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Letter to the Editor

Adaptive control for modified projective synchronization of a four-dimensional chaotic system with uncertain parameters

Ju H. Park

Robust Control and Nonlinear Dynamics Laboratory, Department of Electrical Engineering, Yeungnam University, 214-1 Dae-Dong, Kyongsan 712-749, Republic of Korea

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Abstract

This article is concerned with the modified projective synchronization problem for a class of four-dimensional chaotic system with uncertain parameters. By utilizing Lyapunov method, an adaptive control scheme for the synchronization has been presented. The control performances are verified by a numerical simulation.

MSC: 93C40; 93C10

Keywords: Modified projective synchronization; Adaptive control; Chaotic systems; Lyapunov method

1. Introduction

A chaotic system has complex dynamical behaviors that possess some special features such as being extremely sensitive to tiny variations of initial conditions, and having bounded trajectories in the phase space with a positive leading Lyapunov exponent and so on. In particular, chaos synchronization has attracted a great deal of attention from various scientific fields since Pecora and Carroll [18] introduced a method to synchronize two identical chaotic systems with different initial conditions [1–6,9–15,17,22,23]. The idea of synchronization is to use the output of the master system to control the slave system so that the output of the response system follows the output of the master system asymptotically. Many methods and techniques for handling chaos control and synchronization of various chaotic systems have been developed, such as PC method [18], OGY method [12], time-delay feedback approach [17], feedback approach [6,9], backstepping design technique [23], adaptive method [4,5,10,13,22], linear control method [11,14], nonlinear control scheme [1,3,15] and so on.

Recently, Qi et al. [19] developed a new four-dimensional (4D) continuous autonomous chaotic system, in which each equation in the system contains a 3-term cross product, and analyzed basic properties of the system by means of Lyapunov exponents and bifurcation diagrams. More recently, the adaptive synchronization problem of the 4D chaotic system with uncertain parameters is studied by Park [16].

On the other hand, most of research on chaos synchronization efforts mentioned above have concentrated on studying complete synchronization (CS), identical synchronization or conventional synchronization, where two coupled chaotic systems exhibit identical oscillations. In the practical applications, CS only occurs at a certain point in the parameter

E-mail address: jessie@yu.ac.kr.

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space, and it is difficult to achieve CS except under ideal conditions. Recently, thus a more general form of synchronization scheme, called generalized synchronization (GS), has been extensively investigated [20,21,7,8], where the drive and response systems could be synchronized up to a scaling factor α. It suggests that one can achieve control of this synchronization in general classes of chaotic systems including non-partially linear systems. More recently, Li [8] consider a new GS method, called modified projective synchronization (MPS), where the responses of the synchronized dynamical states synchronize up to a constant scaling matrix.

In this paper, the problem of MPS of the 4D chaotic system with uncertain parameters is investigated based on the result [16]. For the chaotic synchronization, a class of adaptive feedback control scheme is proposed using the Lyapunov's direct method.

The organization of this paper is as follows. In Section 2, the problem statement and master–slave synchronization scheme are presented for the 4D chaotic system. In Section 3, we provide a numerical example to demonstrate the effectiveness of the proposed method. Finally, concluding remark is given in Section 4.

2. Adaptive control for modified projective synchronization of a 4D chaotic system

Consider the following 4D chaotic system [19] described by

$$\begin{cases} \dot{z}_1 = a(z_2 - z_1) + z_2 z_3 z_4, \\ \dot{z}_2 = b(z_1 + z_2) - z_1 z_3 z_4, \\ \dot{z}_3 = -c z_3 + z_1 z_2 z_4, \\ \dot{z}_4 = -d z_4 + z_1 z_2 z_3, \end{cases} \tag{1}$$

where z_1, z_2, z_3 and z_4 are state variables and a, b, c and d are all positive real constant parameters.

For the dynamic properties such as chaotic behavior, bifurcation and so on with respect to system (1), see Reference [19]. For example, the system is chaotic when

(i) $a = 35, \quad b = 10, \quad c = 1, \quad d \in (0, 21.88]$,
(ii) $b = 10, \quad c = 1, \quad d = 10, \quad 23.98 \leqslant a \leqslant 80.65$,
(iii) $a = 30, \quad b = 10, \quad d = 10, \quad 0 < c \leqslant 15.3$,
(iv) $a = 30, \quad c = 1, \quad d = 10, \quad 4.25 < b < 14.3$.

The MPS means that the state vectors of master system with uncertain parameters and slave systems with estimate parameters synchronize up to some nonzero scaling factor α_i, that is, the state vectors of the systems become proportional. Our goal is to make MPS between two 4D chaotic systems by using adaptive control scheme when the parameter of the master system is unknown and different with those of the slave system. For the 4D chaotic system (1), the master (or drive) and slave (or response) systems are defined below, respectively,

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1) + x_2 x_3 x_4, \\ \dot{x}_2 = b(x_1 + x_2) - x_1 x_3 x_4, \\ \dot{x}_3 = -c x_3 + x_1 x_2 x_4, \\ \dot{x}_4 = -d x_4 + x_1 x_2 x_3, \end{cases} \tag{2}$$

and

$$\begin{cases} \dot{y}_1 = a_1(y_2 - y_1) + y_2 y_3 y_4 + u_1, \\ \dot{y}_2 = b_1(y_1 + y_2) - y_1 y_3 y_4 + u_2, \\ \dot{y}_3 = -c_1 y_3 + y_1 y_2 y_4 + u_3, \\ \dot{y}_4 = -d_1 y_4 + y_1 y_2 y_3 + u_4, \end{cases} \tag{3}$$

where x_i and y_i stand for state variables of the master system and the slave one, respectively, a_1, b_1, c_1 and d_1 are uncertain parameters of the slave system which needs to be estimated, and u_1, u_2, u_3 and u_4 are the nonlinear control laws such that two chaotic systems can be synchronized in the sense of MPS, i.e.,

$$\begin{cases} \lim_{t\to\infty} \|x_1 - \alpha_1 y_1\| = 0, \\ \lim_{t\to\infty} \|x_2 - \alpha_2 y_2\| = 0, \\ \lim_{t\to\infty} \|x_3 - \alpha_3 y_3\| = 0, \\ \lim_{t\to\infty} \|x_4 - \alpha_4 y_4\| = 0. \end{cases} \tag{4}$$

Now, define the error signals as

$$\begin{cases} e_1(t) = x_1 - \alpha_1 y_1, \\ e_2(t) = x_2 - \alpha_2 y_2, \\ e_3(t) = x_3 - \alpha_3 y_3, \\ e_4(t) = x_4 - \alpha_4 y_4. \end{cases} \tag{5}$$

From Eq. (5), we have the following error dynamics:

$$\begin{aligned} \dot{e}_1(t) &= a(x_2 - x_1) - \alpha_1 a_1 (y_2 - y_1) + x_2 x_3 x_4 - \alpha_1 y_2 y_3 y_4 - \alpha_1 u_1, \\ \dot{e}_2(t) &= b(x_1 + x_2) - \alpha_2 b_1 (y_1 + y_2) - x_1 x_3 x_4 + \alpha_2 y_1 y_3 y_4 - \alpha_2 u_2, \\ \dot{e}_3(t) &= -c x_3 + \alpha_3 c_1 y_3 + x_1 x_2 x_4 - \alpha_3 y_1 y_2 y_4 - \alpha_3 u_3, \\ \dot{e}_4(t) &= -d x_4 + \alpha_4 d_1 y_4 + x_1 x_2 x_3 - \alpha_4 y_1 y_2 y_3 - \alpha_4 u_4. \end{aligned} \tag{6}$$

For two identical chaotic systems without control ($u_i = 0$), if the initial condition of two systems is different, i.e., $x_i(0) \neq y_i(0)$, the trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two controlled chaotic systems, the two systems will approach synchronization for any initial condition by appropriate control gain and update laws for uncertain parameters. For this goal, the following control laws and update laws for system (3) are designed:

$$\begin{aligned} u_1 &= \frac{1}{\alpha_1} [x_2 x_3 x_4 - \alpha_1 y_2 y_3 y_4 - a_1(\alpha_1 - \alpha_2) y_2 + a_1 e_2 + (k_1 - a_1) e_1], \\ u_2 &= \frac{1}{\alpha_2} [-x_1 x_3 x_4 + \alpha_2 y_1 y_3 y_4 + b_1 e_1 + b_1(\alpha_1 - \alpha_2) y_1 + (b_1 + k_2) e_2], \\ u_3 &= \frac{1}{\alpha_3} [x_1 x_2 x_4 - \alpha_3 y_1 y_2 y_4 + (k_3 - c_1) e_3], \\ u_4 &= \frac{1}{\alpha_4} [x_1 x_2 x_3 - \alpha_4 y_1 y_2 y_3 + (k_4 - d_1) e_4], \end{aligned} \tag{7}$$

and

$$\begin{aligned} \dot{a}_1 &= (x_2 - x_1) e_1, \\ \dot{b}_1 &= (x_1 + x_2) e_2, \\ \dot{c}_1 &= -x_3 e_3, \\ \dot{d}_1 &= -x_4 e_4, \end{aligned} \tag{8}$$

where k_i is the control gains of positive scalars.

Then, we have the following theorem.

Theorem. *For given nonzero scalars* α_i $(i = 1, 2, 3, 4)$, *MPS between two systems* (2) *and* (3) *will occur by the adaptive control law* (7) *and update law* (8).

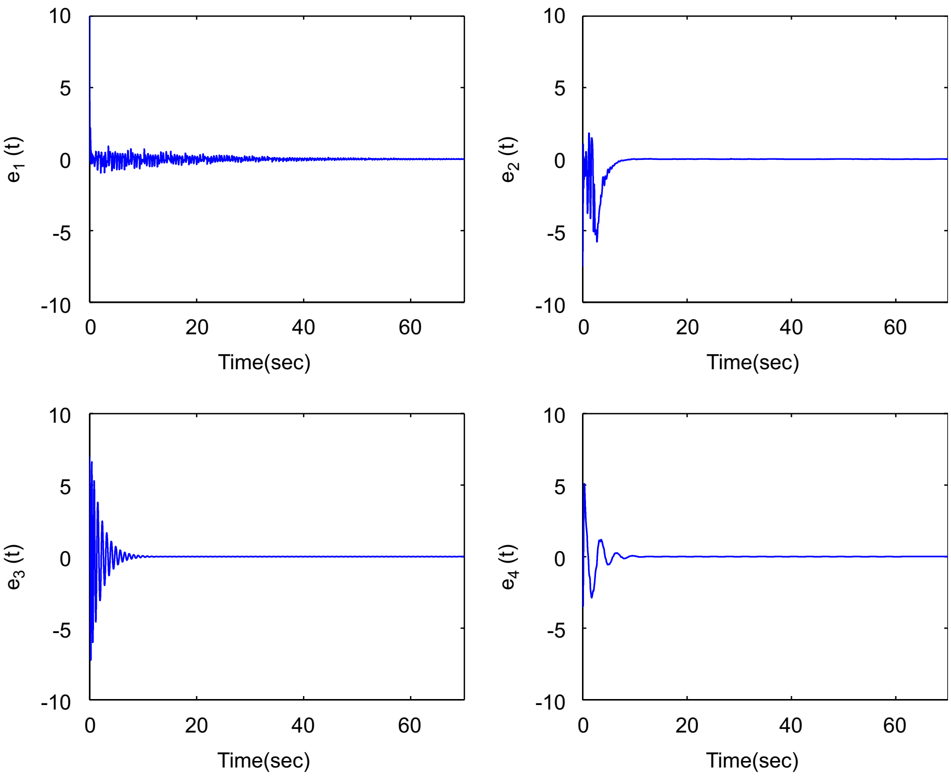

Fig. 1. Synchronization errors.

Proof. Define a Lyapunov candidate

$$V = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + e_4^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2), \tag{9}$$

where $e_a = a_1 - a$, $e_b = b_1 - b$, $e_c = c_1 - c$ and $e_d = d_1 - d$.

The time derivative of the Lyapunov function along the trajectory of error system (5) is

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &= \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + \dot{e}_4 e_4 + \dot{e}_a e_a + \dot{e}_b e_b + \dot{e}_c e_c + \dot{e}_d e_d \\ &= e_1[a(x_2 - x_1) - \alpha_1 a_1 (y_2 - y_1) + x_2 x_3 x_4 - \alpha_1 y_2 y_3 y_4 - \alpha_1 u_1] \\ &\quad + e_2[b(x_1 + x_2) - \alpha_2 b_1 (y_1 + y_2) - x_1 x_3 x_4 + \alpha_2 y_1 y_3 y_4 - \alpha_2 u_2] \\ &\quad + e_3[-c x_3 + \alpha_3 c_1 y_3 + x_1 x_2 x_4 - \alpha_3 y_1 y_2 y_4 - \alpha_3 u_3] \\ &\quad + e_4[-d x_4 + \alpha_4 d_1 y_4 + x_1 x_2 x_3 - \alpha_4 y_1 y_2 y_3 - \alpha_4 u_4] \\ &\quad + \dot{a}_1 (a_1 - a) + \dot{b}_1 (b_1 - b) + \dot{c}_1 (c_1 - c) + \dot{d}_1 (d_1 - d).\end{aligned} \tag{10}$$

By substituting Eqs. (7) and (8) into Eq. (10), we have

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -e^{\mathrm{T}} P e, \tag{11}$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \\ e_3 \\ e_4 \end{bmatrix}, \quad P = \begin{bmatrix} k_1 & 0 & 0 & 0 \\ 0 & k_2 & 0 & 0 \\ 0 & 0 & k_3 & 0 \\ 0 & 0 & 0 & k_4 \end{bmatrix}.$$

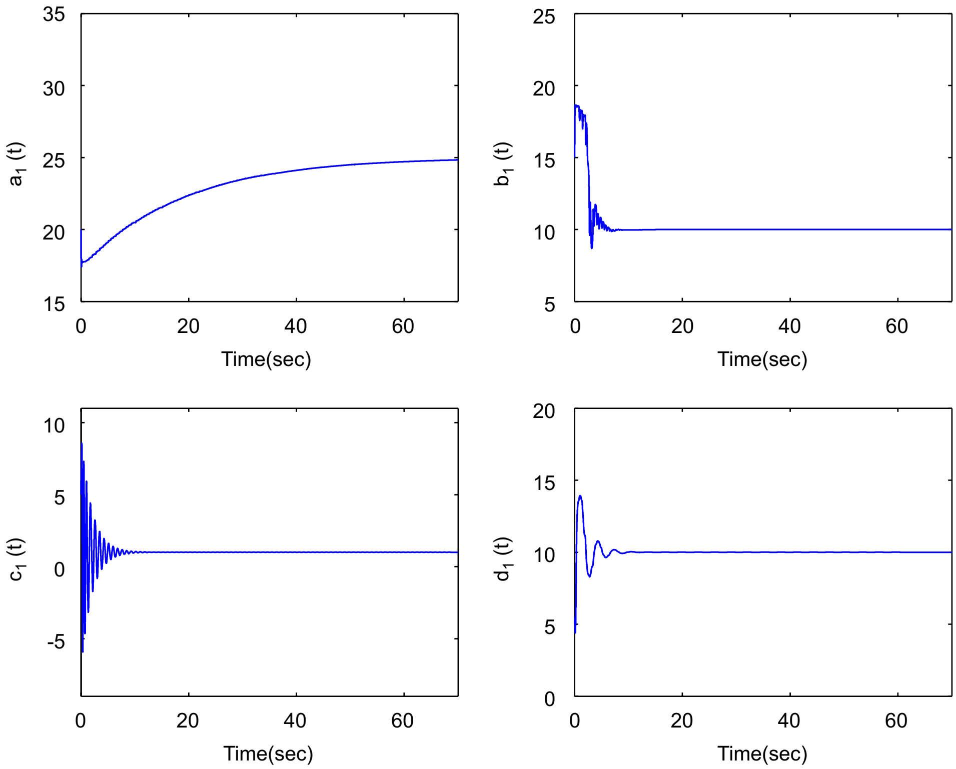

Fig. 2. Estimation of uncertain parameters.

Since $\dot{V}$ is negative semidefinite, we cannot immediately obtain that the origin of error system (5) is asymptotically stable. In fact, as $\dot{V} \leqslant 0$, then $e_1, e_2, e_3, e_4, e_a, e_b, e_c, e_d \in \mathscr{L}_\infty$. From the error system (5), we have $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4 \in \mathscr{L}_\infty$. Since $\dot{V} = -e^{\mathrm{T}} P e$ and P is a positive-definite matrix, then we have

$$\int_0^t \lambda_{\min}(P)\|e\|^2 \,\mathrm{d}t \leqslant \int_0^t e^{\mathrm{T}} P e \,\mathrm{d}t \leqslant \int_0^t -\dot{V} \,\mathrm{d}t = V(0) - V(t) \leqslant V(0),$$

where $\lambda_{\min}(P)$ is the minimum eigenvalue of positive-definite matrix P. Thus $e_1, e_2, e_3, e_4 \in \mathscr{L}_2$. According to the Barbalat's lemma, we have $e_1(t), e_2(t), e_3(t), e_4(t) \to 0$ as $t \to \infty$. Therefore, the slave system (3) synchronizes the master system (2) in the sense of MPS. This completes the proof. □

Remark. The convergence rate of error signals can be adjusted by the control gains k_i.

3. Numerical example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation results for the 4D chaotic system (1). In the numerical simulations, the fourth-order Runge–Kutta method is used to solve the systems with time step size 0.001.

For this numerical simulation, we assume that the initial condition, $(x_1(0), x_2(0), x_3(0), x_4(0)) = (5, -5, -3, 2)$ and $(y_1(0), y_2(0), y_3(0), y_4(0)) = (-5, 5, 5, -5)$, and control gains, $(k_1, k_2, k_3, k_4) = (15, 1, 1, 1)$, are employed. As a test for verification of MPS of the system, let us take $\alpha_1 = 1$, $\alpha_2 = 0.5$, $\alpha_3 = -2$ and $\alpha_4 = -1$. Hence the error system has the initial values $e_1(0) = 10$, $e_2(0) = -7.5$, $e_3(0) = 7$ and $e_4(0) = -3$. The four unknown parameters are chosen as $a = 25$, $b = 10$, $c = 1$ and $d = 10$ in simulations so that system (1) exhibits a chaotic behavior. Synchronization of systems (2) and (3) via adaptive control law (7) and (8) with the initial estimated parameters $a_1(0) = 20$, $b_1(0) = 15$, $c_1(0) = 5$ and $d_1(0) = 5$ are shown in Figs. 1 and 2. Fig. 1 displays the synchronization errors between systems (2) and (3). Fig. 2

shows that the estimates $a_1(t)$, $b_1(t)$, $c_1(t)$, $d_1(t)$ of the unknown parameters converges to $a = 25$, $b = 10$, $c = 1$ and $d = 10$ as $t \to \infty$.

4. Concluding remark

In this article, the MPS for a 4D chaotic system, in which each equation in the system has a 3-term cross product, is investigated. An adaptive control scheme and parameter update rule have been proposed for the synchronization. Then, using the Lyapunov analysis, the stability of error signals for chaos synchronization is proved. Finally, a numerical simulation has been done to show the effectiveness of control scheme proposed.

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