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Global synchronization of complex networks perturbed by the Poisson noise

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ABSTRACT

In this paper, the problem of stochastic synchronization analysis is investigated for complex networks perturbed by the Poisson noise. By using the key tool such as the infinitesimal operator for stochastic differential equations driven by the Poisson process, this paper proposes a globally exponentially synchronization criterion in mean square for complex networks perturbed by the Poisson noise. Finally, numerical examples are provided to demonstrate the effectiveness of the proposed approach.

1. Introduction

As is known to all, complex dynamical networks (CDNs) widely exist in the real world, including food-webs, ecosystems, metabolic pathways, the Internet, the World Wide Web, social networks and global economic markets [1,2]. Since the discoveries of the small-world feature [3] and the scale-free feature [4] of complex networks, the analysis and the control of the dynamical behaviors in complex networks have been extensively investigated in the past decades. As a significant collective behavior, the studies on the synchronization phenomena of complex dynamical networks have gained considerable research interests [5–12].

On the other hand, in the real world, complex networks are often subject to environmental disturbances; especially the signal transfer within complex networks is always affected by the stochastic perturbations. Therefore, in order to reflect more realistic dynamical behaviors, many researchers have recently investigated the synchronization problems of complex networks perturbed by stochastic noises. For instance, complex networks perturbed by Brown noises have been discussed in [13–16]. The synchronization problems of discrete-time stochastic complex networks with Brown noises were investigated in [13,14]. As to the continuous case, the global exponential synchronization problem for complex dynamical networks with nonidentical nodes and Brown perturbations was studied in [15]. And the synchronization control problem for the competitive complex networks with Brown noises was investigated in [16].

However, it is well known that in the real world, beside Brown noises, there is a very common but important kind of random noises: Poisson noises. Poisson noises can be widely found in various applications such as neurophysiology systems, storage systems, queueing systems, economic systems, and so on [17,18]. It should be pointed out that, unlike the Brown process whose almost all sample paths are continuous, the Poisson process $\mathcal{N}(t)$ is a jump process and has the sample paths which are right-continuous and have left limits (i.e. càdlàg). Therefore, there is a great difference between the stochastic integral with respect to the Brown process and the one with respect to the Poisson process. As a result, the dynamical

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behaviors of the stochastic systems driven by the Poisson process are essential different from the stochastic systems driven by the Brown process. Thus, it is very important to investigate the dynamic behaviors, such as the synchronization phenomena, for complex networks perturbed by the Poisson process. However, to the best of our knowledge, there is still no paper to discuss the synchronization problem for this kind of systems.

Motivated by above reasons, this paper investigates the synchronization problem for stochastic complex networks perturbed by the Poisson noise. By using the key tool such as the infinitesimal operator for stochastic differential equations driven by the Poisson process, this paper presents a globally exponentially synchronization criterion in mean square for complex networks perturbed by the poisson noise. Finally, numerical examples are provided to demonstrate the effectiveness of the proposed approach.

Notation: Throughout the paper, unless otherwise specified, we will employ the following notation. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geqslant 0}, \mathbb{P})$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geqslant 0}$ and $\mathbb{E}(\cdot)$ be the expectation operator with respect to the probability measure. If A is a vector or matrix, its transpose is denoted by A^T. If P is a square matrix, $P > 0$ ($P < 0$) means that is a symmetric positive (negative) definite matrix of appropriate dimensions while $P \geqslant 0$ ($P \leqslant 0$) is a symmetric positive (negative) semidefinite matrix. I stands for the identity matrix of appropriate dimensions. Let $|\cdot|$ denote the Euclidean norm of a vector and its induced norm of a matrix. Unless explicitly specified, matrices are assumed to have real entries and compatible dimensions. $L^2(\Omega)$ denotes the space of all random variables X with $\mathbb{E}|X|^2 < \infty$, it is a Banach space with norm $\|X\|_2 = (\mathbb{E}|X|^2)^{1/2}$. The symbol '$*$' within a matrix represents the symmetric terms of the matrix, e.g. $\begin{pmatrix} X & Y \\ * & Z \end{pmatrix} = \begin{pmatrix} X & Y \\ Y^T & Z \end{pmatrix}$.

2. Problem formulation and preliminaries

Consider the following complex dynamical networks consisting of N nodes perturbed by the Poisson noise:

$$dx_i(t) = \left[Ax_i(t) + Bf(x_i(t)) + \sum_{j=1}^{N} g_{ij}\Gamma x_j(t)\right]dt + \sigma_i(t, x_i(t))d\mathcal{N}(t), \quad i = 1, 2, \ldots, N \tag{1}$$

where $x_i(t) = [x_{i1}(t), x_{i2}(t), \ldots, x_{in}(t)]^T \in \mathcal{R}^n$ is the state vector of the ith network at time t; A denotes a known connection matrix, B denotes the connection weight matrix; $\Gamma \in \mathcal{R}^{n\times n}$ is the matrix describing the inner-coupling between the subsystems at time t; $G = (g_{ij})_{N\times N}$ is the out-coupling configuration matrix representing the coupling strength and the topological structure of the complex networks. $\sigma_i(\cdot,\cdot) : \mathcal{R} \times \mathcal{R}^n \to \mathcal{R}^n$ is the noise intensity function vector and $\{\mathcal{N}(t)\}_{t\geqslant 0}$ is a one-dimension $\{\mathcal{F}_t\}_{t\geqslant 0}$ adapted Poisson process with parameter $\lambda > 0$. And $f(x_i(t)) = (f_1(x_{i1}(t)), \ldots, f_n(x_{in}(t)))^T$ is an unknown but sector-bounded nonlinear function.

The initial conditions associated with system (1) are given by

$$x_i(0) = \varphi_i, \quad i = 1, 2, \ldots, N, \tag{2}$$

where φ_i is the $\mathcal{F}_0$-measurable random variable and independent of $\{\mathcal{N}(t)\}_{t\geqslant 0}$ such that $\mathbb{E}(\varphi_i^2) < \infty$.

Let

$$x(t) = \left(x_1(t)^T, \ldots, x_N(t)^T\right)^T,$$
$$F(x(t)) = \left(f(x_1(t))^T, \ldots, f(x_N(t))^T\right)^T,$$
$$\sigma(t, x(t)) = \left(\sigma_1(t, x_1(t))^T, \ldots, \sigma_N(t, x_N(t))^T\right)^T.$$

With the Kronecker product '$\otimes$' for matrices, system (1) can be rearranged as

$$dx(t) = y(t, x(t))dt + \sigma(t, x(t))d\mathcal{N}(t), \tag{3}$$

where $y(t, x(t)) = (I_N \otimes A + G \otimes \Gamma)x(t) + (I_N \otimes B)F(x(t))$.

In this paper, we will mainly be concerned with the globally exponentially synchronization criterion in mean square. For this, we need the infinitesimal operator $\mathcal{D}$ associated to Eq. (3) (see [18–21]).

$$\mathcal{D}V(t, x) = \frac{\partial V(t, x)}{\partial t} + \frac{\partial V(t, x)}{\partial x} y(t, x) + \lambda(V(t, x + \sigma(t, x)) - V(t, x)), \tag{4}$$

where $V(t, x)$ is any non-negative function on $\mathcal{R} \times \mathcal{R}^{n\times N}$ and is continuously twice differentiable with respect to x and once differentiable with respect to t. Throughout this paper, the following assumptions, definitions and propositions are needed to prove our main results.

Definition 1 [15]. The stochastic complex network (1) is globally exponentially synchronized in mean square, if there exist constants $\alpha > 0$, $\gamma > 0$, such that for all φ_i, φ_j, the following holds for $t \geqslant 0$:

$$\mathbb{E}\{|x_i(t, \varphi_i) - x_j(t, \varphi_j)|^2\} \leqslant \gamma e^{-\alpha t}, \quad 1 \leqslant i < j \leqslant N.$$

Assumption 1. For $\forall x, y \in \mathcal{R}^n$, the nonlinear function $f(\cdot)$ is assumed to satisfy the following condition:

$$(f(x) - f(y) - U(x - y))^T (f(x) - f(y) - V(x - y)) \leqslant 0, \tag{5}$$

where U and V are known constant real matrices.

Assumption 2. The outer-coupling configuration matrix of the complex networks (1) satisfies

$$g_{ij} = g_{ji} \geqslant 0, \quad (i \neq j),$$
$$g_{ii} = -\sum_{j=1, j\neq i}^{N} g_{ij}, \quad i, j = 1, 2, \ldots, N.$$

Assumption 3. The noise intensity function vector $\sigma_i : \mathcal{R} \times \mathcal{R}^n \to \mathcal{R}^n$ satisfies the Lipschitz condition, i.e., there exists a constant matrix W of appropriate dimension such that

$$|\sigma_i(t, u) - \sigma_j(t, v)|^2 \leqslant |W(u - v)|^2 \tag{6}$$

for all $i, j = 1, 2, \ldots, N$ and $u, v \in \mathcal{R}^n$.

Proposition 1 [15]. *The Kronecker product has the following properties:*

$$(\alpha A) \otimes B = A \otimes (\alpha B),$$
$$(A + B) \otimes C = A \otimes C + B \otimes C,$$
$$(A \otimes B)(C \otimes D) = (AC) \otimes (BD),$$
$$(A \otimes B)^T = A^T \otimes B^T.$$

Proposition 2 [13]. *Let* $\mathcal{U} = (\alpha_{ij})_{n\times n}$, $P \in \mathcal{R}^{m\times m}$, $x = (x_1^T, x_2^T, \ldots, x_n^T)^T$, $y = (y_1^T, y_2^T, \ldots, y_n^T)^T$, *where* $x_i = (x_{i1}, x_{i2}, \ldots, x_{im})^T \in \mathcal{R}^m$, $y_i = (y_{i1}, y_{i2}, \ldots, y_{im})^T \in \mathcal{R}^m$ *(*$i = 1, 2, \ldots, n$*). If* $\mathcal{U} = \mathcal{U}^T$ *and each row sum of* $\mathcal{U}$ *is equal to zero, then*

$$x^T(\mathcal{U} \otimes P)y = -\sum_{1\leqslant i<j\leqslant n} \alpha_{ij}(x_i - x_j)^T P(y_i - y_j). \tag{7}$$

3. Main results

We are in the position to present our main results of the globally exponentially synchronization criterion in mean square for the complex networks perturbed by the Poisson noise.

Theorem 1. *Under Assumptions 1–3, the dynamic system (1) is globally exponentially synchronized in mean square if there exist matrices* $P > 0$ *and scalars* $\epsilon > 0$, $\epsilon_1 > 0$ *such that the following LMI hold for all* $1 \leqslant i < j \leqslant N$

$$\Xi = \begin{pmatrix} \Xi_{11} & \Xi_{12} & \sqrt{\lambda}P & 0 \\ * & -2\epsilon_1 I & 0 & 0 \\ * & * & -P & P \\ * & * & * & -\epsilon I \end{pmatrix} < 0, \tag{8}$$

where

$$\Xi_{11} = PA + A^T P - Ng_{ij}P\Gamma - Ng_{ij}\Gamma^T P + \epsilon\lambda W^T W - \lambda P - \epsilon_1 U^T V - \epsilon_1 V^T U,$$
$$\Xi_{12} = PB + \epsilon_1 U^T + \epsilon_1 V^T.$$

Proof. Firstly, from (8), we can see that the matrix $\Theta = diag(I, I, P^{-1}, I)$ is nonsingular. Thus we can have

$$\Theta^T \Xi \Theta = \begin{pmatrix} \Xi_{11} & \Xi_{12} & \sqrt{\lambda}I & 0 \\ * & -2\epsilon_1 I & 0 & 0 \\ * & * & -P^{-1} & I \\ * & * & * & -\epsilon I \end{pmatrix} < 0. \tag{9}$$

Then by the Schur complement, we can obtain

$$\tilde{\Xi} = \begin{pmatrix} \tilde{\Xi}_{11} & \Xi_{12} \\ * & -2\epsilon_1 I \end{pmatrix} < 0, \tag{10}$$

where

$$\tilde{\Xi}_{11} = \Xi_{11} + \lambda(P^{-1} - \epsilon^{-1}I)^{-1}.$$

Secondly, for the system (3), consider the following Lyapunov function:

$$V(t,x) = x(t)^T(U \otimes P)x(t), \tag{11}$$

where

$$U = \begin{pmatrix} N-1 & -1 & \cdots & -1 \\ -1 & N-1 & \cdots & -1 \\ \cdots & \cdots & \cdots & \cdots \\ -1 & -1 & \cdots & N-1 \end{pmatrix}.$$

Then, by the formula (4), the infinitesimal operator can be obtained

$$\mathcal{D}V(t,x) = 2x(t)^T(U \otimes P)y(t,x(t)) + \lambda(x(t) + \sigma(t,x(t)))^T(U \otimes P)(x(t) + \sigma(t,x(t))) - \lambda x(t)^T(U \otimes P)x(t). \tag{12}$$

By Propositions 1 and 2, it is easy to obtain

$$\begin{aligned}\mathcal{D}V(t,x) = \sum_{1\leqslant i<j\leqslant N} &[2(x_i(t) - x_j(t))^T(PA - Ng_{ij}P\Gamma)(x_i(t) - x_j(t)) + 2(x_i(t) - x_j(t))^T PB(f(x_i(t)) - f(x_j(t))) \\ &+ \lambda\big(x_i(t) - x_j(t) + \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t))\big)^T P\big(x_i(t) - x_j(t) + \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t))\big) - \lambda(x_i(t) \\ &- x_j(t))^T P(x_i(t) - x_j(t))].\end{aligned} \tag{13}$$

By (9), we have

$$P^{-1} - \epsilon^{-1}I > 0, \tag{14}$$

$$\epsilon I - P > 0. \tag{15}$$

Then, we can prove that

$$\begin{aligned}&(x_i(t) - x_j(t) + \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t)))^T P(x_i(t) - x_j(t) + \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t))) \\ &\leqslant (x_i(t) - x_j(t))^T(P^{-1} - \epsilon^{-1}I)^{-1}(x_i(t) - x_j(t)) + \epsilon(\sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t)))^T(\sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t))).\end{aligned} \tag{16}$$

In fact, using the matrix inversion formula, we can easily know that

$$(P^{-1} - \epsilon^{-1}I)^{-1} = P + P(\epsilon I - P)^{-1}P. \tag{17}$$

Let

$$\Upsilon = \begin{pmatrix} (P^{-1} - \epsilon^{-1}I)^{-1} & 0 \\ * & \epsilon I \end{pmatrix} - \begin{pmatrix} P & P \\ * & P \end{pmatrix} = \begin{pmatrix} P(\epsilon I - P)^{-1}P & -P \\ * & \epsilon I - P \end{pmatrix}. \tag{18}$$

Noticing that

$$\Phi = \begin{pmatrix} P^{-1} & 0 \\ (\epsilon I - P)^{-1} & I \end{pmatrix} \tag{19}$$

is nonsingular, we can obtain that

$$\Phi^T \Upsilon \Phi = \begin{pmatrix} 0 & 0 \\ * & \epsilon I - P \end{pmatrix} \geqslant 0, \tag{20}$$

which denotes $\Upsilon \geqslant 0$. Thus, it follows that

$$\begin{aligned}&\begin{pmatrix} x_i(t) - x_j(t) \\ \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t)) \end{pmatrix}^T \Upsilon \begin{pmatrix} x_i(t) - x_j(t) \\ \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t)) \end{pmatrix} = \begin{pmatrix} x_i(t) - x_j(t) \\ \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t)) \end{pmatrix}^T \left(\begin{pmatrix} (P^{-1} - \epsilon^{-1}I)^{-1} & 0 \\ * & \epsilon I \end{pmatrix} \right. \\ &\quad \left. - \begin{pmatrix} P & P \\ * & P \end{pmatrix} \right) \begin{pmatrix} x_i(t) - x_j(t) \\ \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t)) \end{pmatrix} \\ &\quad = (x_i(t) - x_j(t))^T(P^{-1} - \epsilon^{-1}I)^{-1}(x_i(t) - x_j(t)) + \epsilon(\sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t)))^T(\sigma_i(t,x_i(t)) \\ &\qquad - \sigma_j(t,x_j(t))) - (x_i(t) - x_j(t) + \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t)))^T P(x_i(t) - x_j(t) + \sigma_i(t,x_i(t)) - \sigma_j(t,x_j(t))) \geqslant 0,\end{aligned} \tag{21}$$

From (21), it is very easy to obtain (16). Using (16), we can have

$$\begin{aligned}\mathcal{D}V(t,x) \leqslant \sum_{1\leqslant i<j\leqslant N}\Big[&2(x_i(t)-x_j(t))^T(PA-Ng_{ij}P\Gamma)(x_i(t)-x_j(t))+2(x_i(t)-x_j(t))^TPB(f(x_i(t))-f(x_j(t)))\\ &+\lambda(x_i(t)-x_j(t))^T(P^{-1}-\epsilon^{-1}I)^{-1}(x_i(t)-x_j(t))+\lambda\epsilon(\sigma_i(t,x_i(t))-\sigma_j(t,x_j(t)))^T(\sigma_i(t,x_i(t))-\sigma_j(t,x_j(t)))\\ &-\lambda(x_i(t)-x_j(t))^TP(x_i(t)-x_j(t))\Big]. \end{aligned} \tag{22}$$

From Assumption 3, it is clear that

$$(\sigma_i(t,x_i(t))-\sigma_j(t,x_j(t))^T(\sigma_i(t,x_i(t))-\sigma_j(t,x_j(t)))\leqslant(x_i(t)-x_j(t))^TW^TW(x_i(t)-x_j(t)). \tag{23}$$

From Assumption 1, it can be derived that

$$\begin{aligned}0\leqslant\ &2\epsilon_1(x_i(t)-x_j(t))^TU^T\big(f(x_i(t))-f(x_j(t))\big)+2\epsilon_1(f(x_i(t))-f(x_j(t)))^TV\big(x_i(t)-x_j(t)\big)\\ &-2\epsilon_1\big(x_i(t)-x_j(t)\big)^TU^TV\big(x_i(t)-x_j(t)\big)-2\epsilon_1(f(x_i(t))-f(x_j(t)))^T(f(x_i(t))-f(x_j(t))).\end{aligned} \tag{24}$$

Combining (22)–(24), we have

$$\mathcal{D}V(t,x)\leqslant\sum_{1\leqslant i<j\leqslant N}\xi_{ij}^T\tilde{\Xi}\xi_{ij}, \tag{25}$$

where

$$\xi_{ij}=\begin{pmatrix}x_i(t)-x_j(t)\\ f(x_i(t))-f(x_j(t))\end{pmatrix}.$$

From (10), it is easy to prove that there exists a scalar $c>0$ such that

$$\mathcal{D}V(t,x))\leqslant -c\sum_{1\leqslant i<j\leqslant N}(x_i(t)-x_j(t))^T(x_i(t)-x_j(t)). \tag{26}$$

Finally, using a method similar to that used to prove the stability of stochastic differential equations driven by the Poisson process in [18,20,21], we can prove that all the subsystems in (1) are globally asymptotically synchronized in the mean square. The proof is completed. □

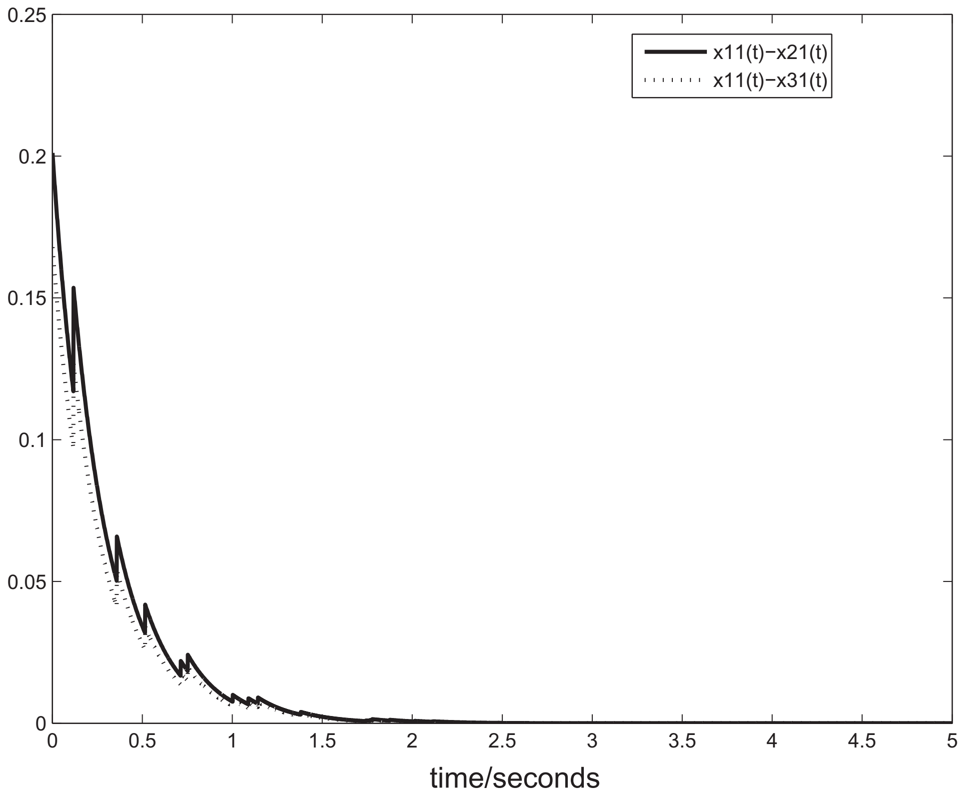

Fig. 1. State error of $x_{11}(t)-x_{i1}(t)$, $i=2,3$.

Remark 1. In recent years, the synchronization problems for stochastic complex networks have been studied extensively in [13–16]. It should be pointed out that most of stochastic complex networks in above papers are perturbed by the Brown noises, and stochastic complex networks perturbed by the Poisson noise are still not investigated. Theorem 1 gives a globally exponentially synchronization criterion in mean square for complex networks perturbed by the Poisson noise by using the infinitesimal operator of stochastic differential equations driven by the Poisson process.

4. Numerical examples

In this section, we present two simulation examples to illustrate the effectiveness of our approach.

Example 1. Consider the following complex network consisting of three nodes.

$$dx_i(t) = \left[Ax_i(t) + Bf(x_i(t)) + \sum_{j=1}^{3} g_{ij}\Gamma x_j(t)\right]dt + \sigma_i(t, x_i(t))d\mathcal{N}(t)$$

for all $i = 1, 2, 3$, where $x_i(t) = [x_{i1}(t), x_{i2}(t)]^T \in \mathcal{R}^2$ is the state vector of the ith subsystem, $\{\mathcal{N}(t)\}_{t\geqslant 0}$ is a one-dimension $\{\mathcal{F}_t\}_{t\geqslant 0}$ adapted Poisson process with parameter $\lambda = 6.5$. Let

$$A = \begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}, \quad B = \begin{pmatrix} -1 & 0.1 \\ 0.2 & -1 \end{pmatrix}.$$

The out-coupling configuration matrices G and inner-coupling matrices Γ are chosen as

$$G = \begin{pmatrix} -3 & 1 & 2 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 0.2 & 0 \\ 0.1 & 0.2 \end{pmatrix}.$$

The noise intensity function vector $\sigma_i(\cdot,\cdot)$ is of the following form:

$$\sigma_i(t, x_i(t)) = \begin{pmatrix} \sqrt{0.1} & 0 \\ 0 & \sqrt{0.2} \end{pmatrix} x_i(t),$$

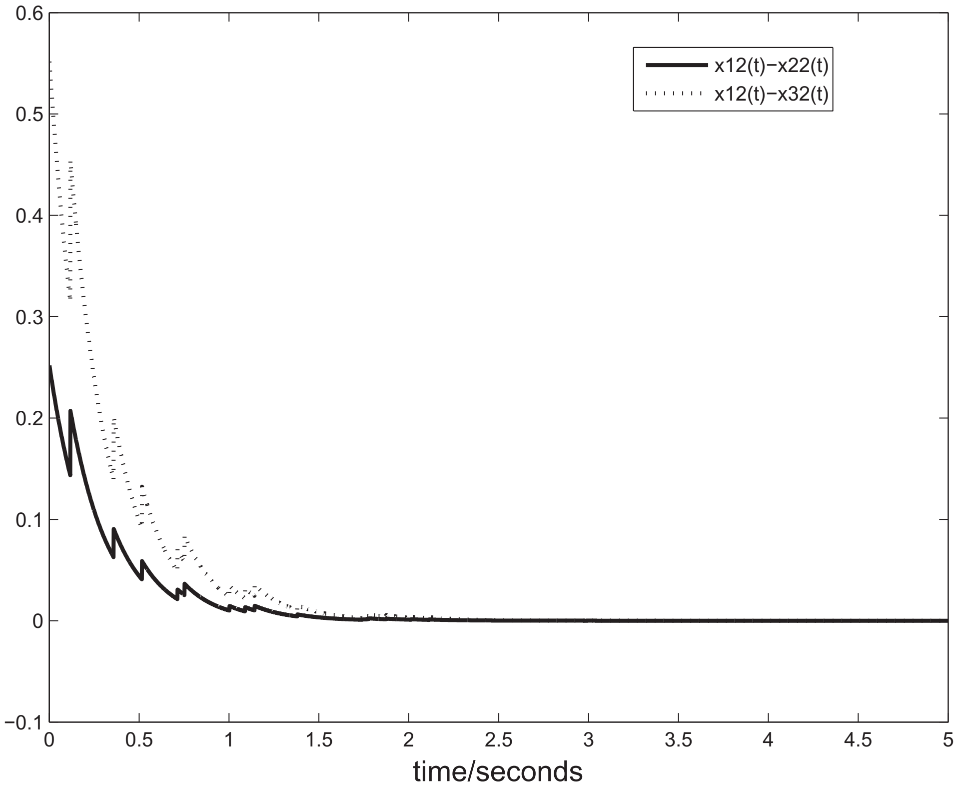

Fig. 2. State error of $x_{12}(t) - x_{i2}(t)$, $i = 2, 3$.

and the nonlinear function $f(x_i(t)) = (f_1(x_{i1}(t)), f_2(x_{i2}(t)))^T = (\tanh(x_{i1}(t)), \tanh(x_{i2}(t)))^T$. Thus, the matrices U, V, W in Assumptions 1 and 3 are

$$U = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad W = \begin{pmatrix} \sqrt{0.1} & 0 \\ 0 & \sqrt{0.2} \end{pmatrix}.$$

According to Theorem 1, we can know that this complex network is globally exponentially synchronized in mean square. When we randomly choose the initial states in $[0,1] \times [0,1]$, the synchronization errors are plotted in Figs. 1 and 2, which can confirm that the stochastic complex dynamical system (1) is globally exponentially synchronized in mean square.

Example 2. Consider the following complex network consisting of three nodes.

$$dx_i(t) = \left[Ax_i(t) + Bf(x_i(t)) + \sum_{j=1}^{3} g_{ij}\Gamma x_j(t)\right]dt + \sigma_i(t, x_i(t))d\mathcal{N}(t)$$

for all $i = 1, 2, 3$, where $x_i(t) = [x_{i1}(t), x_{i2}(t)]^T \in \mathcal{R}^2$ is the state vector of the ith subsystem, $\{\mathcal{N}(t)\}_{t \geqslant 0}$ is a one-dimension $\{\mathcal{F}_t\}_{t \geqslant 0}$ adapted Poisson process with parameter $\lambda = 5$. Let

$$A = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -0.1 \\ -5 & 3 \end{pmatrix}.$$

The out-coupling configuration matrices G and inner-coupling matrices Γ are chosen as

$$G = \begin{pmatrix} -3 & 1 & 2 \\ 1 & -2 & 1 \\ 2 & 1 & -3 \end{pmatrix}, \quad \Gamma = \begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}.$$

The noise intensity function vector $\sigma_i(\cdot, \cdot)$ is of the following form:

$$\sigma_i(t, x_i(t)) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} x_i(t),$$

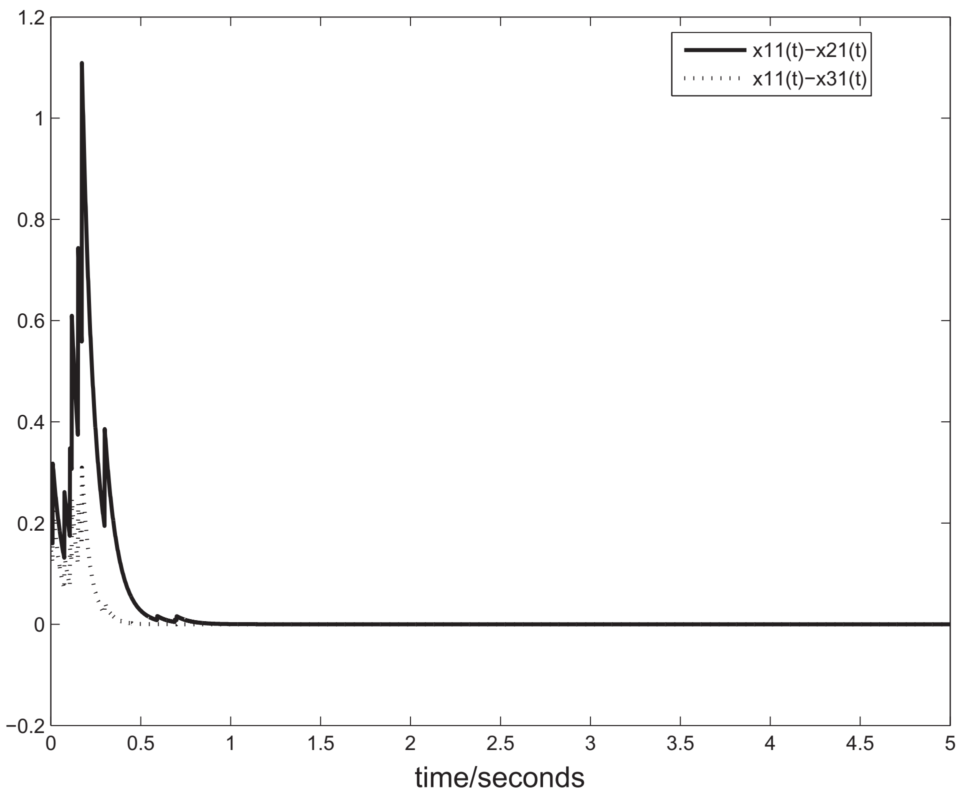

Fig. 3. State error of $x_{11}(t) - x_{i1}(t)$, $i = 2, 3$.

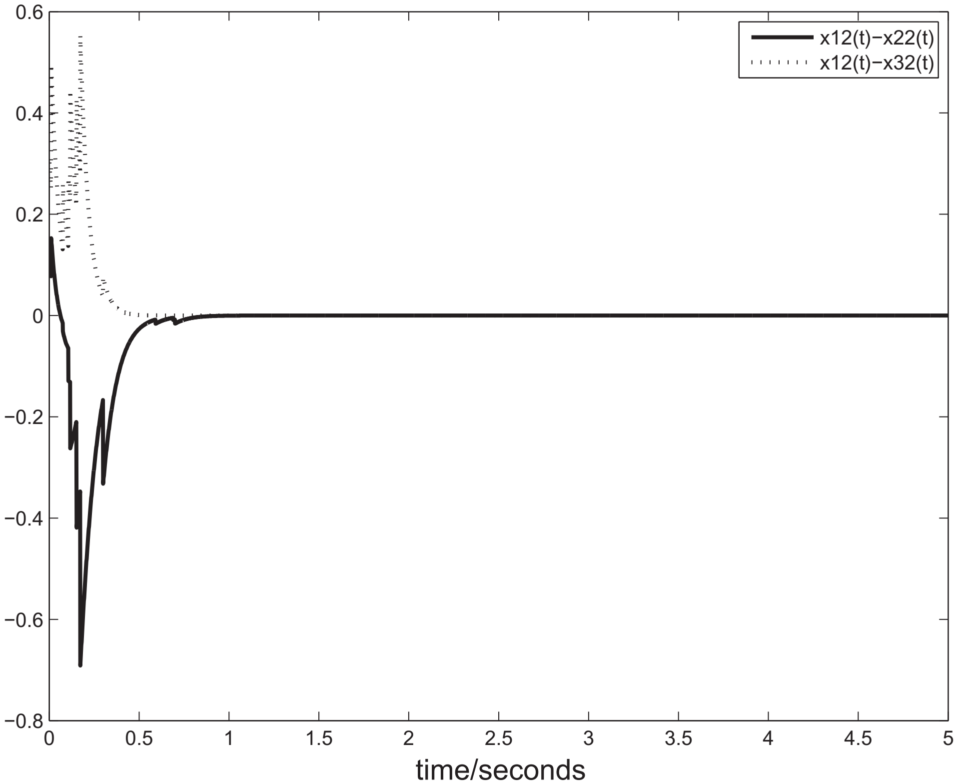

Fig. 4. State error of $x_{12}(t) - x_{i2}(t)$, $i = 2, 3$.

and the nonlinear function $f(x_i(t)) = (f_1(x_{i1}(t)), f_2(x_{i2}(t)))^T = (\tanh(x_{i1}(t)), \tanh(x_{i2}(t)))^T$. Thus, the matrices U, V, W in Assumptions 1 and 3 are

$$U = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad V = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad W = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

According to Theorem 1, we can know that this complex network is globally exponentially synchronized in mean square. When we randomly choose the initial states in $[0, 1] \times [0, 1]$, the synchronization errors are plotted in Figs. 3 and 4, which can confirm that the stochastic complex dynamical system (1) is globally exponentially synchronized in mean square.

5. Conclusions

This paper is concerned with the problem of stochastic synchronization analysis for complex networks perturbed by the Poisson noise. Using the infinitesimal operator for stochastic differential equations driven by the Poisson process, this paper gives a globally exponentially synchronization criterion in mean square. Finally, numerical examples are provided to demonstrate the effectiveness of the proposed approach.

On the other hand, it is worth mentioning that there are still some important problems to solve for stochastic complex networks perturbed by the Poisson noise. (1) When the whole network cannot synchronize by itself, some controllers may be designed and applied to force the network to synchronize. Therefore, it is necessary to consider the control problem, such as the adaptive control and pinning control, for synchronization of stochastic complex networks perturbed by the Poisson noise in the future. (2) It has now been well realized that in spreading information through complex networks, there always exist time delays, which may decrease the quality of the system and even lead to oscillation, divergence, and instability. Accordingly, the synchronization problems for stochastic delayed complex networks perturbed by the Poisson noise should be studied in the future researches.

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