. In this simulation, we applied an novel robust MPC algorithm with iterative step so that $\bar{P}_d$ is updated [3].

An Robust MPC is designed and the simulation results are shown in Figs. 1 and 2. In Figs. 1 and 2, the simulated states and inputs of the resulting closed-loop system are illustrated when the time delay is given by $d = 4$. It is shown that the closed-loop system is stable and impulse-free. In Fig 3, the upper bound $\gamma(k)$ of the cost function monotonically decreases. In order to see the this results in detail, we plotted the $\gamma(k)$ by logarithmic scale in Fig.4.

5. CONCLUSIONS

In this paper, we proposed new cost monotonicity condition of robust MPC algorithm for uncertain singular systems with state delay. The controller is designed by solving the min-max problem with the infinite horizon cost function subject to a new LMI condition for cost monotonicity and input constraints using zero equation with free variable matrices. The synthesis conditions were represented in the form of a finite number of LMIs. Numerical examples demonstrated a significant performance over existing approaches.

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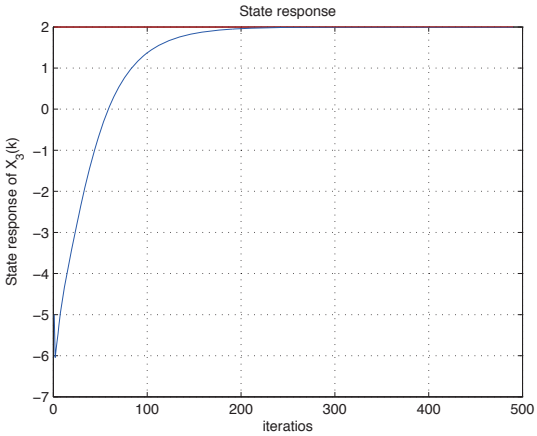


Fig. 1 The set-point tracking performance

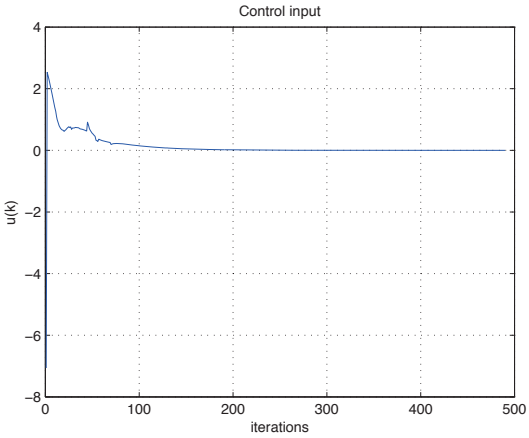


Fig. 2 Input trajectories

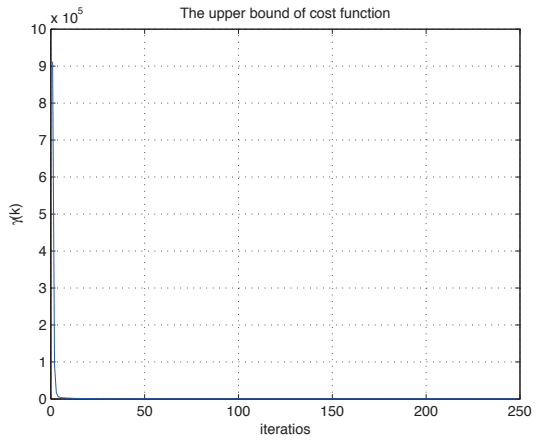


Fig. 3 Upper bound of cost $\gamma(k)$

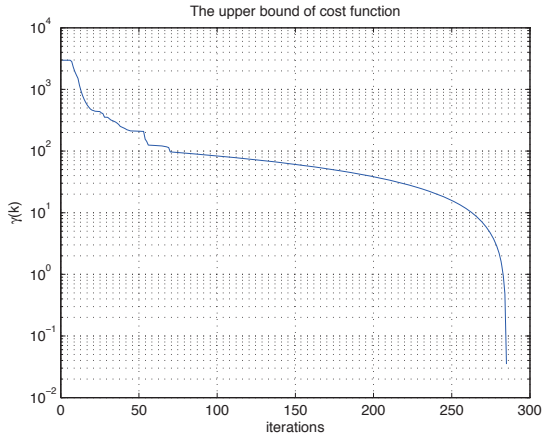


Fig. 4 Upper bound of cost $\gamma(k)$: logarithmic scale

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