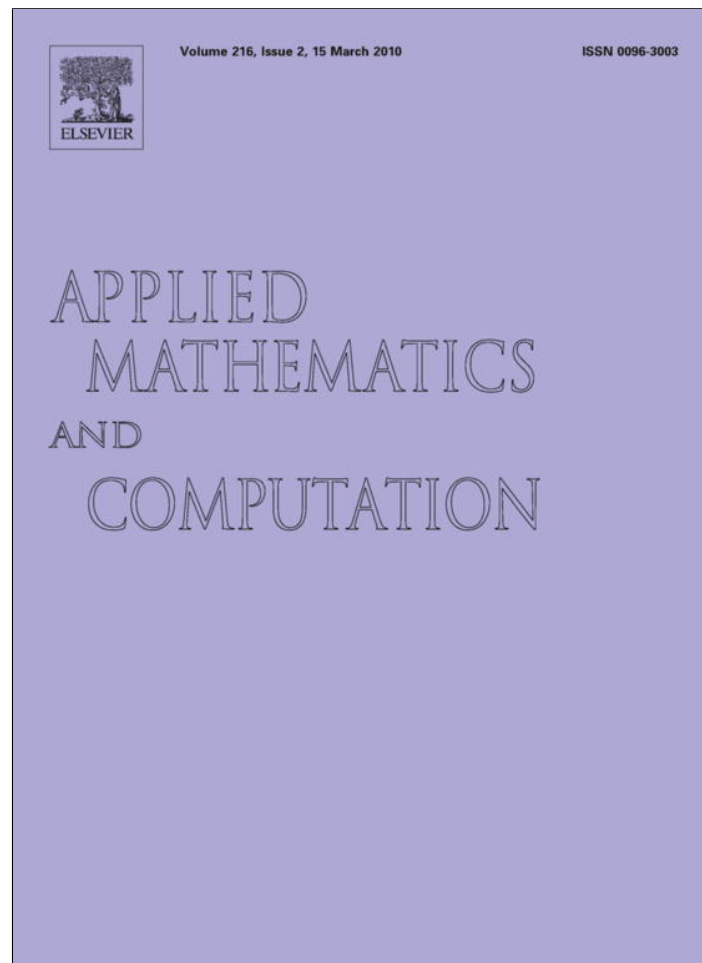


Provided for non-commercial research and education use.
Not for reproduction, distribution or commercial use.



This article appeared in a journal published by Elsevier. The attached copy is furnished to the author for internal non-commercial research and education use, including for instruction at the authors institution and sharing with colleagues.

Other uses, including reproduction and distribution, or selling or licensing copies, or posting to personal, institutional or third party websites are prohibited.

In most cases authors are permitted to post their version of the article (e.g. in Word or Tex form) to their personal website or institutional repository. Authors requiring further information regarding Elsevier's archiving and manuscript policies are encouraged to visit:

<http://www.elsevier.com/copyright>



Contents lists available at ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

3-D mesh sequence compression using wavelet-based multi-resolution analysis

Jae W. Cho^a, S. Valette^b, Ju H. Park^c, Ho Y. Jung^{d,*}, R. Prost^b

^a Media Solution Center, Samsung Electronics, Co. Ltd., 416, Maetan-3-dong, Yeongtong-gu, Suwon, Gyeonggi-do, 443-742, Republic of Korea

^b CREATIS, CNRS #5520, Inserm U630, INSA-UCB, Université de Lyon, Villeurbanne, France

^c Department of Electrical Engineering, Yeungnam University, 214-1, Dae-dong, Gyeongsan-si, Gyeongsangbuk-do, 712-749, Republic of Korea

^d School of Electronic Engineering, Communication Engineering and Computer Science, Yeungnam University, 214-1, Dae-dong, Gyeongsan-si, Gyeongsangbuk-do, 712-749, Republic of Korea

ARTICLE INFO

Keywords:

Data compression
Image compression
Wavelet transform

ABSTRACT

In this paper, we present two compression methods for irregular three-dimensional (3-D) mesh sequences with constant connectivity. The proposed methods mainly use an exact integer spatial wavelet analysis (SWA) technique to efficiently decorrelate the spatial coherence of each mesh frame and also to adaptively transmit mesh frames with various spatial resolutions. To reduce the temporal redundancy, the first proposed method applies multi-order differential coding (MDC) to the temporal sequences obtained from SWA. MDC determines the optimal order of the differential coder by analyzing the variance of prediction errors. Comparing with the first order differential coding (FDC) scheme, the method can improve the compression performance. The second proposed method applies temporal wavelet analysis (TWA) to the temporal sequences. In particular, this method offers spatio-temporal multi-resolution coding. Through simulations, we prove that our methods enable efficient lossy-to-lossless compression for 3-D mesh sequences in a single frame work.

© 2010 Elsevier Inc. All rights reserved.

1. Introduction

With the remarkable progress of multimedia and information technologies, three-dimensional (3-D) data has been more and more widely used in various applications such as virtual reality, video games, animation movies and medical images.

Polygonal meshes provide an efficient representation of 3-D objects [1,2], since they can be rapidly rendered by existing graphics hardware. Like as the categorization of 2-D still images and motion pictures, they can be classified into static meshes and mesh sequences. Static meshes contain two kinds of principal information, the locations of vertices and their topological connections – geometry and connectivity, respectively. Similar to motion pictures, a 3-D mesh sequence consists of consecutive static meshes. The motion of meshes is usually represented by vertex displacements. These kinds of mesh sequences have constant connectivity information over all mesh frames. On the other hand, some mesh sequences might have variable connectivity over all or partial mesh frames. In this paper, we address only mesh sequences with constant connectivity.

Generally, mesh sequences obtained by 3-D scanners or mesh design tools such as 3-D Studio MAX require huge capacity or enormous bandwidth to be stored or transmitted. For that reason, it has become an important issue to develop efficient compression methods for 3-D mesh sequences. Similar to 2-D motion picture compression, spatial and temporal redundancies are mainly exploited to minimize data size. To reduce the spatial redundancy, the geometry and connectivity

* Corresponding author.

E-mail addresses: jaewon79.cho@samsung.com (J.W. Cho), sebastien.valette@creatis.insa-lyon.fr (S. Valette), jessie@yu.ac.kr (J.H. Park), hoyoul@yu.ac.kr (H.Y. Jung), remy.prost@creatis.insa-lyon.fr (R. Prost).

information of a single mesh frame can be modeled for entropy coding. The geometrical coherence in temporal direction between consecutive mesh frames can be used to reduce the temporal redundancy. Clearly, other attributes such as normal vectors or texture information could be also regarded as important components to be compressed. Note that we focus on geometry coding in this paper.

Since Lengyel [3] proposed a geometry compression method for 3-D mesh sequences, there have been several attempts to reduce the spatial and geometry redundancies [3–10]. In [3], the original meshes are segmented into small rigid body meshes. The motion of each rigid body mesh is represented by affine transform coefficients, then the coefficients and residuals are quantized and encoded by an entropy coder. This algorithm uses temporal coherence of rigid body meshes to reduce temporal redundancy. However, this method has difficulties to obtain precise segmentation and cannot have good coding performance for the mesh sequences with high geometrical complexity. A quantization based method was also presented by Zhang and Owen [4]. They proposed a hybrid compression method combining delta and octree coding schemes. For each mesh frame, the geometry information is encoded by using selectively one of two coding schemes which has smaller prediction errors. This technique requires high processing time, because it iterates the encoding processes until the predetermined visual quality of the decoded mesh sequences. Some principal component analysis (PCA) based methods have been presented [5,6]. Alexa and Müller [5] represented 3-D mesh sequences using several principal bases obtained by PCA. Karni and Gotsman [6] expanded it to a hybrid method combining PCA and linear prediction coding (LPC). However, these methods essentially require high computational complexity to calculate the eigenvectors. Ibarria and Rossignac [7] introduced an efficient compression method which can simultaneously reduce the temporal and spatial redundancies by using a space-time replica predictor. Recently, scalability has become an important issues in video coding, as it facilitates to adaptively manage bit-rates according to different conditions of bandwidth or capacity [11]. From the viewpoint of scalable coding, wavelet transform – spatial wavelet analysis (SWA) and/or temporal wavelet analysis (TWA) – is suitable for 3-D mesh sequences. Some wavelet-based methods have been introduced [10,8,9]. Payan and Antonini [10] used a TWA to reduce temporal redundancy. Although they achieved good compression performance by using their optimal bit allocation scheme, they did not consider the spatial redundancy. Guskov and Khodakovsky [8] introduced a SWA-based compression algorithm. They encoded the differential errors between the wavelet coefficients of previous and current frames. Here, the wavelet coefficients are obtained from the Burt–Adelson style pyramid scheme. The method can provide the spatial resolution scalability.

In this paper, we propose two compression techniques of the mesh geometry for 3-D mesh sequences with constant connectivity. To reduce the spatial redundancy, both proposed methods use the SWA technique which employs an exact integer analysis and synthesis filter bank [12]. The filters can be directly applied to irregular meshes. Besides, they can easily achieve lossy-to-lossless compression.¹ In order to reduce the temporal redundancy, we consider two different techniques, multi-order differential coding (MDC) and TWA. The first method uses SWA and MDC schemes. In our previous work [9], we used a first order differential coding (FDC) technique employing IPPP frame pattern coding which combines Intra-mesh and Predicted-mesh coding. To improve the coding efficiency, we introduce a more sophisticated approach in which the variances of prediction errors are analyzed to find the optimal order. The second method employs both SWA and TWA schemes. Although TWA scheme was applied in a previous algorithm [10], there has been no attempt to apply SWA and TWA, simultaneously. Both proposed methods can provide lossy-to-lossless compression if input mesh sequences have integer coordinates. The first method can reconstruct mesh sequences with various spatial resolutions, and the second enables temporal multi-resolution coding as well as spatial one.

The rest of this paper is organized as follows. In Section 2, SWA and TWA techniques, which are used in our compression schemes, are introduced. Section 3 shows that the entropy coding efficiency can be estimated from the variance of the prediction model. Two compression methods using wavelet-based multi-resolution analysis are proposed in Section 4. Section 5 shows the simulation results of the proposed methods in terms of lossless and lossy compression performances. Finally, Section 6 concludes this paper.

2. Overview of wavelet-based multi-resolution analysis

Early compression methods for multimedia data have been mainly concentrated on the development of single-rate coding system. Although single-rate coding has enough performance in a network environment with fixed bandwidth, it might be difficult to be promptly applied to variable bandwidth conditions. For that reasons, scalable coding techniques such as the annexed functionalities of motion picture experts group (MPEG)-2 and -4 have been intensively researched. In general scalable decoding frameworks, the coarsest version is first reconstructed from the base layer, and higher resolution versions are adaptively produced from the enhancement layers depending on channel conditions. It has been well-known that wavelet-based multi-resolution analysis techniques are useful for scalable coding. Besides, they provide good coding performance, as the probability density function (PDF) of the wavelet coefficients can be approximated to Laplacian distribution with a sharp peak [9]. These are the reasons why we use SWA and TWA in order to design efficient 3-D mesh sequence compression systems. In the following sub-sections, SWA and TWA schemes are summarized.

¹ In general, lossy compression gain is determined by quantization via Rate-Distortion (R-D) optimization. Note that, in this paper, we regard 'multi-resolution transmission (or representation)' as 'lossy compression' because it could also reduce data size.

2.1. Spatial wavelet analysis (SWA) and its synthesis

The wavelet-based multi-resolution scheme for 3-D static meshes was firstly introduced by Lounsbery [13]. Fig. 1 shows an example of SWA and its synthesis processes. From the original mesh C^j , the SWA is performed by two analysis filters, A^j (low-pass filter) and B^j (high-pass filter) as follows

$$C^{j-1} = A^j C^j \tag{1}$$

$$D^j = B^j C^j \text{ for } 0 \leq j \leq J \tag{2}$$

where j is the spatial resolution level, and C^j is a $v^j \times 3$ matrix representing the vertex coordinates (x -, y -, and z -coordinates) of the input mesh having v^j vertices. A fine mesh C^j is decomposed into a coarse mesh C^{j-1} and wavelet coefficients D^{j-1} . The wavelet coefficients represent the lost details. We obtain a hierarchy of meshes from the original C^j to the simplest one, C^0 , so-called base mesh.

The reconstruction is done by two synthesis filters, P^j and Q^j . It is formulated as

$$C^j = P^j C^{j-1} + Q^j D^{j-1} \tag{3}$$

A fine mesh is reconstructed from the coarse one and the corresponding wavelet coefficients. If the filter-banks satisfy the following constraint, we can achieve perfect reconstruction [12,14].

$$\begin{bmatrix} A^j \\ B^j \end{bmatrix} = [P^j | Q^j]^{-1} \tag{4}$$

Lounsbery's scheme handles meshes with one-to-four (1:4) subdivision connectivity. The mesh hierarchy can be considered as successive quadrisections of a base mesh (C^0) faces followed by deformation of edge midpoints to fit the surface to be approximated. The vertices of coarse mesh have arbitrary valences while the subdivided mesh interiors and boundary vertices have valence six and four, respectively. Conversely, four-to-one (4:1) face coarsening in Eq. (1) is the inverse operation of quadrisection. The wavelets functions, in this scheme, are hat functions associated with odd vertices of the mesh at resolution j and linearly vanishing on the opposite edges. This wavelet is often called the 'Lazy wavelet'. The scaling functions are also hat function but with a twice wider support and are associated with the even vertices. However, wavelets are not orthogonal to scaling functions. Then a primal 2-ring lifting is used to construct new wavelets which are more orthogonal to the scaling functions. These wavelets produce the coarse meshes with good quality in terms of approximation.

Recently the wavelet multi-resolution analysis has been extended to irregular mesh (vertices can have any valence) by Valette and Prost [12]. In [15], they also introduced an exact integer analysis and synthesis with the lifting scheme based on Lazy filter-banks and the Rounding transform [16,17]. Now, the analysis is sequentially performed by the lifted Lazy filter-banks.

$$D^{j-1} = [B^j_{lazy} C^j] \tag{5}$$

$$C^{j-1} = A^j_{lazy} C^j + [\alpha^j D^{j-1}] \tag{6}$$

where A^j_{lazy} and B^j_{lazy} are Lazy analysis filters, and α^j is a $v^{j-1} \times (v^j - v^{j-1})$ matrix chosen to ensure that C^{j-1} is the best approximation of C^j . The synthesis is done by

$$C^j = [P^j_{lazy} (C^{j-1} - [\alpha^j D^{j-1}])] + Q^j_{lazy} D^{j-1} \tag{7}$$

where P^j_{lazy} and Q^j_{lazy} are Lazy synthesis filters, and $[\cdot]$ and $\lceil \cdot \rceil$ are the floor and ceiling operators, respectively. These modified filter-banks make it possible to implement a lossless compression for the given meshes with integer coordinates. In addition, they can be applied to irregular meshes by using an irregular subdivision scheme [15]. Note that the Lounsbery's method based on regular subdivision scheme [13] cannot work on irregular ones. These are the reasons why our methods use this exact integer analysis/synthesis scheme and an irregular coarsening approach. The notation 'SWA' indicates the exact integer spatial wavelet analysis in the rest of this paper. For more details about the SWA, refer to [12,15] and [18].

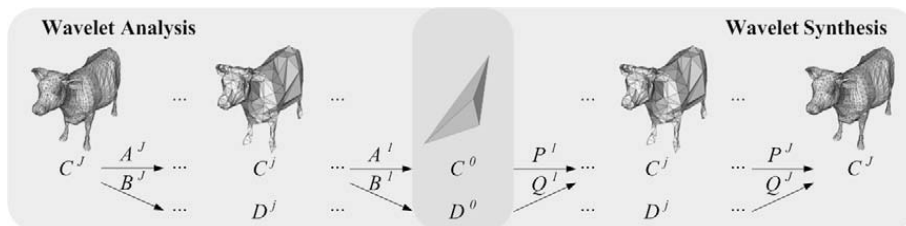


Fig. 1. Spatial wavelet analysis (SWA) and its synthesis processes.

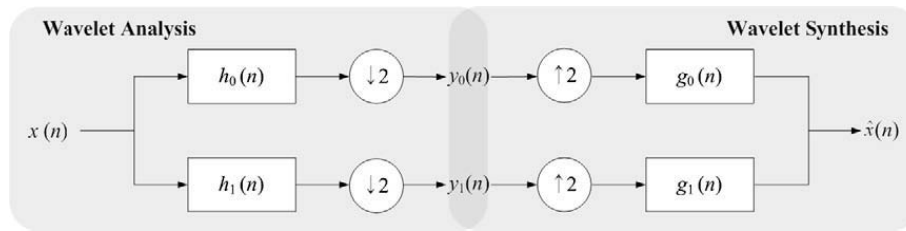


Fig. 2. 2-Channel temporal wavelet analysis (TWA) and its synthesis processes.

2.2. Temporal wavelet analysis (TWA) and its synthesis

Fig. 2 shows an example of the TWA and its synthesis processes. In the wavelet analysis process, the original signal $x(n)$ (for $1 \leq n \leq N$, and N is the number of samples) is decomposed into low and high frequency band signals, $y_0(n)$ and $y_1(n)$, by an analysis filter-bank, $h_0(n)$ and $h_1(n)$. Here, N should be an integer power of two. If a practical temporal sequence has arbitrary number of samples, it can be classified into several groups (each group consists of N samples). And then TWA is applied to each group. Note that the extra samples can be added by zeros in order that the last group has N samples. From the view point of coding efficiency, we need to carefully determine the number of groups, since the bit-rate can increase as the number of extra sample does. Low and high frequency band signals correspond to the coarse version of the original signal and its details, respectively. To obtain more resolution levels, the analysis process can be repeatedly applied to the low frequency band signal. In the wavelet synthesis process, the two sub-band signals are transformed into a reconstructed signal $\hat{x}(n)$ by a synthesis filter-bank, $g_0(n)$ and $g_1(n)$. Note that the implementation allows lossless compression using a lifting scheme [19,16,20].

In our second proposed method (see Section 4.2), TWA is applied to whole temporal sequences. Here, temporal movement of each coordinate is regarded as a 1-D signal as in [10].

3. Estimation of entropy coding efficiency

To improve the entropy coding efficiency, there have been many trials to produce a good prediction model whose distribution exhibits one sharp peak. The more the distribution concentrates on a specific value, the better the coding efficiency can be expected. A good statistical model for the prediction errors is a Laplacian distribution. The coding efficiency can be estimated by using the variance of the distribution.

Consider a continuous random variable X with zero mean Laplacian distribution for which the PDF is defined by

$$p_X(x) = \frac{\lambda}{2} e^{-\lambda|x|}. \tag{8}$$

Here, the sharpness of the distribution can be determined by the parameter λ . Clearly, the entropy² of the random variable, $H(X)$, is obtained as [21]

$$H(X) = \int_{-\infty}^{\infty} p_X(x) \log_2 \frac{1}{p_X(x)} dx = \log_2 \frac{2}{\lambda} + \frac{1}{\ln 2}. \tag{9}$$

The variance, σ^2 , is given by the second moment $E[X^2]$:

$$\sigma^2 = E[X^2] = \int_{-\infty}^{\infty} x^2 p_X(x) dx = \frac{2}{\lambda^2}. \tag{10}$$

From Eqs. (9) and (10), both the entropy and variance are functions of λ . Therefore Eq. (9) can be rewritten as

$$H(X) = \log_2 \sigma \sqrt{2} + \frac{1}{\ln 2}. \tag{11}$$

Fig. 3 shows a relationship between the entropy and the variance according to different σ . As shown in this figure, the variance is highly correlated with the entropy. In our approaches, the entropy coding efficiency is estimated using the variance of prediction model.

² This type of entropy is named ‘continuous or differential entropy’. It could sometimes have negative values and then be inefficient to measure the amount of information comparing with Shannon entropy. Note that we just use continuous entropy to demonstrate the inter-relationship between entropy and variance focusing on a statistical model, Laplacian distribution.

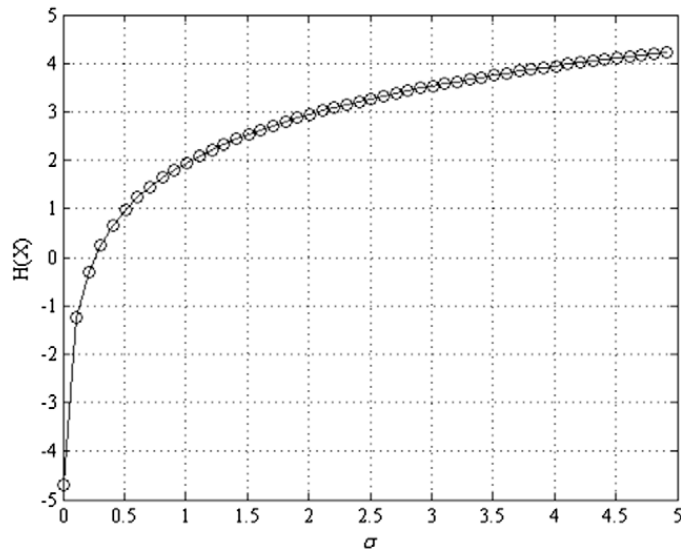


Fig. 3. Relationship between the entropy and the variance according to different σ .

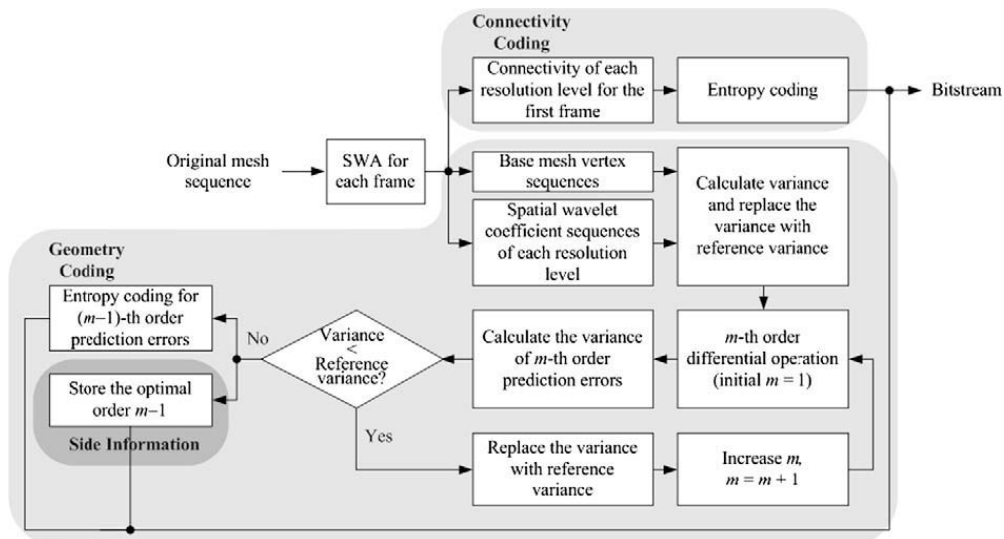


Fig. 4. The encoding process of the proposed method using SWA and MDC techniques.

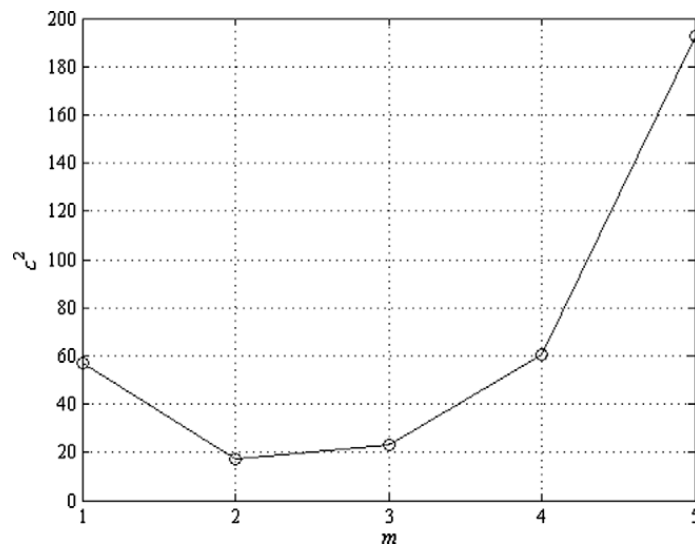


Fig. 5. Variances (σ^2) of prediction errors of Face model according to different orders (m) of MDC. The x-coordinate of the first base mesh vertex sequence of this model is designated for a practical example.

4. Proposed compression methods

The proposed methods mainly use the exact integer SWA scheme [15] introduced in Section 2.1. The SWA can efficiently reduce the spatial redundancy and also offer a progressive transmission from the base mesh to the original one for static

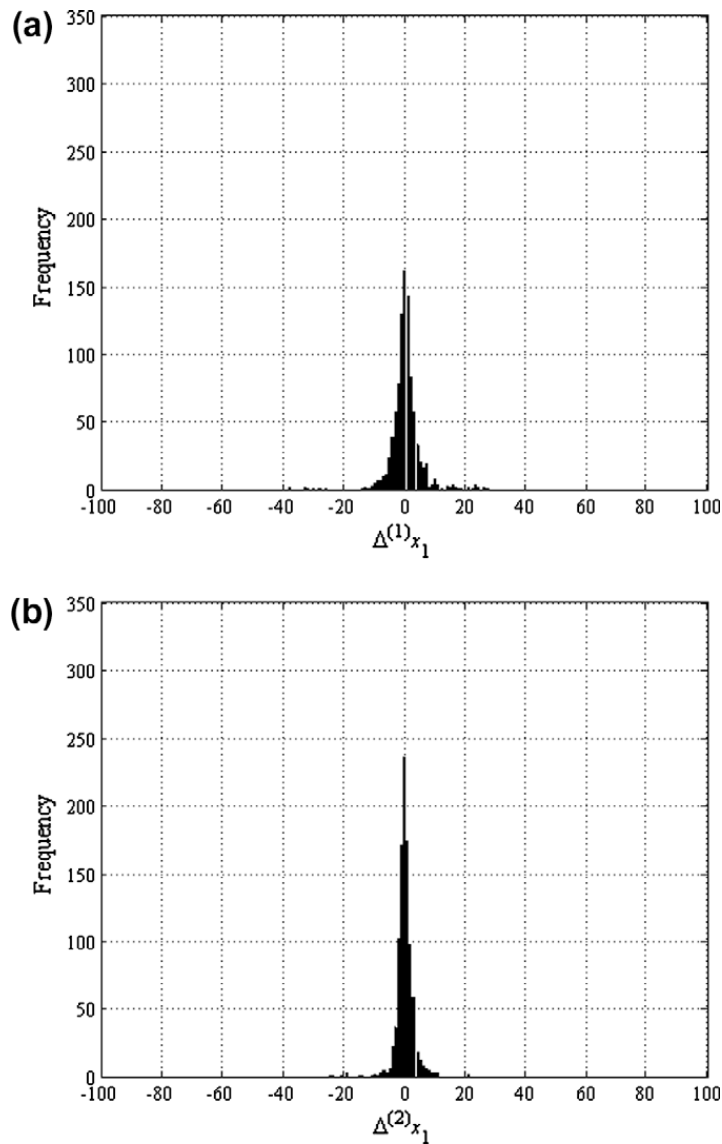


Fig. 6. Prediction error distributions of Face model in terms of (a) FDC and (b) MDC.

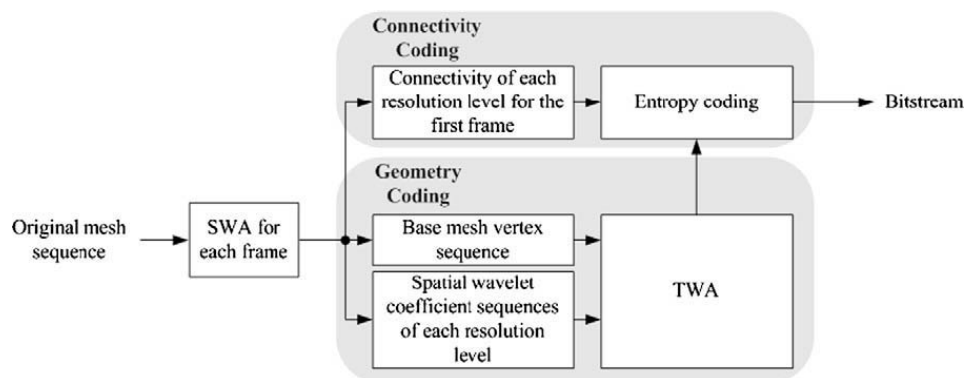


Fig. 7. The encoding process of the proposed method using SWA and TWA techniques.

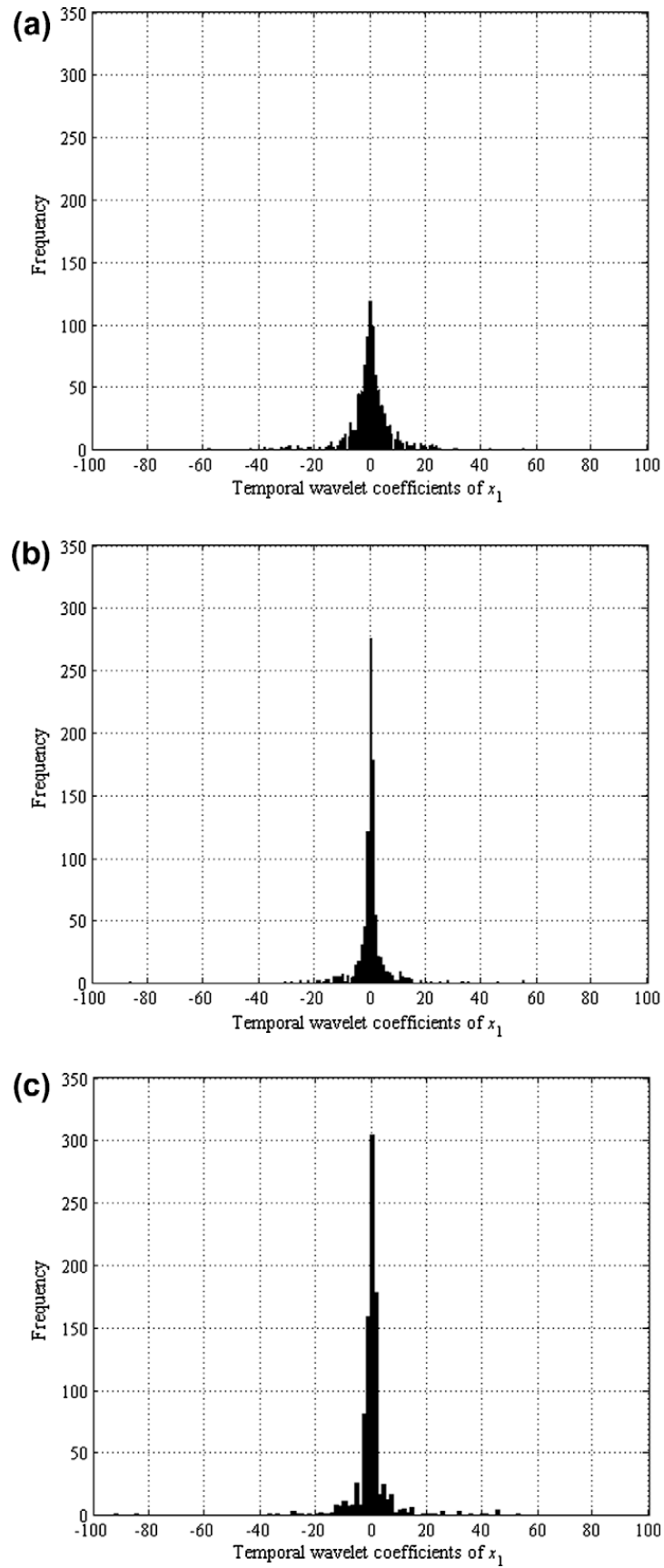


Fig. 8. Distributions of wavelet coefficients of the x -coordinate of the first base mesh vertex sequence of Face model using (a) Haar (2/2 tap), (b) Le Gall (5/3 tap) (c) Daubechies (9/7 tap) filters.

irregular meshes. Clearly, this scheme can be applied to each frame of 3-D mesh sequences, and therefore provide an adaptive transmission for mesh sequences when the bandwidth is not fixed because it can produce mesh sequences with various spatial resolutions.

To reduce the temporal redundancy existing in the base mesh and spatial wavelet coefficients of mesh sequences, two different techniques are used in our proposed methods. The first proposed method employs the MDC which determines the optimal order of the differential encoder by evaluating the variance of the prediction error. The MDC provides simple and adaptive differential coding technique. The second employs the TWA scheme. This approach allows spatio-temporal multi-resolution coding in a single frame work. Although TWA was already used for geometry compression in temporal domain [10], there has been no attempt to use simultaneously both SWA and TWA. In the following sub-sections, we describe our proposed methods in details.

4.1. The proposed compression method using SWA and MDC

Fig. 4 shows the encoding process of the proposed method using SWA and MDC techniques. We assume the original mesh sequences are represented by integer coordinates. First, each mesh frame is transformed by the exact integer SWA [15]. Two kinds of major information are obtained from this transform for each frame, namely the connectivity and the geometry. The geometry information contains the coordinates of the base mesh and of wavelet coefficients corresponding to each spatial resolution level. The connectivity is the topological connections of the vertices.

The second step is connectivity coding. The connectivity information obtained from the first step is entropy coded by an arithmetic coder [22]. Note that this process is performed only for the first frame, because the mesh sequence has constant connectivity. The reader can refer to [15] for more details of the connectivity coding.

The third step is geometry coding to reduce the temporal redundancy. Each coordinate of the base mesh vertices and of the spatial wavelet coefficients is processed independently as 1-D signal with N samples along temporal direction. Here, N is the number of frames. For a given r th base mesh vertex $c_r^0 = (x_r, y_r, z_r)$ ($c_r^0 \in C^0$ for $1 \leq r \leq R$, and R is the number of base mesh vertices), each coordinate is independently treated as 'base mesh vertex sequence': $x_r(n), y_r(n)$, and $z_r(n)$ (for $1 \leq n \leq N$). Similarly, for a given s th wavelet coefficient of the j th spatial resolution level $d_s^j = (x_s^j, y_s^j, z_s^j)$ ($d_s^j \in D^j$ for $1 \leq s \leq S$, and S is the number of wavelet coefficients in each spatial resolution level), each coordinate is independently treated as 'spatial wavelet coefficient sequence': $x_s^j(n), y_s^j(n)$, and $z_s^j(n)$. Consequently, the 'temporal sequences' consist of the base mesh vertex sequences and the spatial wavelet coefficient sequences. Note that MDC is applied to each temporal sequence.

Before discussing MDC technique, the first order differential operation for the temporal sequences is formulated by

$$\Delta^{(1)}u(n) = u(n) - u(n - 1), u \in \{x_r, y_r, z_r, x_s^j, y_s^j, z_s^j\} \tag{12}$$

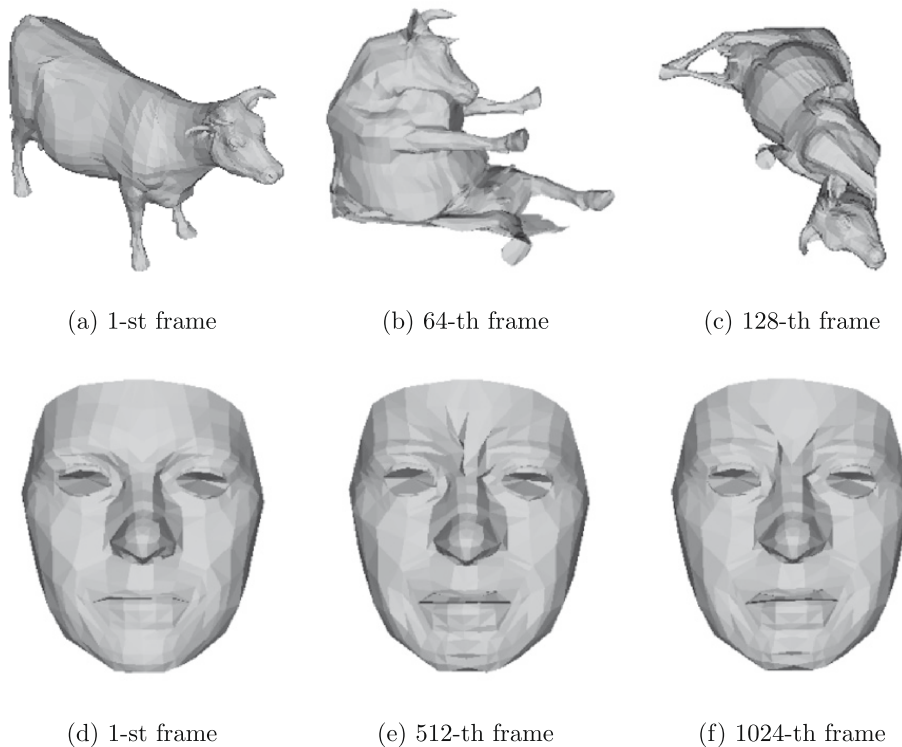


Fig. 9. Original mesh sequences, (a)–(c) Cow and (d)–(f) Face models.

The prediction errors obtained from Eq. (12) might still have some redundancy. Then, the remaining redundancy could be reduced by repeatedly applying the differential operation to the prediction errors. This is named MDC and given by

$$\Delta^{(m)}u(n) = \Delta^{(m-1)}u(n) - \Delta^{(m-1)}u(n - 1) \tag{13}$$

where $m(m \geq 2)$ is the order of differential coding. Then this order can be easily determined by finding the optimal order with smaller entropy via analyzing the variance of the prediction errors. In order to prove the correctness of our idea, we consider that the input sequence is modeled as wide sense stationary markov process (WSSMP). As proved in Appendix A, the first order predictor can be applied for a first order Markov process (a first order autoregressive process, AR(1)) with

Table 1

Lossless compression results of SWA + MDC method compared with SWA and SWA + FDC methods.

Method	Model	Bitrate (bits/vertex/frame)
SWA	Cow	22.40
	Face	30.04
SWA + FDC	Cow	14.22
	Face	11.49
SWA + MDC	Cow	13.57
	Face	10.52

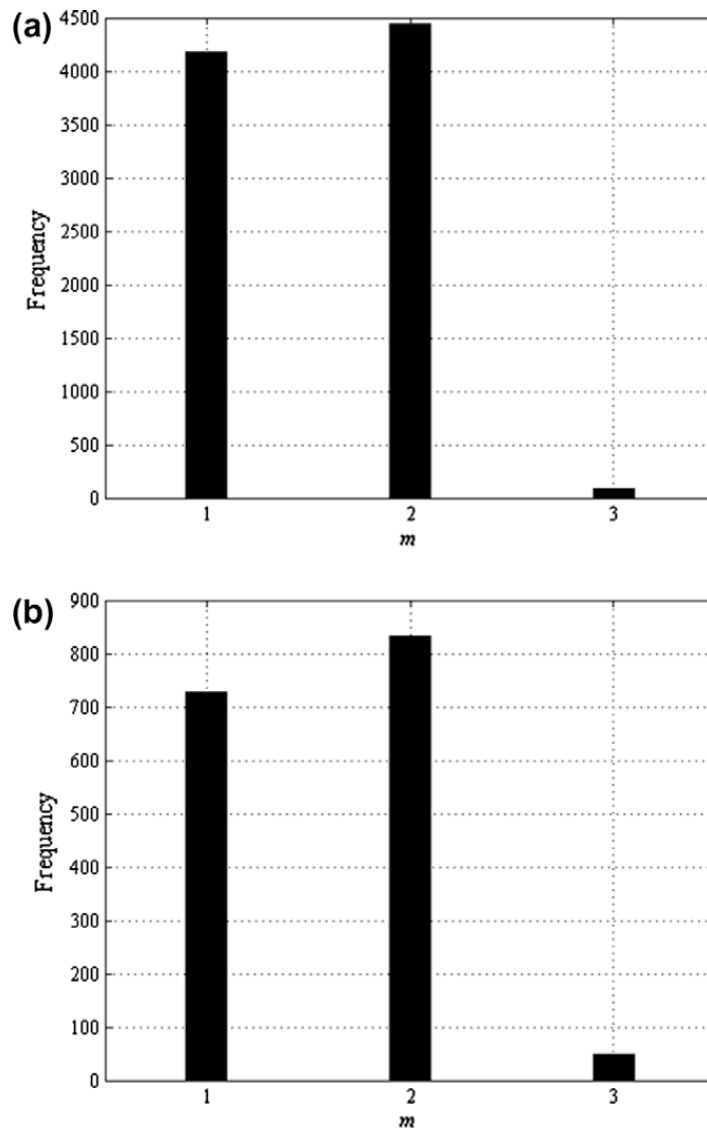


Fig. 10. Distributions of practical optimal orders of the differential coder in SWA + MDC for (a) Cow and (b) Face models.

a correlation coefficient existing in the range of $0.5 < \rho < 1$. The more the input samples are correlated (p th order AR process, $AR(p)$), the higher the efficiency of entropy coding is expected with higher order (p th order) differential coding.

To apply the MDC technique to practical 1-D temporal sequences, we use an iterative approach as follows.

- (1) calculate the variance of input signal σ_u^2 and let it be a reference variance such as $\sigma_{ref}^2 = \sigma_u^2$;
- (2) initialize the parameter m as 1;
- (3) perform the m th order differential operation via

$$\Delta^{(m)}u(n) = \begin{cases} u(n) - u(n-1) & \text{if } m = 1; \\ \Delta^{(m-1)}u(n) - \Delta^{(m-1)}u(n-1) & \text{if } m \geq 2; \end{cases}$$

- (4) calculate the variance of prediction errors through $\sigma_{\Delta^{(m)}u}^2 = 1/N \sum_{n=1}^N (\Delta^{(m)}u(n) - \mu_{\Delta^{(m)}u})^2$, where μ is mean value;
- (5) if $\sigma_{\Delta^{(m)}u}^2 < \sigma_{ref}^2$, increase $m(m = m + 1)$, replace σ_{ref}^2 with $\sigma_{\Delta^{(m)}u}^2$ and go back to (3);
- (6) or else stop and encode the $(m - 1)$ th order differential errors using the arithmetic coder.

We can automatically find the optimal order by using this iterative approach. Note that the entropy coding efficiency can be estimated using the variance of the prediction errors, as mentioned in Section 3. Fig. 5 shows the variances of prediction errors according to different orders of MDC. Here, the x -coordinate of the first base mesh vertex sequence,

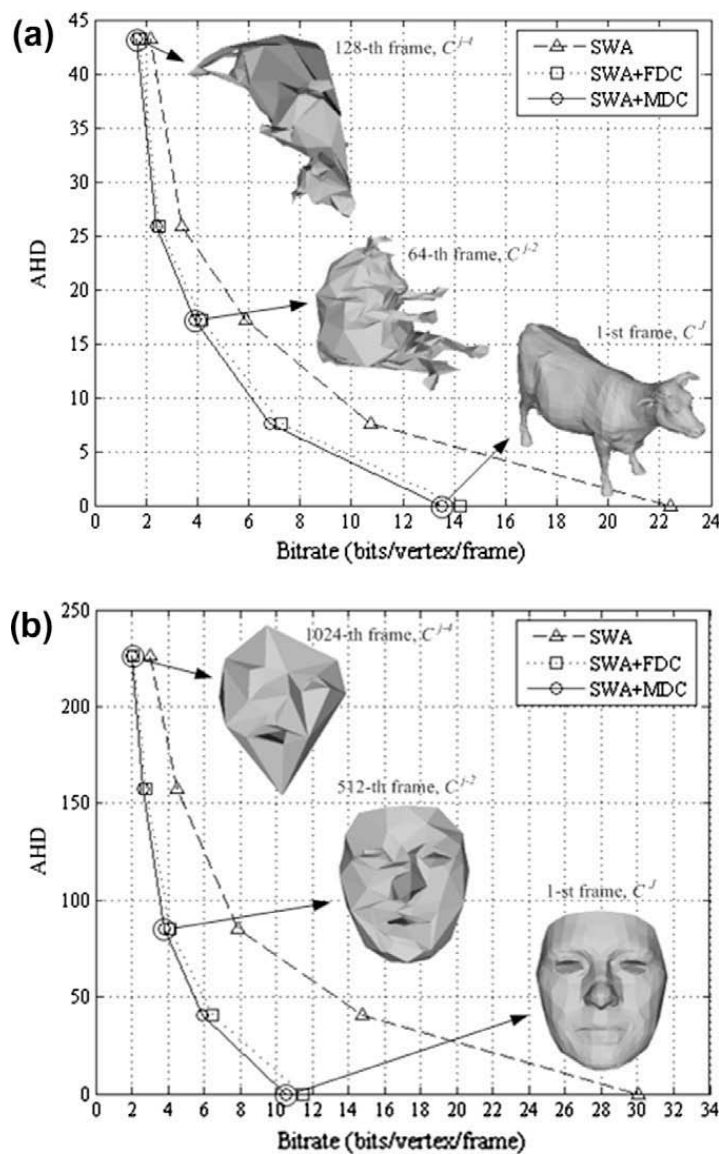


Fig. 11. R-D curves of SWA, SWA + FDC and SWA + MDC methods for (a) Cow and (b) Face models.

$x_1(n)$ ($1 \leq n \leq 1024$), of Face model is shown for a practical example. In this case, the second order is selected for MDC, as its corresponding variance is the smallest. Fig. 6 depicts the distributions of prediction error for the first order and the second order differential coding. From this figure, we can easily estimate that MDC is more efficient than FDC. In terms of computational complexity, the m th order differential coding requires $N_{(m-1)}$ additional add operation for each temporal sequence compared to the $(m - 1)$ th order differential coding, where $N_{(m-1)}$ is the number of samples from the $(m - 1)$ th order differential coding. Even though MDC costs slightly additional computational resources, this drawback could be overcome by applying MDC to multiple temporal sequences in parallel via multi-core processor.

In the third step, the optimal order of each sequence should be also transmitted as side-information whose bitrate is given by

$$\frac{\lceil \log_2 o_{max} \rceil \times 3}{N} \text{ (bits/vertex/frame)} \tag{14}$$

where o_{max} is the maximum optimal order. Note that the side-information is very negligible.

The entropy coded connectivity and geometry including the side-information are finally merged into the compressed bit-stream. Note that the transmitted bit-stream, in the decoder side, can be sequentially reconstructed from the coarsest signals to finer ones with various spatial resolutions. We call this approach SWA + MDC method.

4.2. The proposed compression method using SWA and TWA

Fig. 7 shows the encoding process of the proposed compression method using SWA and TWA techniques. As the first two steps of this encoding process are identical to those mentioned in Section 4.1, the peculiar steps of this scheme are described in detail. Note that the same notations with the previous section are used for the sake of simplicity.

Up to the second step, each frame of the original mesh sequence is transformed by the exact integer SWA [15], and the connectivity information for only the first frame is entropy coded by the arithmetic coder.

The third step performs the geometry coding to reduce the temporal redundancy. For geometry coding, the proposed method applies TWA to the temporal sequences of the base mesh vertices ($x_r(n)$, $y_r(n)$, and $z_r(n)$) and of the spatial wavelet coefficients ($x_s^j(n)$, $y_s^j(n)$, and $z_s^j(n)$). Similar to MDC, TWA can be also applied simultaneously to multiple temporal sequences in parallel via multi-core processor, since each temporal sequence can be independently transformed. Note that the efficiency of entropy coding depends on the performance of frequency decomposition according to temporal wavelet filter-banks. Many analysis and synthesis filter-banks have been developed [20]. We consider three well-known filter-banks such as Haar (2/2 tap) [23], Le Gall (5/3 tap) and Daubechies (9/7 tap) [24] filters. These filter-banks can be applied for lossless compression of 3-D mesh sequences by implementing them in integer lifting form, because the input sequences have integer coordinates. Fig. 8 shows the distributions of temporal wavelet coefficients. Here, the x -coordinate of the first base mesh vertex sequence, $x_1(n)$ ($1 \leq n \leq 1024$), of Face model is chosen for a practical example. From this figure, we can expect Le Gall and Daubechies filters to be more efficient than Haar filter. We experimentally evaluate the coding efficiency according to these filters in Section 5.2.

Table 2

The lossless compression results of SWA + TWA method compared with TWA scheme according to temporal wavelet decomposition levels and three temporal wavelet filter-banks: Haar (2/2 tap), Le Gall (5/3 tap) and Daubechies (9/7 tap) filters.

# of TWA levels	TWA filter	Model	Bitrate (bits/vertex/frame)	
			TWA	SWA + TWA
1	Haar	Cow	32.78	18.21
		Face	24.84	20.77
	Le Gall	Cow	30.19	16.39
		Face	22.05	19.01
	Daubechies	Cow	29.93	16.53
		Face	22.13	19.44
3	Haar	Cow	30.60	16.30
		Face	17.52	14.99
	Le Gall	Cow	27.16	13.92
		Face	13.86	12.58
	Daubechies	Cow	26.49	13.93
		Face	13.47	12.95
5	Haar	Cow	30.43	16.11
		Face	16.06	13.83
	Le Gall	Cow	27.08	13.79
		Face	12.52	11.54
	Daubechies	Cow	26.34	13.73
		Face	12.12	11.83

In the third step, the low and high frequency band signals obtained from both SWA and TWA are entropy coded by the arithmetic coder.

The entropy coded connectivity and geometry information are finally merged into the compressed bit-stream. Note that the transmitted bit-stream, in the decoder side, can be sequentially reconstructed from the coarsest signals to finer ones with various spatial and temporal resolutions, simultaneously. We call the approach, SWA + TWA method.

5. Simulation results

Simulations are carried out on two 3-D irregular triangle mesh sequences, Cow (with 204 frames and 2904 vertices/frame) and Face (with 10,002 frames and 539 vertices/frame). The number of vertices and their connectivity information are fixed over all frames. To apply TWA to mesh sequences, the number of frames should be an integer power of two. Therefore, we use only the first 128 and 1024 frames of Cow and Face, respectively. In the pre-processing step of SWA, each coordinate is uniformly quantized, coded to 12 bits, and used for the original like as in [6]. Fig. 9 shows several frames of the original mesh sequences as examples.

To measure the quality distortion between the original mesh sequence and decompressed one, we use Metro [25] which provides the Hausdorff Distance (HD) between two static surfaces modeled by triangular meshes. It first evaluates two one-sided distances, $e_n(\mathbf{V}_n, \mathbf{V}'_n)$ and $e_n(\mathbf{V}'_n, \mathbf{V}_n)$ (\mathbf{V}_n and \mathbf{V}'_n represent the original and decompressed surfaces of meshes at the n th frame, respectively). Note that there exist surfaces such that $e_n(\mathbf{V}_n, \mathbf{V}'_n) \neq e_n(\mathbf{V}'_n, \mathbf{V}_n)$. For that reason, the HD of n th mesh frame, $E_n(\mathbf{V}_n, \mathbf{V}'_n)$, is obtained by taking the maximum value of two one-sided distances:

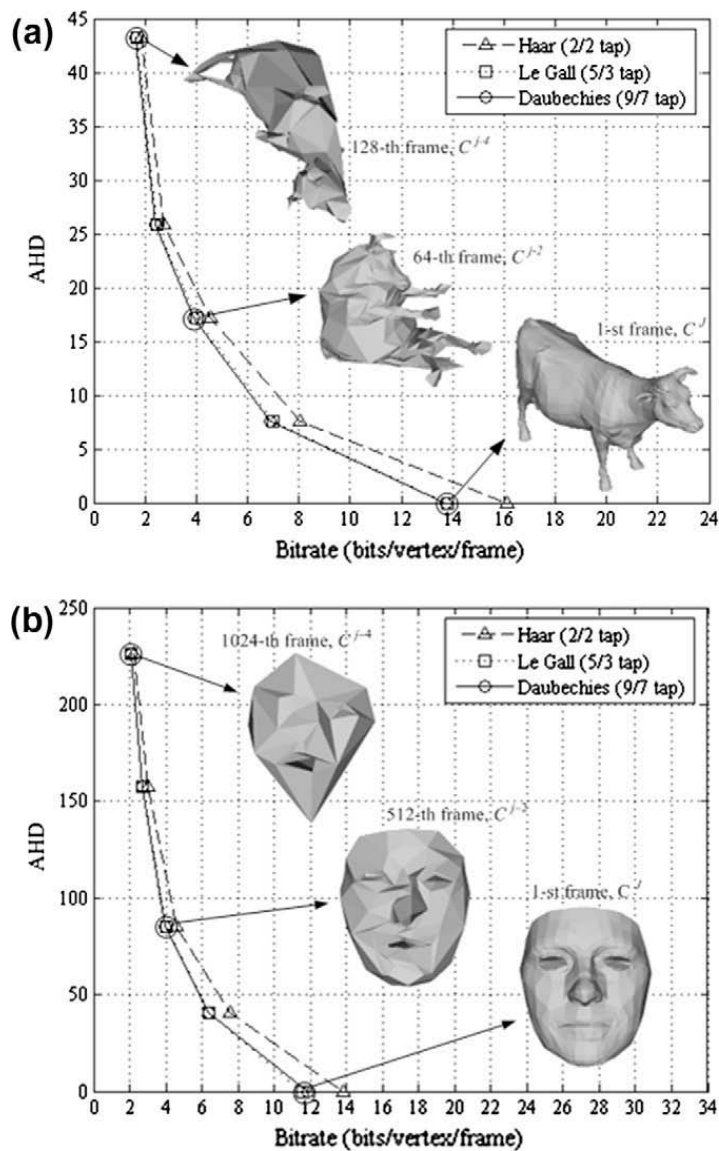


Fig. 12. R-D curves of SWA + TWA method at different spatial resolutions where three temporal wavelet filter-banks are used for (a) Cow and (b) Face models.

$$E_n(\mathbf{V}_n, \mathbf{V}'_n) = \max \{e_n(\mathbf{V}_n, \mathbf{V}'_n), e_n(\mathbf{V}'_n, \mathbf{V}_n)\} \tag{15}$$

The error metric is defined as the average of HD over all frames, called Average HD (AHD), $\bar{E}(\mathbf{V}_n, \mathbf{V}'_n)$:

$$\bar{E}(\mathbf{V}_n, \mathbf{V}'_n) = \frac{1}{N} \sum_{n=0}^{N-1} E_n(\mathbf{V}_n, \mathbf{V}'_n) \tag{16}$$

5.1. SWA + MDC method

To evaluate the coding efficiency of the proposed SWA + MDC method, we perform also two other methods, SWA method and SWA + FDC method. Here, the SWA method does not consider the temporal redundancy. Table 1 shows the lossless compression results. As shown in this table, both SWA + FDC and SWA + MDC methods achieve quite high compression performance comparing to SWA method, because they exploit the temporal coherence. SWA + MDC method is more efficient than SWA + FDC method. It shows that the first order of differential coder is not good enough to reduce the temporal redundancy. Fig. 10 shows the distribution of practical optimal orders. Actually, 56% and 55% of the temporal sequences obtained from SWA need second or third order differential coding in Cow and Face models, respectively. Although the proposed method requires side-information to transmit the optimal order of each sequence, the amount is so small as to be negligible. In this simulation, two bits per temporal sequence are assigned to transmit the order. Cow and Face models need 4.69×10^{-2} and 5.86×10^{-3} (bits/vertex/frame) for the side-information, respectively.

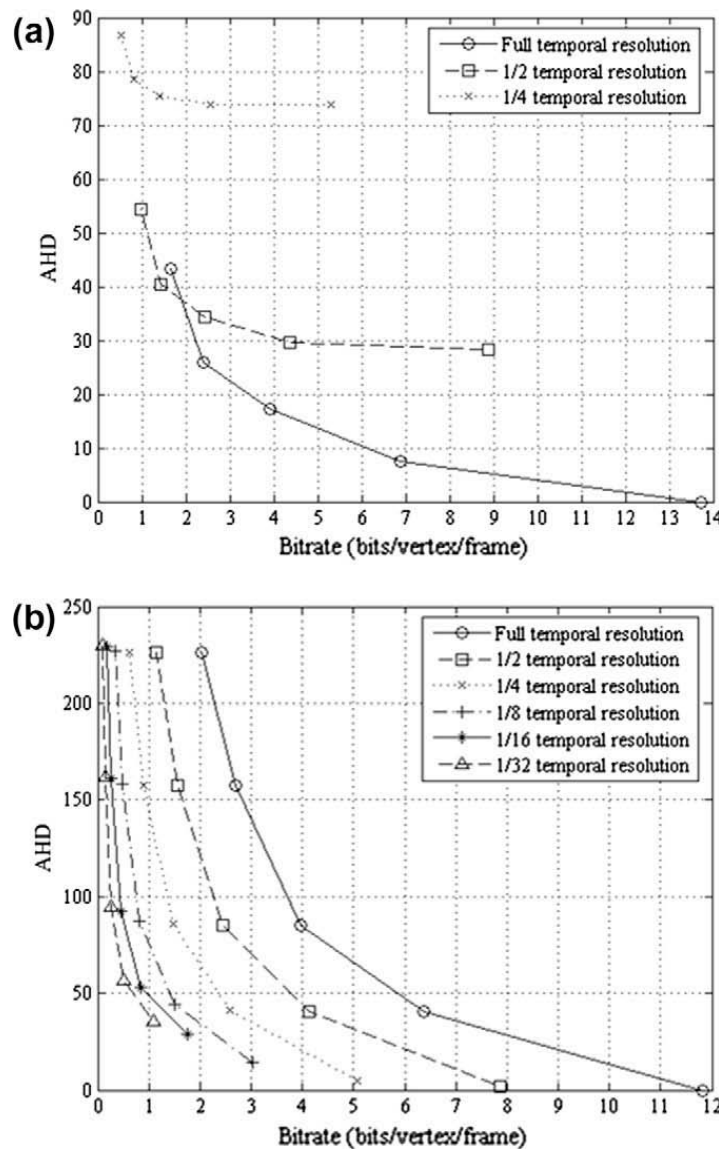


Fig. 13. R-D curves of SWA + TWA method at different spatio-temporal resolutions, where the temporal sequences are decomposed into five levels using Daubechies filter-banks for (a) Cow and (b) Face models.

The lossy-to-lossless compression performances according to various spatial resolutions are evaluated in terms of AHD and bitrates. Fig. 11 depicts the Rate-Distortion (R-D) curves of SWA, SWA + FDC and SWA + MDC methods. Cow and Face models are decomposed into 19 and 11 spatial levels. Here, we present the results of five highest resolution levels. Note that the worst case is the base mesh sequences. As shown in Fig. 11, the proposed method enables to reconstruct the mesh sequences at various spatial resolutions. Similar to lossless compression results, SWA + MDC method has better coding efficiency than the others.

5.2. SWA + TWA method

For the evaluation of SWA + TWA method, three different temporal wavelet filter-banks – Haar, Le Gall and Daubechies filters – are applied to the temporal sequences obtained from SWA. Each temporal sequence is decomposed into several sub-bands in the dyadic form using lifting scheme [26]. The lossless coding efficiency is evaluated according to different temporal wavelet decomposition levels as shown in Table 2. For comparison, we also present the compression results of the method which applies only TWA to original mesh sequences. We denote it TWA method. As shown in this table, TWA method has poor performance. It means that the spatial redundancy should also be exploited. In particular, Cow model have relatively high spatial redundancy. SWA + TWA method has lower bitrates than TWA method. The coding efficiency depends on the kind of temporal wavelet filter-banks. Le Gall and Daubechies filter-banks are more efficient than Haar filter-banks. The bitrate decreases as a function of the decomposition level. However the performance of SWA + TWA is slightly lower than that of SWA + MDC.

Lossy-to-lossless compression performances are evaluated by R-D curves. Fig. 12 shows the R-D curves according to different spatial resolutions and three temporal wavelet filter-banks. Here, temporal sequences are reconstructed in full temporal resolution. As shown in Fig. 12, Le Gall and Daubechies filters have similar coding efficiency and are more efficient than Haar filter at all spatial resolutions. Although SWA + TWA method has slightly lower performance than SWA + MDC, it provides both spatial and temporal scalability. Fig. 13 shows the R-D curves according to different spatio-temporal resolutions. Here, temporal sequences are decomposed into five levels using Daubechies filter-banks. This figure demonstrates that we can properly select the bitrates at various spatio-temporal resolutions. However, the bitrates should be carefully selected for the mesh sequences having large movement such as Cow model.

6. Conclusions

In this paper, we proposed two geometry compression methods for irregular three-dimensional (3-D) mesh sequences with constant connectivity. To reduce the spatial redundancy, both methods employ an exact integer spatial wavelet analysis (SWA). Temporal redundancy is reduced by multi-order differential coding (MDC) and temporal wavelet analysis (TWA), respectively, in two proposed methods. The method SWA + MDC offers spatial scalability and the method SWA + TWA provides spatio-temporal multi-resolution coding. In addition, both methods enable lossy-to-lossless compression in a single framework. The method SWA + MDC has slightly better performances than SWA + TWA method in both lossless and lossy compressions.

Acknowledgment

This research was supported by the Yeungnam University Research grants in 2008. In addition, the Cow and Face models were kindly offered by Matthias Muller and Demetri Terzopoulos, respectively.

Appendix A. In this Appendix A, we discuss the efficiency of the multi-order differential coding (MDC) according to the correlation coefficient of input signal.

Let an input data sequence $u(n)$ (for $1 \leq n \leq N$) be a wide sense stationary Markov process (WSSMP) with zero mean. The m th order differential error sequence $y^{(m)}(n)$ can be expressed by

$$\begin{aligned} y^{(1)}(n) &= u(n) - u(n-1) \\ y^{(2)}(n) &= y^{(1)}(n) - y^{(1)}(n-1) \\ &\vdots \\ y^{(m)}(n) &= y^{(m-1)}(n) - y^{(m-1)}(n-1) \end{aligned} \tag{A.1}$$

The impulse response of the differential coding system is defined as

$$h(n) = \delta(n) - \delta(n-1) \tag{A.2}$$

Using Eq. (A.1), the m th order differential error sequence can be rewritten as

$$y^{(m)}(n) = h(n) * y^{(m-1)}(n) \tag{A.3}$$

where, * denotes the convolution operator.

The auto-correlation function of Eq. (A.3) can be calculated as

$$\varphi_{y^{(m)}}(k) = \varphi_h(k) * \varphi_{y^{(m-1)}}(k) \tag{A.4}$$

where,

$$\varphi_h(k) = h(k) * h(-k) \quad \text{and} \quad \varphi_{y^{(m-1)}}(k) = E[y^{(m-1)}(k)y^{(m-1)}(n+k)] \tag{A.5}$$

where, $E[\cdot]$ is expectation. According to Eqs. (A.2) and (A.5) is given by

$$\varphi_h(k) = -\delta(k-1) + 2\delta(k) - \delta(k+1) \tag{A.6}$$

Considering the first order differential coders, from Eq. (A.4)

$$\varphi_{y^{(1)}}(k) = -\varphi_u(k-1) + 2\varphi_u(k) - \varphi_u(k+1) \tag{A.7}$$

From Eq. (A.7):

$$\varphi_{y^{(1)}}(0) = 2(\varphi_u(0) - \varphi_u(1)) \tag{A.8}$$

where $\varphi_u(0) = \sigma_u^2$ and if we write the auto-correlation function of the input data sequence as $\varphi_u(k) = \sigma_u^2\psi(k)$, ($\psi(k) < 1, \forall k$), $\varphi_u(1) = \sigma_u^2\psi(1)$. Clearly, from Eq. (A.8), the output variance of the first order differential coder is

$$\sigma_{y^{(1)}}^2 = \varphi_{y^{(1)}}(0) = 2\sigma_u^2(1 - \psi(1)) \tag{A.9}$$

If the input sequence $u(n)$ is a first order stationary Markov process, $\varphi_u(k) = E[u(n)u(n+k)] = \sigma_u^2\psi(k)$ with $\psi(k) = \rho^{|k|}$. Here, ρ ($|\rho| \leq 1$) is the correlation coefficient. Therefore, Eq. (A.9) can be rewritten as follow:

$$\sigma_{y^{(1)}}^2 = \varphi_{y^{(1)}}(0) = 2\sigma_u^2(1 - \rho) \tag{A.10}$$

From Eq. (A.10), $\sigma_{y^{(1)}}^2 < \sigma_u^2$, if and only if $0.5 < \rho \leq 1$. Note that the WSSMP is a first order autoregressive (AR(1)) process of the form:

$$u(n) = \rho u(n-1) + \sqrt{1-\rho^2}w(n) \tag{A.11}$$

where $w(n)$ is a white noise with zero mean and variance σ_w^2 .

Clearly, for an AR(1) process, the optimal predictor is:

$$y_{opt}^1(n) = u(n) - \rho u(n-1) \tag{A.12}$$

Then, its output variance is

$$\sigma_{y_{opt}^1}^2 = \sigma_u^2(1 - \rho)^2 \tag{A.13}$$

It follows that the output variance is more reduced for a predictor having larger correlation coefficient. However, the (optimal) predictor requires both computational costs to calculate the correlation factor and side-information to be transmitted.

For an AR(1) process, it is easy to prove that the output variance of the MDC is reduced if

$$0.7752 < \rho \leq 1 \text{ for the second order}$$

$$0.9199 < \rho \leq 1 \text{ for the third order}$$

$$0.9740 < \rho \leq 1 \text{ for the fourth order}$$

...

Considering that the input sequence is modeled by a p th order AR process (AR(p)), the optimal differential coder has the order p :

$$y_{opt}^1(n) = u(n) - \sum_{k=1}^p a_k u(n-k) \tag{A.14}$$

Clearly, the proposed m th order MDC technique is a good alternative. In addition, the method requires just to evaluate the optimal order of the differential coder in terms of output variance and to transmit it.

References

- [1] Q. Du, M. Gunzburger, Grid generation and optimization based on centroidal Voronoi tessellations, Applied Mathematics and Computation 133 (2002) 591–607.
- [2] W.M. Chan, J.L. Steger, Enhancements of a three-dimensional hyperbolic grid generation scheme, Applied Mathematics and Computation 51 (1992) 181–205.
- [3] J.E. Lengyel, Compression of time-dependent geometry, in: Proceeding of SI3D '99, ACM Press, New York, NY, USA, pp. 89–95.

- [4] J. Zhang, C.B. Owen, Hybrid coding for animated polygonal meshes: combining delta and octree, in: *Proceedings of the International Conference on Information Technology: Coding and Computing*, vol. 1, 2005, pp. 68–73.
- [5] M. Alexa, W. Müller, Representing animations by principal components, in: *Proceedings of Computer Graphics Forum*, vol. 19, 2000.
- [6] Z. Karni, C. Gotsman, Compression of soft-body animation sequences, *Computer and Graphics* 28 (1) (2004) 25–34.
- [7] L. Ibarria, J. Rossignac, Dynapack: space-time compression of the 3D animations of triangle meshes with fixed connectivity, in: *SCA '03*, 2003, pp.126–135.
- [8] I. Guskov, A. Khodakovsky, Wavelet compression of parametrically coherent mesh sequences, in: *Proceedings of SCA '04*, 2004, pp. 183–192.
- [9] J. Cho, M. Kim, S. Valette, H. Jung, R. Prost, 3-D dynamic mesh compression using wavelet-based multi-resolution analysis, in: *Proceedings of ICIP 2006*, 2006, pp. 529–532.
- [10] F. Payan, M. Antonini, Wavelet-based compression of 3D mesh sequences, in: *Proceedings of IEEE ACIDCA-ICMI'2005*, Tozeur, Tunisia, 2005.
- [11] H. Sun, X. Chen, T. Chiang, Digital video transcoding for transmission and storage, No. 0-8493-1694-4, CRC Press, 2005, pp. 54–57 (Chapter 2.4.1.2).
- [12] S. Valette, R. Prost, Wavelet-based multi-resolution analysis of irregular surface meshes, *IEEE Transactions on Visualization and Computer Graphics* 10 (2) (2004) 113–122.
- [13] M. Lounsbery, Multiresolution analysis for surfaces of arbitrary topological type, Ph.D. Thesis, Dept. of Computer Science and Engineering, University of Washington (1994).
- [14] T. Lin, S. Xu, Q. Shi, P. Hao, An algebraic construction of orthonormal M-band wavelets with perfect reconstruction, *Applied Mathematics and Computation* 172 (2006) 717–730.
- [15] S. Valette, R. Prost, Wavelet-based progressive compression scheme for triangle meshes: wavemesh, *IEEE Transactions on Visualization and Computer Graphics* 10 (2) (2004) 123–129.
- [16] H. Jung, R. Prost, Lossless sub-band coding system based on rounding transform, *IEEE Transactions on Signal Processing* 46 (9) (1998) 2535–2540.
- [17] R. Calderbank, I. Daubechies, W. Sweldens, B.-L. Yeo, Wavelet transforms that map integers-to-integers, *Applied and Computational Harmonic Analysis* 5 (3) (1998) 332–369.
- [18] S. Valette, Deformable 2D mesh models and wavelet-based multi-resolution for 3D surfaces, Ph.D. Thesis, Dept. of Electrical Engineering, National Institute for Applied Sciences of Lyon, INSA-Lyon, France, 2002.
- [19] W. Sweldens, The lifting scheme: a custom-design construction of bi-orthogonal wavelets, *Applied and Computational Harmonic Analysis* 3 (2) (1996) 186–200.
- [20] M.D. Adams, F. Kossentini, Reversible integer-to-integer wavelet transforms for image compression: performance evaluation and analysis, *IEEE Transaction on Image Processing* 9 (6) (2000) 1010–1024.
- [21] A. Papoulis, *Probability, Random Variables and Stochastic Process*, McGraw-Hill, 1965. Chapter 15.3, pp. 558–569.
- [22] M. Schindler, A fast renormalisation for arithmetic coding, in: *Proceedings of Data Compression Conference*, 1998, p. 572.
- [23] S. Hongli, C. Yuanli, Q. Zulian, On design of multiwavelet prefilters, *Applied Mathematics and Computation* 172 (2006) 1175–1187.
- [24] K. Maleknejad, M. Yousefi, K. Nouri, Computational methods for integrals involving functions and Daubechies wavelets, *Applied Mathematics and Computation* 189 (2007) 1828–1840.
- [25] P. Cignoni, C. Rocchini, R. Scopigno, Metro: measuring error on simplified surfaces, *Computer Graphic Forum* 17 (2) (1998) 167–174.
- [26] I. Daubechies, W. Sweldens, Factoring wavelet transforms into lifting steps, *Journal of Fourier Analysis and Applications* 4 (3) (1998) 247–269.