



Adaptive lag synchronization for uncertain complex dynamical network with delayed coupling

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ABSTRACT

This paper proposes an adaptive control method to achieve the lag synchronization between uncertain complex dynamical network having delayed coupling and a non-identical reference node. Unknown parameters of both the network and reference node are estimated by adaptive laws obtained by Lyapunov stability theory. With the estimated parameters, the proposed method guarantees the globally asymptotical synchronization of the network in spite of unknown bounded disturbances. The effectiveness of our work is verified through a numerical example and simulation.

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1. Introduction

A complex dynamical network (CDN) is a set of coupled nodes interconnected by edges, in which each node represents a dynamical system. The structure of many real systems in nature can be described by the CDNs, such as social relationship networks, metabolic networks, food chain, disease transmission networks, Internet, the World-Wide-Web, power grids, and so on [1–3]. This has led to much interest to the studies of the CDNs. In particular, synchronization of the network has been one of the main topics due to its various applications. In the literature, a number of researchers have proposed many synchronization methods including linear state feedback control [4], pinning control [5,6], state observer based control [7], control of CDN with impulsive effect [8,9], and adaptive control methods [10–14]. It should be noted that these studies dealt with complete synchronization scheme. However, several different types of synchronization phenomena have been reported, such as generalized [15,16], lag [17], phase [18], projective [19], anticipating synchronization [20], and so on. Among them, lag synchronization can be a reasonable scheme from the viewpoint of engineering applications and characteristics of channel. This is why the time delay is inevitable when signals between systems are transferred. Therefore, lag synchronization has become a hot topic and attracted much attention from authors in many fields [17,21–23]. Unfortunately, there exist few results of lag synchronization method for CDNs [24]. In [24], a control method was proposed to lag synchronize the network with an identical node. Although the approach achieved the lag synchronization for CDN, there are still some problems which should be studied. They include: (1) coupling delay, (2) parameter uncertainty and external disturbance, and (3) synchronization with non-identical node. (1) Coupling delay between nodes is an inevitable factor in the network. Because the speed of signal travel between nodes is limited and the network nodes may be required to have non-local interconnections such as telecommunications [25,26]. (2) It is well-known that parameter uncertainty and external disturbance are unavoidable factors in many practical situations. Moreover, they can destroy the system stability or can make control of dynamic

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systems more difficult due to their effects. Therefore, some approaches such as updating law for unknown parameters or robust controller have been developed to deal with the uncertainty and disturbance [12,13,27]. (3) It is not realistic to assume that all nodes of the network are synchronized with an identical reference node. In real life applications such as laser array and biological systems, it is recognized that the network synchronization with non-identical node can be demanded [28,29]. Therefore, it is worth proposing a lag synchronization method in which the problems mentioned above are considered.

In this paper, a lag synchronization method between uncertain complex network with delayed coupling and a non-identical reference node has been proposed. Both the network nodes and reference one have parameter uncertainties and bounded external disturbances. All of the unknown parameters are estimated by adaptive laws derived from Lyapunov stability theory, which are used in the proposed synchronization method. By use of the updating laws, a robust controller is designed to synchronize the network despite the disturbances bounded by unknown constants. In the end, the network is globally asymptotically synchronized with the proposed method. Results of numerical example show the effectiveness of the proposed approach.

The notation throughout the paper is quite standard. \mathbb{R}^n denotes n -dimensional Euclidean space, and $\mathbb{R}^{n \times m}$ is the set of all $n \times m$ real matrices. The notation $X > 0 (\geq 0)$ means that X is real symmetric and positive definite (semi-definite). $diag(\dots)$ denotes the block diagonal matrix. The superscript T denotes the transpose of the matrix. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. Problem statement

Consider a controlled complex dynamic network consisting of N linearly and diffusively non-delayed and delayed coupled nodes with both parameter uncertainty and disturbance. The i th node can be described as follows:

$$\dot{x}_i(t) = f_i(x_i(t)) + F_i(x_i(t))\theta_i + c \sum_{j=1}^N a_{ij}\Gamma x_j(t) + c \sum_{j=1}^N b_{ij}\Gamma x_j(t - d_i) + \Delta_i(t) + u_i(t), \tag{1}$$

where $i = 1, 2, \dots, N$, $x_i(t) = [x_{i1}(t), x_{i2}(t), \dots, x_{in}(t)]^T \in \mathbb{R}^n$ is the state vector of node i , $u_i(t) \in \mathbb{R}^n$ is input vector, $f_i : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $F_i : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m_i}$ are the known continuous nonlinear function matrices, $\theta_i \in \mathbb{R}^{m_i}$ is the unknown constant parameter vector, $c > 0$ is the coupling strength, $d_i \geq 0$ is unknown coupling delay, and $\Delta_i \in \mathbb{R}^n$ is the disturbance. Coupling matrices $A = (a_{ij}) \in \mathbb{R}^{N \times N}$ and $B = (b_{ij}) \in \mathbb{R}^{N \times N}$ with zero-sum rows represent the non-delayed and delayed coupling configuration of the network, respectively. If there is a connection between i and j node ($i \neq j$), $a_{ij} = 1$ (or $b_{ij} = 1$), otherwise $a_{ij} = 0$ (or $b_{ij} = 0$) ($i \neq j$) for $i, j = 1, 2, \dots, N$. $\Gamma = diag(\gamma_1, \gamma_2, \dots, \gamma_n)$ is a positive matrix with $\gamma_i = 1$ for a particular i and $\gamma_j = 0$ for $j \neq i$, which means two coupled nodes are linked through their i th state variables.

The reference node is described as

$$\dot{x}_r(t) = f_r(x_r(t)) + F_r(x_r(t))\theta_r + \Delta_r(t), \tag{2}$$

where $x_r(t) = [x_{r1}(t), x_{r2}(t), \dots, x_{rm}(t)]^T \in \mathbb{R}^n$ is the state vector, $f_r : \mathbb{R}^n \rightarrow \mathbb{R}^n$ and $F_r : \mathbb{R}^n \rightarrow \mathbb{R}^{n \times m_r}$ are the known continuous nonlinear function matrices, $\theta_r \in \mathbb{R}^{m_r}$ is the unknown constant parameter vector, and $\Delta_r \in \mathbb{R}^n$ is the disturbance.

Let us define the error signal for lag synchronization as

$$e_i(t) = x_i(t) - x_r(t - \tau(t)) = \begin{bmatrix} x_{i1}(t) - x_{r1}(t - \tau(t)) \\ x_{i2}(t) - x_{r2}(t - \tau(t)) \\ \vdots \\ x_{in}(t) - x_{rn}(t - \tau(t)) \end{bmatrix} = \begin{bmatrix} e_{i1}(t) \\ e_{i2}(t) \\ \vdots \\ e_{in}(t) \end{bmatrix} \text{ for } i = 1, \dots, N, \tag{3}$$

where $\tau(t) \geq 0$ is a given channel propagation delay or channel time-delay.

Our objective is to design the controller $u_i(t)$ which makes the error $e_i(t)$ globally asymptotically stabilized, i.e.

$$\lim_{t \rightarrow \infty} \|e_i(t)\| = \lim_{t \rightarrow \infty} \|x_i(t) - x_r(t - \tau(t))\| = 0, \quad i = 1, 2, \dots, N. \tag{4}$$

This means that the network (1) is lag synchronized with the reference node (2).

Throughout this paper, following hypotheses are given:

Assumption 1. The channel propagation delay $0 \leq \tau(t) < \infty$ is a differentiable function with $|\dot{\tau}(t)| \leq v < \infty$ for any t where v is a positive constant.

Assumption 2. For any positive constants Ψ_i and Ψ_r , the time-varying disturbances $\Delta_i(t)$ and $\Delta_r(t)$ are bounded, i.e. $\|\Delta_i(t)\| \leq \Psi_i$, $\|\Delta_r(t)\| \leq \Psi_r$.

In many practical cases, it is difficult to know the upper bounds Ψ_i , Ψ_r of disturbances for the network and reference node. In this paper, we enable to achieve the lag synchronization by using adaptive scheme to estimate the unknown upper bounds.

3. Controller design for lag synchronization

In this section, we propose an adaptive lag synchronization method for the uncertain complex dynamical network (1) with delayed coupling.

From (1) and (2), the error dynamics for lag synchronization is obtained as

$$\begin{aligned} \dot{e}_i(t) = & f_i(x_i(t)) + F_i(x_i(t))\theta_i + c \sum_{j=1}^N a_{ij}\Gamma e_j(t) + c \sum_{j=1}^N b_{ij}\Gamma e_j(t-d_i) + \Delta_i(t) + u_i(t) - (1-\hat{\tau}(t))\{f_r(x_r(t-\tau(t))) \\ & + F_r(x_r(t-\tau(t)))\theta_r + \Delta_r(t)\}. \end{aligned} \quad (5)$$

The following theorem provides the control input and adaptive laws design method to make the errors $e_i(t)$ for $i = 1, \dots, N$ globally asymptotically stabilized.

Theorem 1. Consider the lag synchronization error (3) between the complex dynamical network (1) and the reference node (2). The error is globally asymptotically stabilized with a given propagation delay $\tau(t)$, if the control input and the adaptive laws are chosen as

$$u_i(t) = -\alpha_i(t)e_i(t) - \beta_i(t)\operatorname{sgn}(e_i(t)) - f_i(x_i(t)) - F_i(x_i(t))\hat{\theta}_i(t) + (1-\hat{\tau}(t))\{f_r(x_r(t-\tau(t))) + F_r(x_r(t-\tau(t)))\hat{\theta}_r(t)\}, \quad (6)$$

$$\dot{\hat{\theta}}_i(t) = k_1 F_i^T(x_i(t))e_i(t), \quad (7)$$

$$\dot{\hat{\theta}}_r(t) = -k_2(1-\hat{\tau}(t))F_r^T(x_r(t-\tau(t)))e_i(t), \quad (8)$$

$$\dot{\alpha}_i(t) = k_3 e_i^T(t)e_i(t), \quad (9)$$

$$\dot{\beta}_i(t) = k_4 e_i^T(t)\operatorname{sgn}(e_i(t)) \quad \text{for } i = 1, \dots, N, \quad (10)$$

where k_1, k_2, k_3 , and k_4 are positive constants, and $\hat{\theta}_i(t)$ and $\hat{\theta}_r(t)$ are the estimated parameters for the network (1) and reference node (2), respectively.

Proof. Choose the following Lyapunov function candidate

$$V(t) = \frac{1}{2} \sum_{i=1}^N e_i^T(t)e_i(t) + \frac{1}{2k_1} \sum_{i=1}^N \tilde{\theta}_i^T(t)\tilde{\theta}_i(t) + \frac{1}{2k_2} \sum_{i=1}^N \tilde{\theta}_r^T(t)\tilde{\theta}_r(t) + \frac{1}{2k_3} \sum_{i=1}^N \tilde{\alpha}_i^2(t) + \frac{1}{2k_4} \sum_{i=1}^N \tilde{\beta}_i^2(t) + \sum_{i=1}^N \int_{t-d_i}^t e_i^T(s)Qe_i(s)ds, \quad (11)$$

where $\tilde{\theta}_i(t) = \hat{\theta}_i(t) - \theta_i$, $\tilde{\theta}_r(t) = \hat{\theta}_r(t) - \theta_r$, $\tilde{\alpha}_i(t) = \alpha_i(t) - \alpha_i^*$, $\tilde{\beta}_i(t) = \beta_i(t) - \beta_i^*$, $Q = \operatorname{diag}(q_1, \dots, q_n) > 0$, and α_i^* and β_i^* are designed positive constants.

Then, the time derivative of $V(t)$ along the error dynamics (5) is derived as

$$\dot{V} = \sum_{i=1}^N \left[e_i^T(t)\dot{e}_i(t) + \frac{1}{k_1} \dot{\tilde{\theta}}_i^T(t)\tilde{\theta}_i(t) + \frac{1}{k_2} \dot{\tilde{\theta}}_r^T(t)\tilde{\theta}_r(t) + \frac{1}{k_3} \dot{\alpha}_i(t)\tilde{\alpha}_i(t) + \frac{1}{k_4} \dot{\beta}_i(t)\tilde{\beta}_i(t) + e_i^T(t)Qe_i(t) - e_i^T(t-d_i)Qe_i(t-d_i) \right]. \quad (12)$$

By application of the control input (6) to error dynamics $\dot{e}(t)$, we have

$$\begin{aligned} \dot{V} = & \sum_{i=1}^N \left[e_i^T(t)\{-\alpha_i(t)e_i(t) - \beta_i(t)\operatorname{sgn}(e_i(t)) - f_i(x_i(t))\tilde{\theta}_i(t) + (1-\hat{\tau}(t))F_r(x_r(t-\tau))\tilde{\theta}_r(t)\} \right] \\ & + \sum_{i=1}^N \left[e_i^T(t) \left\{ c \sum_{j=1}^N a_{ij}\Gamma e_j(t) + c \sum_{j=1}^N b_{ij}\Gamma e_j(t-d_i) + \Delta_i(t) - (1-\hat{\tau}(t))\Delta_r(t) \right\} \right] \\ & + \sum_{i=1}^N \left[\frac{1}{k_1} \dot{\tilde{\theta}}_i^T(t)\tilde{\theta}_i(t) + \frac{1}{k_2} \dot{\tilde{\theta}}_r^T(t)\tilde{\theta}_r(t) + \frac{1}{k_3} \dot{\alpha}_i(t)\tilde{\alpha}_i(t) + \frac{1}{k_4} \dot{\beta}_i(t)\tilde{\beta}_i(t) \right] + \sum_{i=1}^N e_i^T(t)Qe_i(t) - \sum_{i=1}^N e_i^T(t-d_i)Qe_i(t-d_i). \end{aligned} \quad (13)$$

From the adaptation laws (7) and (8), \dot{V} is led as follows:

$$\begin{aligned} \dot{V} = & - \sum_{i=1}^N \alpha_i(t)e_i^T(t)e_i(t) - \sum_{i=1}^N \beta_i(t)e_i^T(t)\operatorname{sgn}(e_i(t)) + c \sum_{i=1}^N e_i^T(t) \sum_{j=1}^N a_{ij}\Gamma e_j(t) + c \sum_{i=1}^N e_i^T(t) \sum_{j=1}^N b_{ij}\Gamma e_j(t-d_i) \\ & + \sum_{i=1}^N e_i^T(t)(\Delta_i(t) - (1-\hat{\tau}(t))\Delta_r(t)) + \sum_{i=1}^N e_i^T(t)Qe_i(t) - \sum_{i=1}^N e_i^T(t-d_i)Qe_i(t-d_i) + \sum_{i=1}^N \frac{1}{k_3} \dot{\alpha}_i(t)\tilde{\alpha}_i(t) + \sum_{i=1}^N \frac{1}{k_4} \dot{\beta}_i(t)\tilde{\beta}_i(t). \end{aligned} \quad (14)$$

Let us define $\Omega = \operatorname{diag}(\alpha_1, \dots, \alpha_N)$, $\Omega^* = \operatorname{diag}(\alpha_1^*, \dots, \alpha_N^*)$,

$$\tilde{e}_j(t) = \begin{bmatrix} e_{1j}(t) \\ \vdots \\ e_{Nj}(t) \end{bmatrix}, \quad \text{and} \quad \tilde{e}_j(t-d) = \begin{bmatrix} e_{1j}(t-d_1) \\ \vdots \\ e_{Nj}(t-d_N) \end{bmatrix} \quad \text{for } j = 1, \dots, n. \tag{15}$$

Then, applying (9)–(14) yields

$$\begin{aligned} \dot{V} = & - \sum_{j=1}^n \tilde{e}_j^T(t) \Omega^* \tilde{e}_j(t) + c \sum_{j=1}^n \gamma_j \tilde{e}_j^T(t) A \tilde{e}_j(t) + c \sum_{j=1}^n \gamma_j \tilde{e}_j^T(t) B \tilde{e}_j(t-d) + \sum_{j=1}^n q_j \tilde{e}_j^T(t) \tilde{e}_j(t) - \sum_{j=1}^n q_j \tilde{e}_j^T(t-d) \tilde{e}_j(t-d) \\ & + \sum_{i=1}^N e_i^T(t) (\Delta_i(t) - (1 - \dot{\tau}(t)) \Delta_r(t)) - \sum_{i=1}^N \beta_i^* e_i^T(t) \text{sgn}(e_i(t)). \end{aligned} \tag{16}$$

By use of the fact that $2x^T y \leq x^T \Xi x + y^T \Xi^{-1} y$ for any vectors $x, y \in \mathbb{R}^m$, and a positive definite matrix $\Xi \in \mathbb{R}^{m \times m}$, we have

$$\begin{aligned} \dot{V} \leq & - \sum_{j=1}^n \tilde{e}_j^T(t) \Omega^* \tilde{e}_j(t) + c \sum_{j=1}^n \gamma_j \tilde{e}_j^T(t) A \tilde{e}_j(t) + \sum_{j=1}^n q_j \tilde{e}_j^T(t) \tilde{e}_j(t) + \sum_{j=1}^n \frac{(c\gamma_j)^2}{4q_j} \tilde{e}_j^T(t) B B^T \tilde{e}_j(t) + \sum_{i=1}^N e_i^T(t) (\Delta_i(t) \\ & - (1 - \dot{\tau}(t)) \Delta_r(t)) - \sum_{i=1}^N \beta_i^* e_i^T(t) \text{sgn}(e_i(t)). \end{aligned} \tag{17}$$

From Assumptions 1 and 2, the following inequality is led

$$\Delta_i(t) - (1 - \dot{\tau}(t)) \Delta_r(t) \leq \|\Delta_i(t) - (1 - \dot{\tau}(t)) \Delta_r(t)\| \leq \|\Delta_i(t)\| + |1 - \dot{\tau}(t)| \cdot \|\Delta_r(t)\| \leq \varepsilon_i, \tag{18}$$

where ε_i is a positive constant.

Eventually, we obtain

$$\dot{V} \leq \sum_{j=1}^n \tilde{e}_j^T(t) \left[c\gamma_j A^s + \frac{(c\gamma_j)^2}{4q_j} B B^T + q_j I_N - \Omega^* \right] \tilde{e}_j(t) + \sum_{i=1}^N [\varepsilon_i - \beta_i^*] \|e_i(t)\|, \tag{19}$$

where $A^s = \frac{A+A^T}{2}$.

Therefore, by taking appropriate α_i^* and β_i^* such that

$$c\gamma_j A^s + \frac{(c\gamma_j)^2}{4q_j} B B^T + q_j I_N - \Omega^* < 0, \quad \text{for } j = 1, \dots, n, \tag{20}$$

$$\varepsilon_i - \beta_i^* < 0, \quad \text{for } i = 1, \dots, N, \tag{21}$$

we can obtain $\dot{V} \leq 0$. Noticing the positive differentiable and radially unbounded Lyapunov function V , we can observe that the set $S = \{e_i(t) \in \mathbb{R}^n | \dot{V}(t) = 0\} = \{e_i(t) \in \mathbb{R}^n | e_i(t) = 0\}$ contains no solutions other than the trivial solution $e_i(t) = 0$. According to Lasalle’s invariance principle [30], the error $e_i(t)$ is globally asymptotically stable, i.e. $\lim_{t \rightarrow \infty} \|e_i(t)\| = 0$.

Finally, this means that the lag synchronization between the network (1) and the reference node (2) is achieved by the control (6) and the update laws (7)–(10). This completes the proof. \square

Remark 1. In much literature [21–23], the propagation delay $\tau(t)$ is assumed to be a constant value and it results in that $\dot{\tau}(t) = 0$. In this paper, we consider the situation where the propagation delay is a time varying function. Therefore, it can be said that the proposed method is more general and realistic than the works in [21–23].

Remark 2. The proposed method can be applied to the situations where the network (1) has only delayed or non-delayed coupling. In other words, the control input (6) and adaptive laws (7)–(10) are still held when $A = 0$ or $B = 0$. This is easily derived from the proof of Theorem 1 by setting A and B as zero matrix, respectively.

Remark 3. Although the inclusion of the $\text{sgn}(e_i(t))$ function in (6) provides the robustness against unknown disturbances, it inevitably cause chattering phenomenon due to the delay of the control input. It is well-known that the phenomenon may degrade the performance of the controlled system and even lead to instability. In order to alleviate the chattering, the boundary layer approach is used by replacing the $\text{sgn}(e_i(t))$ function with the following saturation function

$$\text{sat}\left(\frac{e_i(t)}{\delta}\right) = \begin{cases} \text{sgn}(e_i(t)), & \text{if } \|e_i(t)\| > \delta, \\ \frac{e_i(t)}{\delta}, & \text{if } \|e_i(t)\| \leq \delta, \end{cases} \tag{22}$$

where δ is a small positive constant [30]. This saturation function (22) can approach the $\text{sgn}(\cdot)$ function, as enough small δ is chosen. However, the error $e_i(t)$ is only driven into a small bounded region $\{\|e_i(t)\| \leq \delta\}$, which means that the property of

asymptotical stability is lost. Therefore, considering the relation between the chattering phenomenon and stability property is required when designing the controller.

4. Numerical simulation

Let us consider an example to demonstrate the effectiveness of the proposed lag synchronization method. The reference node is described as Chua's circuit (Fig. 1(a)) with unknown parameters and disturbance

$$\begin{bmatrix} \dot{x}_{r1}(t) \\ \dot{x}_{r2}(t) \\ \dot{x}_{r3}(t) \end{bmatrix} = \begin{bmatrix} 0 \\ x_{r1}(t) - x_{r2}(t) + x_{r3}(t) \\ 0 \end{bmatrix} + \begin{bmatrix} x_{r2}(t) - x_{r1}(t) - h(x_{r1}(t)) & 0 \\ 0 & 0 \\ 0 & -x_{r2}(t) \end{bmatrix} \begin{bmatrix} \theta_{r1} \\ \theta_{r2} \end{bmatrix} + \Delta_r(t), \quad (23)$$

where $h(x_{r1}(t)) = \eta_2 x_{r1}(t) + \frac{1}{2}(\eta_1 - \eta_2)(|x_{r1}(t) + 1| - |x_{r1}(t) - 1|)$ with $\eta_1 = -1.4325$ and $\eta_2 = -0.7831$, and the parameter vector and disturbance signal are chosen as

$$\theta_r = [\theta_{r1} \ \theta_{r2}]^T = [10 \ 15]^T,$$

$$\Delta_r(t) = [0.3 \sin(t) \ \cos(t) \ 0.1 \sin(t) \ 0.5 \cos(t)]^T.$$

Rössler attractor (Fig. 1(b)) is chosen as the i th network node with delayed coupling

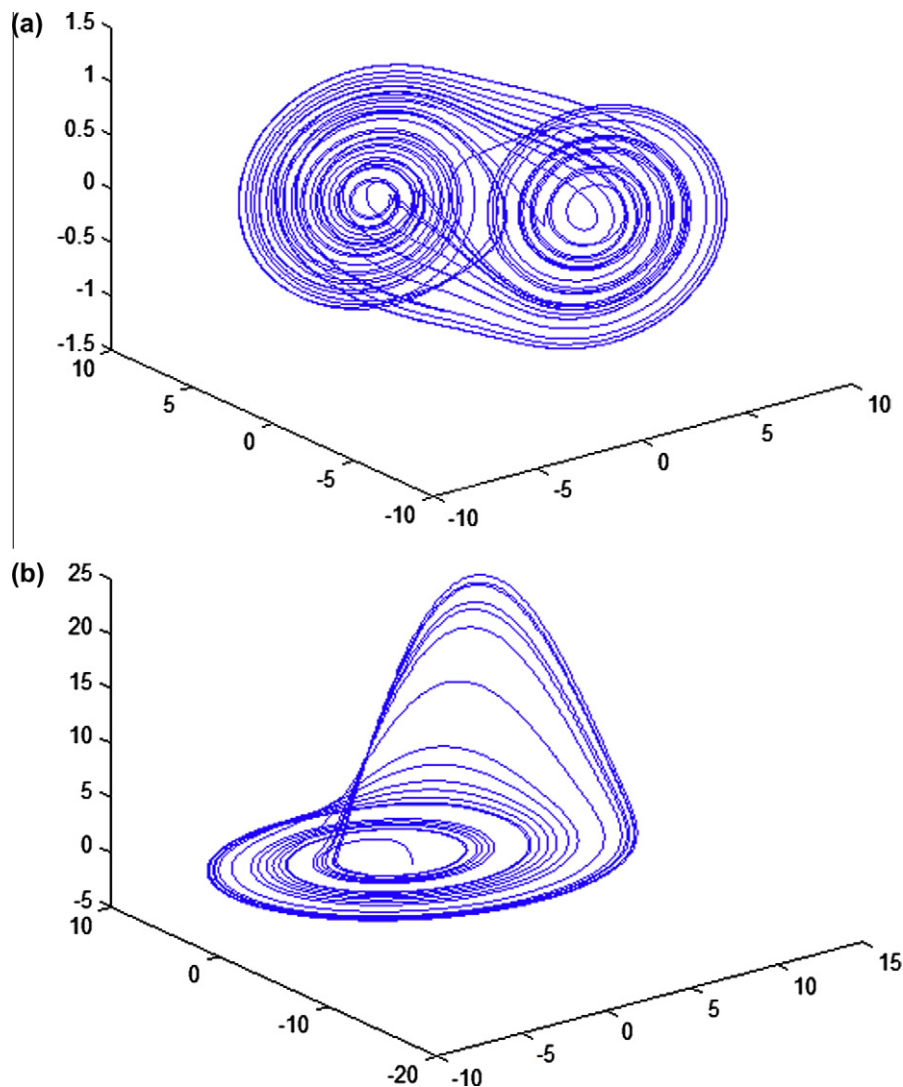


Fig. 1. The trajectories of Chua circuit (a) and Rössler attractor (b).

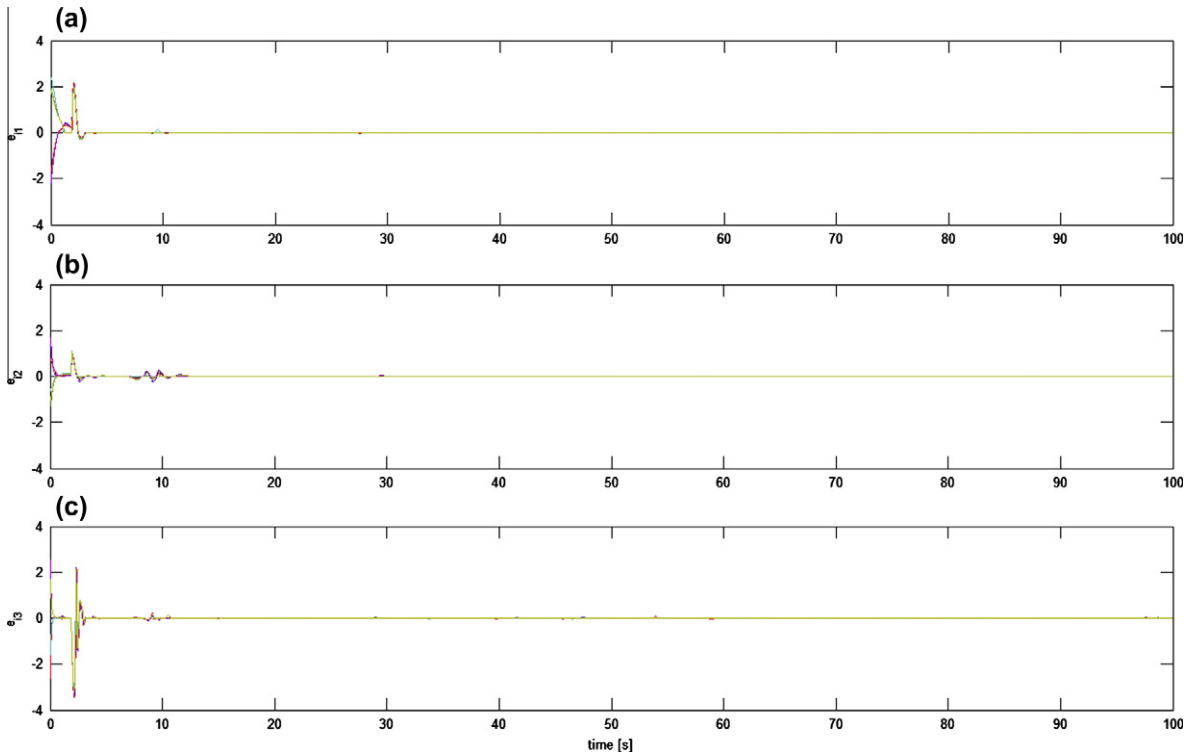


Fig. 2. The synchronization error $e_i(t) = x_i(t) - x_i(t - \tau(t))$ for $i = 1, \dots, 6$.

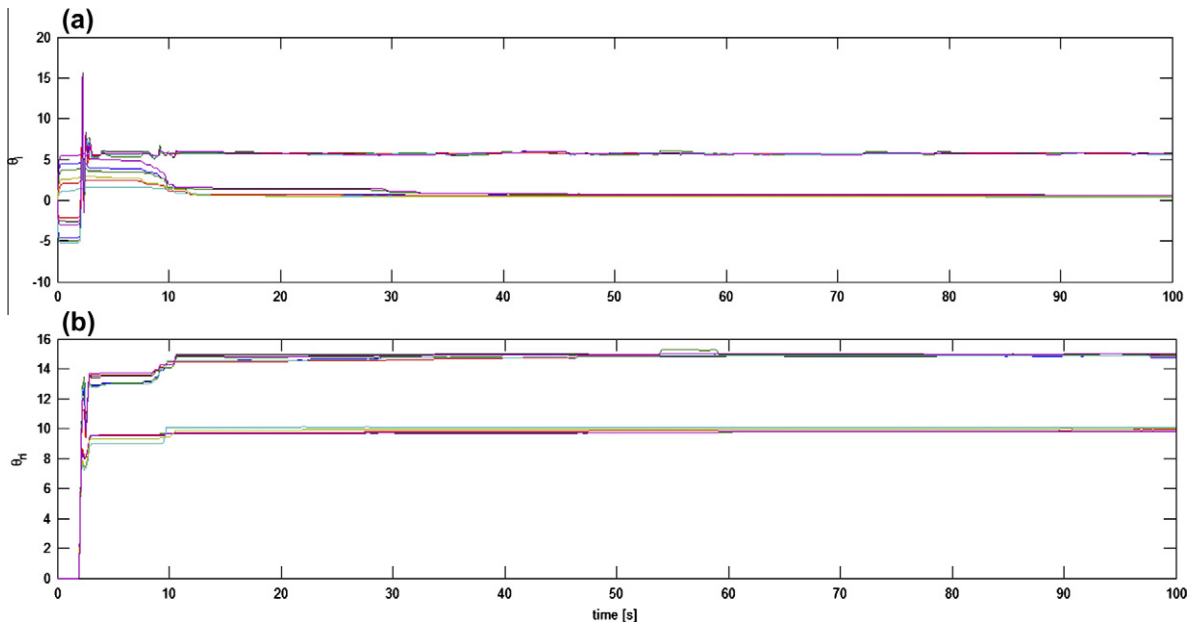


Fig. 3. The estimated parameters $\hat{\theta}_i$ (a) and $\hat{\theta}_{ri}$ (b).

$$\begin{bmatrix} \dot{x}_{i1}(t) \\ \dot{x}_{i2}(t) \\ \dot{x}_{i3}(t) \end{bmatrix} = \begin{bmatrix} -x_{i2}(t) - x_{i3}(t) \\ x_{i1}(t) \\ x_{i1}(t)x_{i3}(t) + 0.2 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ x_{i2}(t) & 0 \\ 0 & -x_{i3}(t) \end{bmatrix} \begin{bmatrix} \theta_{i1} \\ \theta_{i2} \end{bmatrix} + c \sum_{j=1}^N a_{ij} \Gamma x_j(t) + c \sum_{j=1}^N b_{ij} \Gamma x_j(t - d_i) + \Delta_i(t) + u_i(t) \quad \text{for } i = 1, 2, \dots, N,$$

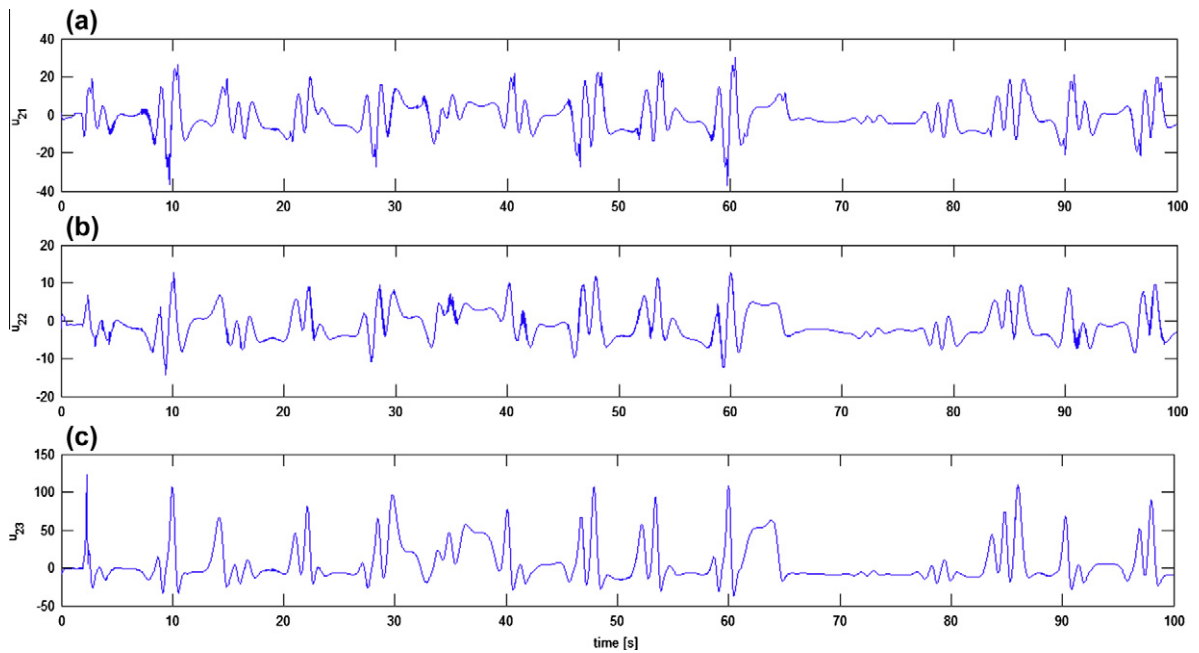


Fig. 4. The control input signal of node 2, $u_2(t) = [u_{21}(t) \ u_{22}(t) \ u_{23}(t)]^T$.

where the unknown parameter vector $\theta_i = [\theta_{i1} \ \theta_{i2}]^T = [0.2 \ 5.7]^T$, $c = 0.2$, $d_i = 1$, $N = 6$, and $\Gamma = I_{3 \times 3}$. The two coupling matrices are chosen as

$$A = (a_{ij}) = \begin{bmatrix} -5 & 1 & 1 & 1 & 1 & 1 \\ 0 & -3 & 1 & 1 & 1 & 0 \\ 1 & 1 & -4 & 0 & 1 & 1 \\ 0 & 1 & 0 & -2 & 1 & 0 \\ 1 & 0 & 0 & 1 & -3 & 1 \\ 0 & 1 & 1 & 1 & 1 & -4 \end{bmatrix}, \quad B = (b_{ij}) = \begin{bmatrix} -4 & 1 & 1 & 0 & 1 & 1 \\ 1 & -4 & 0 & 1 & 1 & 1 \\ 1 & 0 & -3 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \\ 1 & 0 & 1 & 1 & -4 & 1 \\ 0 & 1 & 1 & 0 & 1 & -3 \end{bmatrix}. \quad (25)$$

We choose the disturbance Δ_i as follows:

$$\Delta_i = \begin{bmatrix} \phi_{i1} \cos(\phi_{i3}t) \\ \phi_{i2} \sin(\phi_{i4}t) \\ \phi_{i1} \cos(\phi_{i4}t) \end{bmatrix}, \quad (26)$$

where

$$\Phi = (\phi_{ij}) = \begin{bmatrix} 0.8 & 0.7 & 0.8 & 1.2 \\ 0.9 & 0.8 & 1 & 0.9 \\ 1 & 0.7 & 0.8 & 0.7 \\ 0.6 & 1 & 0.6 & 1.0 \\ 0.9 & 0.8 & 0.8 & 0.9 \\ 0.7 & 0.9 & 0.9 & 0.6 \end{bmatrix}.$$

The propagation delay is $\tau(t) = 1 + \sin(t)$. The gains of adaptive laws (7)–(10) are $k_1 = k_2 = 15$, $k_3 = 1$, and $k_4 = 0.3$. The initial values are $\hat{\theta}_{i0} = \hat{\theta}_{i0} = \alpha_{i0} = \beta_{i0} = 0$, $x_{r0} = [-1 \ -1 \ 1]^T$, and x_{i0} are chosen in $[-3, 3]$ randomly. As mentioned in Remark 3, we replace the $\text{sgn}(e_i(t))$ function in (6) with (22) with $\delta = 0.002$ to reduce the chattering phenomenon.

Fig. 2 shows the lag synchronization errors $e_i(t) = x_i(t) - x_r(t - \tau(t))$ for $i = 1, 2, \dots, 6$. Moreover, we can observe that the estimated parameters of the network nodes (Fig. 3(a)) and reference one (Fig. 3(b)) converge to their real values. Fig. 4 shows the input signals of node 2 and we can see that they rarely have the chattering phenomenon. The trajectories of $x_r(t)$ and $x_i(t)$ for node 2 and 6 are presented in Fig. 5. These results verify that the proposed controller (6) with adaptation laws (7)–(10) makes the network (1) lag-synchronized, even if both the network and reference node have parameter uncertainties and disturbances.

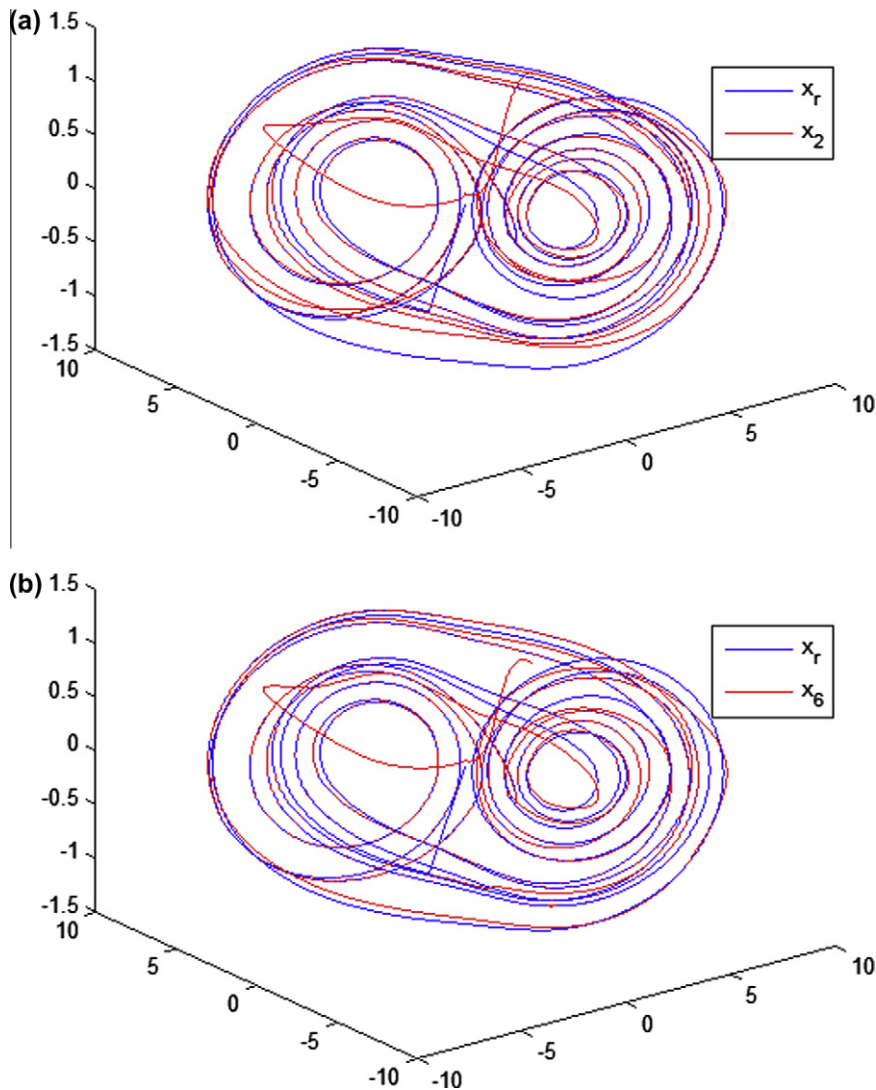


Fig. 5. The state trajectories: (a) $x_2(t)$, (b) $x_6(t)$ with the reference node $x_r(t)$.

5. Conclusion

An adaptive lag synchronization method was presented for uncertain CDNs with delayed coupling. Both the network and a non-identical reference node are affected by parameter uncertainties and disturbances. The unknown parameters were estimated by the adaptive laws obtained from Lyapunov stability theory. Even if there exist unknown bounded disturbances, the proposed controller with the estimated parameters achieved the lag synchronization of the network. Numerical results showed the effectiveness of the proposed approach.

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