Predictive control for sector bounded nonlinear model and its application to solid oxide fuel cell systems

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\textbf{ABSTRACT}

In this paper, a nonlinear model predictive control method is presented for solid oxide fuel cell systems. For realistic modeling, a sector bounded nonlinear model with input constraint is considered. As a performance index, we consider one horizon cost function that is represented by the weighted sum of state, control input and nonlinear function. By minimizing the upper bound of the cost function, the optimized control input sequences are obtained. For cost monotonicity, a terminal inequality condition is derived in terms of a finite number of linear matrix inequalities (LMIs) by using augmented vector feedback law which consists of state and sector bounded nonlinear function. In order to show the effectiveness of the proposed method, the sector bounded nonlinear model is derived for the solid oxide fuel cell systems and the system performance is verified by applying the proposed MPC algorithm to the systems.

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1. Introduction

Solid oxide fuel cell (SOFC) has attracted considerable interest as it offers wide application ranges, flexibility in the choice of fuel, high system efficiency and rapid load following capability [1–4]. In this regard, many researches have been investigated for simulating transient behavior as well as controller design for the SOFC dynamic model. In [1,12], a simple mathematical model and detailed model with temperature dynamics for SOFC stack have been developed to establish controller over the fuel cell voltage. However, these mathematical models have a difficulty on the controller design due to its heavily nonlinear behavior and dependence on disturbances such as load current and inlet temperatures. Thus, numerical modeling techniques based on the experimental input–output data such as support vector machine, T–S fuzzy, and neural network have been proposed in [5–7,13,14]. In [23], a model predictive control method for SOFC systems was presented by use of a simplified dynamic model which is obtained by subspace identification method. Recently, the MPC technique has been applied to the SOFC systems due to its slow dynamics and tight operating constraints [8–14,23].

It should be pointed out that model predictive control (MPC) scheme is very useful since it is possible to handle input constraints and to use the current measurement state at optimization algorithms. But if the model is not accurate, the control technique does not guarantee the stability and performance [21]. Thus, the mathematical model based predictive control method is required to overcome the disadvantage of input–output data based MPC method for SOFC systems.

In this paper, we consider a sector bounded nonlinear model to handle SOFC systems and propose a one horizon robust MPC method for the systems. The sector bounded nonlinear systems, which have a feedback connection with a linear...
dynamical system and nonlinearity satisfying certain sector type constraints, have been extensively studied in control theory research area\cite{15–19}. Since the pioneer work of Lur’ë was presented in 1940’, the notion of absolute stability has played an important role in stability analysis and controller design problems\cite{20}. But few results have been published concerning controller synthesis for systems with sector nonlinearities, see e.g.\cite{22–25}. Especially, previous design methods did not consider nonlinear feedback for sector nonlinearity. To the best of authors’ knowledge, there are no approaches considering state as well as nonlinear function feedback.

The proposed nonlinear MPC method minimizes an upper bound of one horizon cost function subject to the terminal inequality. The solution to this nonlinear MPC, which is solved repeatedly at each sampling times, minimizes an upper bound on the considered finite horizon cost function and the control input stabilizes the systems satisfying the sector condition. In order to handle the sector nonlinearity, we adopt convex representation of sector bounds of the nonlinear function. To guarantee closed-loop stability of the nonlinear MPC, a stabilization criterion is expressed in terms of LMIs which can be solved very efficiently by various convex optimization algorithms\cite{26}. Finally, we demonstrate the effectiveness of the proposed approach via numerical simulations.

2. Problem statement

Consider the following discrete-time sector bounded Lur’ë systems

\[
\begin{align*}
    x(k+1) &= Ax(k) + Ff(q(x(k))) + Bu(k), \\
    q(k) &= Cx(k)
\end{align*}
\]  

(1)

with input constraints

\[-\bar{u} \leq u(k) \leq \bar{u}, \text{ for all } k \in [0, \infty),\]

(2)

where \( x \in \mathbb{R}^n \) is the state vector, \( u \in \mathbb{R}^m \) is the control input, \( A \in \mathbb{R}^{n \times n}, F \in \mathbb{R}^{n \times p}, B \in \mathbb{R}^{n \times m}, C \in \mathbb{R}^{p \times n} \) are constant matrices and \( f(q(k)) \in \mathbb{R}^n \) is memoryless time-invariant nonlinearity with sector restriction such as

\[
\begin{align*}
    a_i &\leq \frac{\partial f(q) \partial q}{q} \leq b_i, \\
\end{align*}
\]

in which \( a_i, b_i \) are lower/upper sector bounds, respectively. For simplicity, let us define

\[
\delta^i(k) = \frac{f(q_i(k))}{q_i(k)},
\]

(4)

then

\[f(q(k)) = \Delta(k)Cx(k),\]

(5)

where \( \Delta(k) = \text{diag}\{\delta^1(k), \ldots, \delta^p(k)\} \).

In order to describe the convexity of the nonlinearity, let us define

\[
\begin{align*}
    \Delta_1 &\triangleq \text{diag}\{a_1, \ldots, a_p\}, & \Delta_2 &\triangleq \text{diag}\{b_1, \ldots, b_p\}
\end{align*}
\]

(6)

then function \( \Delta(k) \) can be represented by:

\[
\Delta(k) \in \text{Co}\{\Delta_1, \Delta_2\},
\]

(7)

where Co denotes convex hull.

Here, the goal of this paper is to design a stabilizing control \( u(k) \) for (1) by the model predictive control strategy. To find such a control, we consider the following performance index

\[
J(k, k+1) \equiv x(k|k)^T Ax(k|k) + u(k|k)^T \mathcal{A} u(k|k) + V(k+1|k)
\]

(8)

where \( V(k+1|k) \) is a terminal cost and \( \mathcal{A} > 0, \mathcal{A} > 0 \). In order to design a control law for the future sampling time, we consider one step predictive controller with the following structure at each time \( k \),

\[
\begin{align*}
    u(k|k) &= u^*(k|k), \\
    u(k+1|k) &= K_1(k)x(k+1|k) + K_2(k)f(q(x(k+1|k))) \\
    &= K(k)x(k+1|k),
\end{align*}
\]

(9)

where \( u^*(k|k) \) is the control input variable that minimizes the performance index (8), \( x_0(k) = \begin{bmatrix} x(k) \\ f(q(k)) \end{bmatrix} \) and \( K(k) = \begin{bmatrix} K_1(k) \\ K_2(k) \end{bmatrix} \).

For cost monotonicity, the following inequality condition should be satisfied as

\[
V(k+j+1|k) - V(k+j|k) < -[x(k+j|k)^T Ax(k+j|k) + u(k+j|k)^T \mathcal{A} u(k+j|k)], \text{ for } j \geq 1
\]

(10)
where
\[ V(k|k) = x_a(k|k)^T P(k) x_a(k|k) > 0. \] (11)

By summing (10) from \( j = 1 \) to \( j = \infty \), we obtain:
\[ -V(k|k) \leq -J(k, k + 1). \] (12)

For simplicity, we define
\[ J'(k) = \text{Minimize} \quad \max_{h_{k+j} > f} J(k, k + 1) \]
subject to (10).

Thus, our goal is redefined to find a nonlinear MPC for minimization of (13) that minimizes an upper bound on the worst case of the cost function \( J(k, k + 1) \).

Before deriving our main results, let define the following matrices for the sake of simplicity on matrix representation:
\[
A = \begin{bmatrix} A & F \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} B \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ 1 \end{bmatrix},
\]
\[
C(k) = CA_k + BK(k),
\]
\[
W(k) = \begin{bmatrix} 2 + K_1(k)^T \otimes K_1(k) & 0 \\ 0 & K_2(k)^T \otimes K_2(k) \end{bmatrix},
\]
\[
Q = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}, \quad A_F = \begin{bmatrix} A & F \end{bmatrix}, \quad Q(k) = P^{-1}(k).
\]

3. Main result

In this section, we introduce one horizon predictive control method for the sector bounded nonlinear model and derive a stability condition of the MPC with the feedback control law (9).

**Theorem 1.** Consider the system (1) at sampling time \( k \). Then the terminal inequality Eq. (10) with the feedback control law (9) is satisfied for \( j \geq 1 \), if there exists \( G(k), L(k), M(k) \) and \( Q(k) > 0 \) subject to

\[
\begin{bmatrix}
-G(k) - G(k)^T + Q(k) & * & * & * \\
\Delta C(A_kG(k) + BL(k)) & -2M(k) & * & * \\
AG(k) + BL(k) & FM(k) & -Q(k) & * \\
Q^2G(k) & 0 & 0 & -I \\
\otimes L(k) & 0 & 0 & 0 & -I
\end{bmatrix} < 0, \quad i = 1, 2,
\] (15)

\[
\begin{bmatrix}
\bar{u}_i^2 \\
L_i(k) \\
L_i^2(k) \\
G(k) + G^T(k) - Q(k)
\end{bmatrix} \geq 0, \quad l = 1, \ldots, m,
\] (16)

where \( L(k) = K(k)G(k) \) and \( L_i(k), \bar{u}_i \) are \( l \)-th row of \( L(k), \bar{u}_i \), respectively.

**Proof.** For \( j \geq 1 \), the inequality (10) with control input \( u(k+j|k) = K(k)x_a(k+j|k) \) is satisfied if and only if
\[
x_a(k+j+1|k)^T P(k) x_a(k+j+1|k) - x_a(k+j|k)^T P(k) x_a(k+j|k) \\
< -x_a(k+j|k)^T 2x(k+j|k) + x_a(k+j|k)^T K(k)^T \otimes K(k) x_a(k+j|k).
\] (17)

Let us define
\[
x_f(k+j|k) = \begin{bmatrix} x_a(k+j|k) \\ f(q(k+j+1|k)) \end{bmatrix}.
\] (18)

Then, the inequality (17) can be written
\[
x_f^T(k+j|k) \begin{bmatrix}
A^T + K(k)^T B^T \\
0
\end{bmatrix} P(k) \begin{bmatrix} A & BK(k) \\ E^T \end{bmatrix} x_f(k+j|k) - x_f^T(k+j|k) P(k) x_a(k+j|k) \]
\[
< -x_f^T(k+j|k) Q + K(k)^T \otimes K(k) x_a(k+j|k)
\] (19)

where \( A, B, \) and \( E \) are defined in Eq. (14).
The following equality holds for all nonzero matrix $N(k)$

$$f^T(q(k + j + 1|k))N(k)j^Tf(q(k + j + 1|k)) = f^T(q(k + j + 1|k))N(k)\Delta(k)(CAx_a(k + j|k) + CKx_i(k + j|k)).$$

Combining the inequality (19) with equality constraints (20) by use of the augmented vector given in Eq. (18), the following inequality is obtained

$$\alpha x_j(k) > 0.$$  

(21)

where

$$II = \begin{bmatrix} (A + BK(k))^TP(k)(A + BK(k)) - P(k) + W(k) & * & * \\ N(k)\Delta(k)C(k) & -2N(k) & * \\ A + BK(k) & E & -Q(k) \end{bmatrix}. $$

By Schur complement [26], inequality (21) is satisfied if and only if

$$\begin{bmatrix} -P(k) + W(k) & * & * \\ N(k)\Delta(k)C(k) & -2N(k) & * \\ A + BK(k) & E & -Q(k) \end{bmatrix} < 0.$$ 

(23)

By use of an auxiliary full block matrix $G(k)$ and the fact that the matrix $(G(k)^T - Q(k))Q^{-1}(k)(G(k) - Q(k))$ is nonnegative definite, we can get the following inequality

$$G(k)^TQ^{-1}(k)G(k) - G(k) - G(k)^T + Q(k) > 0.$$  

(24)

By multiplying the matrix $\text{diag}(G(k)^T, I, I)$ and $\text{diag}(G(k), I, I)$ in the left and right side of Eq. (23), respectively, the following inequality is obtained with Eq. (24),

$$\begin{bmatrix} -G(k) - G(k)^T + Q(k) + G(k)^T W(k)G(k) & * & * \\ N(k)\Delta C(k)G(k) & -2N(k) & * \\ AG(k) + BL(k) & E & -Q(k) \end{bmatrix} < 0.$$ 

(25)

After performing the congruence transformation with

$$\text{diag}(I, M(k), I, I), \quad M(k) = N(k)^{-1}$$

in the LMI (25), the equivalent condition (15) is obtained by Schur complement.

For the invariant set

$$\zeta(P(k)) = \{x|x_a(k + j|k)^TP(k)x_a(k + j|k) < 1, j \geq 1\} = \left\{x|\begin{bmatrix} x(k + j|k) \\ f(q(k + j|k)) \end{bmatrix}^T Q^{-1}(k) \begin{bmatrix} x(k + j|k) \\ f(q(k + j|k)) \end{bmatrix} < 1, j \geq 1\right\}$$.  

(27)

the input constraint $|u(k + j|k)| = |K_i(k)x_a(k + j|k)| < \bar{u}, j \geq 1, i = 1, \ldots, m$, hold that

$$\max_{j \geq 1} |u(k + j|k)|^2 = \max_{j \geq 1} |K_i(k)x_a(k + j|k)|^2 = \max_{j \geq 1} |L(k)G(k)^{-1}x_a(k + j|k)|^2$$

$$= \max_{j \geq 1} \left(|L_1(k) - L_2(k)G(k)^{-1}x_a(k + j|k)|\right)^2 \leq ||(L(k)G^{-1}(k)Q^{-1}(k)G(k)^{-T}L(k))||^2.$$  

(28)

Thus Eq. (28) is equivalent to the Eq. (16) by Schur complement, then it is satisfied that $|u(k + j|k)| < \bar{u}, j \geq 1, i = 1, 2, \ldots, m$. This completes the proof. $\Box$

For $j = 0$, minimization of $J(k, k + 1)$ is equivalent to

$$\begin{array}{ll}
\text{Minimize} & \gamma(k) \\
\text{subject to} & x(k|k)^T A x(k|k) + u(k|k)^T B u(k|k) + \begin{bmatrix} x(k + 1|k) \\ f(q(k + 1|k)) \end{bmatrix}^T P(k) \begin{bmatrix} x(k + 1|k) \\ f(q(k + 1|k)) \end{bmatrix} \leq \gamma(k). \end{array}$$

(29)

By Schur complement, the optimization problem above is equivalent to

$$\begin{array}{ll}
\text{Minimize} & \gamma(k) \\
\text{subject to} & \begin{bmatrix} \gamma(k) \\ \Sigma(k) & Q(k) \\ 0 & I \\ 0 & 0 & I \end{bmatrix} \geq 0 \end{array}$$

(30)

subject to

$$\begin{bmatrix} \gamma(k) \\ \Sigma(k) & Q(k) \\ 0 & I \\ 0 & 0 & I \end{bmatrix} \geq 0 \end{array}$$

(31)

subject to

$$\begin{bmatrix} \gamma(k) \\ \Sigma(k) & Q(k) \\ 0 & I \\ 0 & 0 & I \end{bmatrix} \geq 0 \end{array}$$

(32)
where

$$\Sigma(k) = \begin{bmatrix} Ax(k|k) + Bu(k|k) + Ff(q(k)) \\ \Delta C(Ax(k|k) + Bu(k|k) + Ff(q(k))) \end{bmatrix}, \quad i = 1, 2.$$  

**Remark 1.** Based on Theorem 1, the proposed nonlinear MPC method can be summarized in the following MPC algorithm.

- **Step 0**: Set $k = 0$.
- **Step 1**: Solve the following optimization problem using the measurement $x(k|k)$ and compute $u(k)$.

$$\text{Minimize} \\
\gamma(k)$$

subject to (15), (16) and (32).
- **Step 2**: Apply $u(k)$ to the plant.
- **Step 3**: $k = k + 1$ and go to Step 1.

**Remark 2.** If there exist a feasible solution of the problem in Theorem 1 at time $k = 0$, then the system (1) with the MPC is robustly asymptotically stable. Since $J'(k)$ is greater than or equal to zero and strictly decreases as time goes to infinity, it plays a role of a Lyapunov function. Therefore we conclude that the closed-loop system is asymptotically stable. Also, the optimal solution of the optimization problem at the time $k$ can be the solution at the time $k + 1$, the feasible solution of the optimization problem at time $k$ is also feasible at the next time $k + 1$.

**Remark 3.** When we assume that the nonlinear function is not available in Theorem 1, the control gain $K_2(k)$ becomes zero. Then, to find a feasible solution using the convex optimization technique, the auxiliary matrix $G(k)$ should have non-full block structure, like $G(k) = \begin{bmatrix} X(k) & 0 \\ Y(k) & Z(k) \end{bmatrix}$. Because of the proposed augmented Lyapunov function and nonlinear function feedback control law, the proposed method has a less conservative condition for cost monotonicity. Via comparison with the method without the nonlinear feedback, the effectiveness of the proposed method will be shown in a numerical example.

### 4. Application to SOFC system

In order to show the effectiveness of the proposed method, let us consider the dynamics of an SOFC system [8] and assume that all states are available. As a widely adopted dynamic model of the SOFC system, the average voltage magnitude of the fuel cell stack is determined by the partial pressure of hydrogen, oxide and water. Table 1 contains the parameters of the SOFC model. Applying Nernst’s equation, the output of the SOFC can be modeled as follows:

$$V_0 = N_0 \left[ E_0 + \frac{R_0 T_0}{2F_0} \ln \frac{p_{H_2}(p_{O_2}/101.325)^{1/2}}{p_{H_2}O_2} \right].$$  

where $p_{H_2}, p_{O_2}, p_{H_2}O_2$ are partial pressures of hydrogen, oxygen and water.

Also, the dynamics of fuel processor and the stack voltage are

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Representation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_0$</td>
<td>1273</td>
<td>K</td>
<td>Absolute temperature</td>
</tr>
<tr>
<td>$F_0$</td>
<td>96.485</td>
<td>mol^-1</td>
<td>Faraday’s constant</td>
</tr>
<tr>
<td>$k_0$</td>
<td>8.314</td>
<td>J mol^{-1}K^{-1}</td>
<td>Universal gas constant</td>
</tr>
<tr>
<td>$E_0$</td>
<td>0.9378</td>
<td>V</td>
<td>Ideal standard potential</td>
</tr>
<tr>
<td>$N_0$</td>
<td>384</td>
<td></td>
<td>Number of cells</td>
</tr>
<tr>
<td>$K_r$</td>
<td>$9.95 \times 10^{-3}$</td>
<td>mol^{-1}A^{-1}</td>
<td>Valve molar constant for hydrogen</td>
</tr>
<tr>
<td>$K_d$</td>
<td>$8.32 \times 10^{-3}$</td>
<td>mol^{-1}KPa^{-1}</td>
<td>Valve molar constant for water</td>
</tr>
<tr>
<td>$K_i, O_2$</td>
<td>$2.77 \times 10^{-3}$</td>
<td>mol^{-1}KPa^{-1}</td>
<td>Valve molar constant for oxygen</td>
</tr>
<tr>
<td>$r_{H_2}$</td>
<td>26.1</td>
<td>s</td>
<td>Response time of hydrogen flow</td>
</tr>
<tr>
<td>$r_d$</td>
<td>2.91</td>
<td>s</td>
<td>Response time of Oxygen flow</td>
</tr>
<tr>
<td>$r_i$</td>
<td>1.145</td>
<td></td>
<td>Ratio of hydrogen to oxygen</td>
</tr>
<tr>
<td>$T$</td>
<td>5</td>
<td>s</td>
<td>Time constant of the fuel processor</td>
</tr>
<tr>
<td>$C$</td>
<td>10</td>
<td>F</td>
<td>Ohmic loss</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Unit</th>
<th>Representation</th>
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</thead>
<tbody>
<tr>
<td>$C_{10}$</td>
<td>101.325</td>
<td>J mol</td>
<td>Parallel Capacitance</td>
</tr>
</tbody>
</table>
where $q_F$ is the hydrogen flow rate, $q_T$ is the inlet fuel flow rate, $V_s$ is stack voltage and $I_D$ is external current load that is assumed to be not changed.

The dynamic equation of the partial pressure inside the channel of hydrogen, oxygen and water are as follows:

$$\dot{p}_{H_2} = \frac{1}{\tau_{H_2}}(q_H - 2K_I I_s - p_{H_2} K_{H_2}),$$

$$\dot{p}_O = \frac{1}{\tau_{O_2}}\left(\frac{q_H - K_I I_s - p_{O_2} K_{O_2}}{\tau_{H-O}}\right),$$

$$\dot{p}_{H_2O} = \frac{1}{\tau_{H_2O}}(2K_I I_s - p_{H_2O} K_{H_2O}),$$

where $I_s = (V_o - V_s)/r$ is stack current.

In order to transform states around its equilibrium point, let us define

$$x_1 = p_{H_2} K_{H_2} - p_{H_2} K_{H_2},$$

$$x_2 = p_{O_2} K_{O_2} - p_{O_2} K_{O_2},$$

$$x_3 = q_H - q_T,$$

$$x_4 = V_s - V_o,$$

$$u = q_T - q_T,$$

$I_D = I_D$, $p_{H_2O} = p_{H_2O}$

are operating points of each states. It should be noted that the pressure of water $p_{H_2O}$ has constant value because external current load $I_D$ is assumed to be not changed in this paper.

Then, the variation of the stack current is expressed as follows

$$I_s - I'_s = \frac{1}{r}(V_o - V'_o - (V_s - V'_s)) = \frac{1}{r}\left[\frac{N_0 R_0 T_0}{2F_0} \left(\ln \left(\frac{p_{H_2}}{p_{H_2}} \right) + \frac{1}{2} \ln \left(\frac{p_{O_2}}{p_{O_2}} \right)\right) - (V_s - V'_s)\right].$$

Therefore, the transformed state space model is described as

$$\dot{x}(t) = A_c x(t) + F_c g(x(t)) + B_c u(t),$$

where $x(t) = [x_1 x_2 x_3 x_4]^T \in \mathbb{R}^4$ is the states, $u(t)$ is the fuel flow rate,

$$A_c = \begin{bmatrix} -\frac{1}{\tau_{H_2}} & 0 & \frac{1}{\tau_{O_2}} & \frac{2K_I}{\tau_{H_2O}} \\ 0 & -\frac{1}{\tau_{O_2}} & 0 & \frac{K_{O_2}}{\tau_{O_2}} \\ 0 & 0 & -\frac{1}{\tau_{H-O}} & 0 \\ 0 & 0 & 0 & -\frac{1}{\tau_I} \end{bmatrix}, \quad B_c = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \quad F_c = \begin{bmatrix} -\frac{2K_I}{\tau_{H_2O}} \\ \frac{K_{O_2}}{\tau_{O_2}} \\ 0 \\ \frac{1}{\tau_I} \end{bmatrix}, \quad g(x(t)) = \begin{bmatrix} \frac{N_0 R_0 T_0}{2F_0} \left(\ln \left(\frac{p_{H_2}}{p_{H_2}} \right) + \frac{1}{2} \ln \left(\frac{p_{O_2}}{p_{O_2}} \right)\right) \\ \ln \left(\frac{p_{H_2}}{p_{H_2}} \right) \\ \ln \left(\frac{p_{O_2}}{p_{O_2}} \right) \end{bmatrix}.$$
Fig. 1. Simulation result (stack voltage $x_4(k)$).

Fig. 2. Simulation result (control input $u(k)$).

Fig. 3. Set point tracking of stack voltage (V).
It should be noted that lower and upper sector bounds of the nonlinear function vector \( f(x(k)) \) are obtained as \([-6.67, 2.83]^T\) and \([9.07, 3.52]^T\) in the range of \( x_1 \in [0, 0.1] \). The initial condition is \( x_0 = [0.0867, 0.0777, 0.0898, 14.9727]^T \), the weighting matrices are \( Q = 2I \), \( R = I \) and the input constraint is \( u = 0.1 \). Figs. 1 and 2 show that the comparison of the proposed method with the case of only state feedback control law \( K_2(k) = 0 \). It can be seen that the proposed method provides a fully utilized fuel flow rate within input constraint and better performance than those without considering the sector nonlinearities.

In addition, Figs. 3 and 4 show the stack voltage and fuel flow rate for set-point tracking at three operating points with \( x^* = \{[0.1054, 0.3149, 0.7023, 240.7075], [0.1538, 0.3573, 0.7508, 250.0042], [0.0671, 0.2815, 0.6640, 230.0098]\} \), respectively. This results show that the proposed method can be well applied to the set-point tracking in the presence of input constraint.

5. Conclusions

In this paper, we proposed a nonlinear MPC algorithm for SOFC systems using a sector bounded nonlinear model. For realistic modeling of SOFC systems, we adopted a mathematical sector bounded model which was represented as the convex combination of each vertices. For the performance, one horizon MPC method was considered with an augmented terminal cost which consists of state and sector bounded nonlinear function. Due to the augmented vector feedback, a less conservative cost monotonicity condition was obtained in the terminal inequality. The proposed one step predictive controller design conditions were expressed in the form of a finite number of LMIs. Through numerical simulations, we showed that the proposed method had better performance than those without considering the sector nonlinearities and a good tracking performance.

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References


