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# Stochastic stability analysis for discrete-time singular Markov jump systems with time-varying delay and piecewise-constant transition probabilities

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#### Abstract

This paper concerns the stochastic stability analysis for discrete-time singular Markov jump systems with time-varying delay and time-varying transition probabilities. The time-varying transition probabilities in the underlying systems are assumed to be finite piecewise-constant. Based on the delay partitioning technique, a delay-dependent stochastic stability condition is derived for these systems, which is formulated by linear matrix inequalities and thus can be checked easily. Some special cases are also considered. Finally, two numerical examples are provided to demonstrate the application and less conservativeness of the developed approaches.

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## 1. Introduction

The analysis and controller design for singular systems have received considerable attention in the past decades, because they can better describe the behavior of some physical systems than state-space ones [1-3], and a great number of fundamental notions

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and results based on the theory of regular systems have been extended to the area of singular systems [3]. On the other hand, the study on time-delay systems has became a topic of great theoretic and practical importance because time-delays are often encountered in various practical systems, such as chemical processes, nuclear reactors, and biological systems, and their existence may lead to instability or significantly deteriorated performances for the corresponding closed-loop systems [4–17]. Thus, singular systems with time-delay have attracted particular interest in the literature, see for instance, [18–21], and the references therein.

On the other hand, Markov jump systems described by a set of linear systems with commutations generated by a finite-state Markov chain are very appropriate and powerful to model changes induced by external causes, e.g., random faults, unexpected events, and uncontrolled configuration changes [22]. Therefore, the study of Markov jump systems with or without time-delay is of great significance and value both theoretically and practically, and a lot of relevant results have been reported in the literature over the past decades, see for instance, [22-27], and the references therein. Recently, as a special class of Markov jump systems, some results on singular Markov jump systems have also been given [28-34]. It should be pointed out that in most existing results on singular Markov jump systems, the considered transition probabilities in the Markov process or Markov chain are assumed to be time-invariant, i.e., the considered Markov process or Markov chain is assumed to be homogeneous. However, the assumption cannot always be satisfied in real applications [35-37], and thus the ideal assumption on transition probabilities inevitably limits the applications of the established results to some extent, although such assumption is definitely expected to simplify the study of Markov jump systems. Therefore it is important and necessary to pay attention to the study of Markov jump systems with time-varying transition probabilities. In [35], the problem of  $\mathcal{H}_{\infty}$ estimation has been investigated for a class of Markov jump systems with time-varying transition probabilities in discrete-time domain, and the mode-dependent and variationdependent filter has been designed such that the resulting closed-loop systems are stochastically stable and have a guaranteed  $\mathcal{H}_{\infty}$  filtering error performance index. In [36], the problem of  $\mathcal{H}_{\infty}$  control for discrete-time Markov jump systems with piecewiseconstant transition probabilities has been investigated by using the average dwell time approach. The stability of Markov jump systems characterized by piecewise-constant transition rates and system dynamics has been investigated in [37], where a sufficient condition has been proposed that guarantees mean square stability under constraints on the dwell-time between switching instants. However, no related results have been established for the stochastically stable analysis of discrete-time singular systems with time-varying transition probabilities and time-delay.

In this paper, the stochastic stability analysis problem is studied for discrete-time singular Markov jump systems with time-varying delay and piecewise-constant transition probabilities. In terms of the delay partitioning approach [38], an linear matrix inequality (LMI)-based delay-dependent stability criterion is established for the considered systems. The given results not only depend upon time-varying delay, but also depend upon the number of delay partitions. Two numerical examples are given to demonstrate the validness and the less conservatism of the obtained results.

*Notation*: The notations used throughout this paper are fairly standard.  $\mathbb{R}^n$  and  $\mathbb{R}^{m \times n}$  denote the *n*-dimensional Euclidean space and the set of all  $m \times n$  real matrices, respectively. The notation X > Y ( $X \ge Y$ ), where X and Y are symmetric matrices, means that X - Y is

positive definite (positive semidefinite). *I* and 0 represent the identity matrix and a zero matrix, respectively. The superscript "T" represents the transpose, and  $\|\cdot\|$  denotes the Euclidean norm of a vector and its induced norm of a matrix.  $(\Omega, \mathcal{F}, \mathcal{P})$  is a probability space,  $\Omega$  is the sample space,  $\mathcal{F}$  is the  $\sigma$ -algebra of subsets of the sample space, and  $\mathcal{P}$  is the probability measure on  $\mathcal{P}$ .  $\mathcal{E}[\cdot]$  denotes the expectation operator with respect to some probability measure  $\mathcal{P}$ . For integers *a* and *b* with a < b,  $\mathbb{N}[a,b] = \{a,a+1,\ldots,b-1,b\}$ . For an arbitrary matrix *B* and two symmetric matrices *A* and *C*,

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix}$$

denotes a symmetric matrix, where "\*" denotes the term that is induced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

#### 2. Preliminaries

Fix a probability space  $(\Omega, \mathcal{F}, \mathcal{P})$ , and consider discrete-time singular Markov jump systems with time-varying delay as

$$\begin{cases} Ex(k+1) = A(r(k))x(k) + A_d(r(k))x(k-d(k)) \\ x(k) = \phi(k), \quad k \in \mathbb{N}[-d_2, 0] \end{cases}$$
(1)

where  $x(k) \in \mathbb{R}^n$  is the state vector, and  $\phi(k)$  is a compatible vector valued initial function. The matrix  $E \in \mathbb{R}^{n \times n}$  may be singular and it is assumed that rank  $E = r \le n$ . A(r(k)) and  $A_d(r(k))$  are known real constant matrices with appropriate dimensions. d(k) denotes time-delay and satisfies

$$0 < d_1 \le d(k) \le d_2 \tag{2}$$

where  $d_1$  and  $d_2$  are known integers.

The parameter r(k) represents a Markov chain taking values in a finite set  $\mathcal{N} = \{1, 2, ..., N\}$  with transition probability matrix  $\Pi^{\sigma(k+1)} = \{\pi_{ij}^{\sigma(k+1)}\}$  given by

$$\Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij}^{\sigma(k+1)}$$
(3)

where  $0 \le \pi_{ij}^{\sigma(k+1)} \le 1$  for all  $i, j \in \mathcal{N}$ , and  $\sum_{j=1}^{N} \pi_{ij}^{\sigma(k+1)} = 1$  for all  $i \in \mathcal{N}$ . Similarly, the parameter  $\sigma(k)$  represents a Markov chain taking values in a finite set  $\mathcal{M} = \{1, 2, \dots, M\}$  with transition probability matrix  $\Sigma = \{q_{lp}\}$  given by

$$\Pr\{\sigma(k+1) = p | \sigma(k) = l\} = q_{lp} \tag{4}$$

where  $0 \le q_{lp} \le 1$  for all  $l, p \in \mathcal{M}$ , and  $\sum_{p=1}^{M} q_{lp} = 1$  for all  $l \in \mathcal{M}$ . In this paper, the Markov chain  $\sigma(k)$  is assumed to be independent on  $\mathcal{F}_{k-1} = \sigma\{r(1), r(2), \dots, r(k-1)\}$ , where  $\mathcal{F}_{k-1}$  is a  $\sigma$ -algebra generated by  $\{r(1), r(2), \dots, r(k-1)\}$  [35].

Before presenting the main results of this paper, we first introduce the following definitions and lemma, which will be essential for our derivation.

**Definition 1** (*Xu and Lam* [3], *Wu et al.* [34]).

1. The system (1) is said to be regular and causal, if the pair  $(E, A_i)$  is regular and causal for any  $i \in \mathcal{N}$  and  $l \in \mathcal{M}$ ,

2. The system (1) is said to be stochastically stable, if for any initial state ( $\phi(k), r_0, \sigma_0$ ), the following condition holds:

$$\lim_{k \to +\infty} \mathcal{E}\{\|x(k)\|^2\} = 0$$
<sup>(5)</sup>

**Definition 2** (*Iosifescu [39], Zhang [35]*). A finite Markov chain  $r(k) \in \mathcal{N}$  is said to be homogeneous (respectively, nonhomogeneous) if for all  $k \ge 0$ , the transition probability satisfies  $\Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij}$  (respectively,  $\Pr\{r(k+1) = j | r(k) = i\} = \pi_{ij}(k)$ ), where  $\pi_{ij}$  (or  $\pi_{ij}(k)$ ) denotes a probability function.

**Remark 1.** According to Definition 2, it can be found that the Markov chain  $\sigma(k)$  in system (1) is homogeneous, while the Markov chain r(k) in system (1) is nonhomogeneous. In this paper, the Markov chain r(k) in system (1) is also called as finite piecewise homogeneous Markov chain, because the considered transition probabilities are time-varying but invariant for the same  $\sigma(k)$ , that is, piecewise-constant.

**Lemma 1** (*Zhu et al.* [40]). For any matrix M > 0, integers  $\gamma_1$  and  $\gamma_2$  satisfying  $\gamma_2 > \gamma_1$ , and vector function  $\omega : \mathbb{N}[\gamma_1, \gamma_2] \to \mathbb{R}^n$ , such that the sums concerned are well defined, then

$$(\gamma_2 - \gamma_1 + 1) \sum_{\alpha = \gamma_1}^{\gamma_2} \omega(\alpha)^{\mathrm{T}} M \omega(\alpha) \ge \sum_{\alpha = \gamma_1}^{\gamma_2} \omega(\alpha)^{\mathrm{T}} M \sum_{\alpha = \gamma_1}^{\gamma_2} \omega(\alpha).$$
(6)

**Lemma 2** (*Park et al. [17], Wu et al. [41]*). For any matrix  $\begin{bmatrix} M & S \\ * & M \end{bmatrix} \ge 0$ , integers  $d_1, d_2, d(k)$  satisfying  $d_1 \le d(k) \le d_2$ , and vector function  $x(k + \cdot) : \mathbb{N}[-d_2, -d_1] \to \mathbb{R}^n$ , such that the sums concerned are well defined, then

$$-d_{12}\sum_{\alpha=k-d_2}^{k-d_1-1}\zeta(\alpha)^{\mathrm{T}}M\zeta(\alpha) \le \varpi(k)^{\mathrm{T}}\Omega\varpi(k)$$
(7)

where  $d_{12} = d_2 - d_1$ ,  $\zeta(\alpha) = x(\alpha + 1) - x(\alpha)$  and

$$\varpi(k) = [x(k-d_1)^{\mathrm{T}} \ x(k-d(k))^{\mathrm{T}} \ x(k-d_2)^{\mathrm{T}}]^{\mathrm{T}} \ \Omega = \begin{bmatrix} -M & M-S & S \\ * & -2M+S+S^{\mathrm{T}} & -S+M \\ * & * & -M \end{bmatrix}$$

In this paper, we will focus on the problem of stability analysis for discrete-time singular Markov jump system (1). By using a Lyapunov functional, we will develop an LMI approach to derive sufficient condition under which the underlying system (1) is regular, causal and stochastically stable.

## 3. Main results

In this section, the stability is discussed for system (1) by the delay partitioning approach. Before proceeding further, for the sake's of vector and matrix representation, the followings are denoted:

$$\mathcal{X}(t) = \left[ x(k)^{\mathrm{T}} x \left( k - \frac{1}{m} d_1 \right)^{\mathrm{T}} x \left( k - \frac{2}{m} d_1 \right)^{\mathrm{T}} \dots x \left( k - \frac{m-1}{m} d_1 \right)^{\mathrm{T}} \right]^{\mathrm{T}}$$

$$\eta(k) = [\Upsilon(k)^{T} \ x(k-d_{1})^{T}]^{T}$$
  

$$\zeta(k) = [\eta(k)^{T} \ x(k-d(k))^{T} \ x(k-d_{2})^{T}]^{T}$$
  

$$W_{1} = [I_{mn} \ 0_{mn \times n}]$$
  

$$W_{2} = [0_{mn \times n} \ I_{mn}]$$
  

$$g_{l} = [0_{n \times (l-1)n} \ I_{n} \ 0_{n \times (m-l+1)n}], \quad l = 1, 2, \dots, m+1$$

This way, system (1) can be rewritten as

$$\begin{cases} Ex(k+1) = A_i g_1 \eta(k) + A_{di} x(k-d(k)) \\ x(k) = \phi(k), \quad k \in \mathbb{N}[-d_2, 0] \end{cases}$$
(8)

**Theorem 1.** Given an integer m > 0, system (1) is regular, causal and stochastically stable, if there exist matrices  $P_{i,l} > 0$ , Q > 0,  $Z_1 > 0$ ,  $Z_2 > 0$ , U > 0,  $S_i > 0$  (i = 1, 2, ..., m+),  $\mathcal{Y}_{i,l}$ , and  $W_{i,l}$  such that for any  $i \in \mathcal{N}$  and  $l \in \mathcal{M}$ ,

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & g_1^{\mathrm{T}} A_i^{\mathrm{T}} X_i^{l} \\ * & \Xi_{22} & \Xi_{23} & A_{di}^{\mathrm{T}} \mathcal{D} & A_{di}^{\mathrm{T}} X_i^{l} \\ * & * & \Xi_{33} & 0 & 0 \\ * & * & * & -\mathcal{D} & 0 \\ * & * & * & * & -\mathcal{X}_i^{l} \end{bmatrix} < 0$$

$$\begin{bmatrix} S_{m+1} & \mathcal{Y}_{i,l} \\ * & S_{m+1} \end{bmatrix} \ge 0$$
(10)

where  $d_{12} = d_2 - d_1$ ,  $X_i^l = \sum_{p=1}^M q_{lp} \sum_{j=1}^N \pi_{ij}^p P_{j,p}$ ,  $R \in \mathbb{R}^{n \times (n-r)}$  is any matrix with full column and satisfies  $E^T R = 0$ , and  $\mathcal{D} = (d_1/m)^2 \sum_{i=1}^m S_i + d_{12}^2 S_{m+1}$  and

$$\begin{split} \Xi_{11} &= -g_{1}^{\mathrm{T}} E^{\mathrm{T}} P_{i,l} Eg_{1} + W_{1}^{\mathrm{T}} Q W_{1} - W_{2}^{\mathrm{T}} Q W_{2} - \sum_{i=1}^{m} (g_{i} - g_{i+1})^{\mathrm{T}} E^{\mathrm{T}} S_{i} E(g_{i} - g_{i+1}) \\ &+ g_{m+1}^{\mathrm{T}} Z_{1} g_{m+1} + (d_{12} + 1) g_{m+1}^{\mathrm{T}} Z_{2} g_{m+1} - g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} S_{m+1} E g_{m+1} \\ &+ g_{1}^{\mathrm{T}} W_{i,l} R^{\mathrm{T}} A_{i} g_{1} + g_{1}^{\mathrm{T}} A_{i}^{\mathrm{T}} R W_{i,l}^{\mathrm{T}} g_{1} \\ \Xi_{12} &= g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} S_{m+1} E - g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} \mathcal{Y}_{i,l} E + g_{1}^{\mathrm{T}} W_{i,l} R^{\mathrm{T}} A_{di} \\ \Xi_{13} &= g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} \mathcal{Y}_{i,l} E \\ \Xi_{14} &= g_{1}^{\mathrm{T}} (A_{i} - E)^{\mathrm{T}} \mathcal{D} \\ \Xi_{22} &= -Z_{2} - 2E^{\mathrm{T}} S_{m+1} E + E^{\mathrm{T}} \mathcal{Y}_{i,l} E + E^{\mathrm{T}} \mathcal{Y}_{i,l}^{\mathrm{T}} E \\ \Xi_{23} &= -E^{\mathrm{T}} \mathcal{Y}_{i,l} E + E^{\mathrm{T}} S_{m+1} E \\ \Xi_{33} &= -Z_{1} - E^{\mathrm{T}} S_{m+1} E \end{split}$$

**Proof.** We first proof the regularity and causality of system (1) under the given condition. To this end, we choose two nonsingular matrices M and G such that

$$MEG = \begin{bmatrix} I_r & 0\\ 0 & 0 \end{bmatrix}$$
(11)

Set

$$MA_i G = \begin{bmatrix} A_{1i} & A_{2i} \\ A_{3i} & A_{4i} \end{bmatrix}, \quad G^{\mathsf{T}} W_{i,l} = \begin{bmatrix} W_{i,l}^1 \\ W_{i,l}^2 \end{bmatrix}, \quad M^{-\mathsf{T}} R = \begin{bmatrix} 0 \\ I \end{bmatrix} F$$
(12)

where  $F \in \mathbb{R}^{(n-r) \times (n-r)}$  is any nonsingular matrix. It can be seen that  $\Xi_{11} < 0$  implies

$$-E^{\mathrm{T}}P_{i,l}E + W_{i,l}R^{\mathrm{T}}A_{i} + A_{i}^{\mathrm{T}}RW_{i,l}^{\mathrm{T}} - E^{\mathrm{T}}S_{1}E < 0$$
(13)

Pre-multiplying and post-multiplying Eq. (13) by  $G^{T}$  and G, respectively, we have  $W_{i,l}^{2}F^{T}A_{4i} + A_{4i}^{T}FW_{i,l}^{2T} < 0$ , which implies  $A_{4i}$  is nonsingular. Thus, the pair (E, A) is regular and causal, which implies system (1) is regular and causal.

Next we will show that system (1) is stochastically stable under the given condition. To the end, we define  $\delta(k) = x(k+1)-x(k)$  and consider the following Lyapunov functional for system (1):

$$V(x(k),k,r(k),\sigma(k)) = \sum_{l=1}^{4} V_l(x(k),k,r(k),\sigma(k))$$
(14)

where

$$V_{1}(x(k),k,r(k),\sigma(k)) = x(k)^{\mathrm{T}}E^{\mathrm{T}}P_{r(k),\sigma(k)}Ex(k)$$

$$V_{2}(x(k),k,r(k),\sigma(k)) = \sum_{s=k-d_{1}/m}^{k-1} \Upsilon(s)^{\mathrm{T}}Q\Upsilon(s)$$

$$V_{3}(x(k),k,r(k),\sigma(k)) = \sum_{s=k-d_{2}}^{k-d_{1}-1} x(s)^{\mathrm{T}}Z_{1}x(s) + \sum_{\alpha=-d_{2}+1}^{-d_{1}+1} \sum_{s=k-1+\alpha}^{k-d_{1}-1} x(s)^{\mathrm{T}}Z_{2}x(s)$$

$$V_{4}(x(k),k,r(k),\sigma(k)) = \frac{d_{1}}{m} \sum_{i=1}^{m} \sum_{g=-(i/m)d_{1}}^{-((i-1)/m)d_{1}-1} \sum_{s=k+g}^{k-1} \delta(s)^{\mathrm{T}}E^{\mathrm{T}}S_{i}E\delta(s)$$

$$+ d_{12} \sum_{g=-d_{2}}^{-d_{1}-1} \sum_{s=k+g}^{k-1} \delta(s)^{\mathrm{T}}E^{\mathrm{T}}S_{m+1}E\delta(s)$$

Letting  $\mathcal{E}[\Delta V(k)] = \mathcal{E}[V(k+1,x(k+1),r(k+1),\sigma(k+1)|x(k),r(k) = i,\sigma(k) = l) - V(k,x(k), i,l)]$ , along the solution of system (1), we have that

$$\mathcal{E}[\Delta V_1(k)] = x(k+1)^{\mathrm{T}} E^{\mathrm{T}} \hat{\Lambda}_{il} E x(k+1) - x(k)^{\mathrm{T}} E^{\mathrm{T}} P_{i,l} E x(k)$$
(15)

where

$$\hat{A}_{il} = \sum_{p=1}^{M} \sum_{j=1}^{N} \Pr\{r(k+1) = j, \sigma(k+1) = p | r(k) = i, \sigma(k) = l\} P_{j,p}.$$

It is noted that

 $\Pr\{\sigma(k+1) = p | r(k) = i, \sigma(k) = l\} = q_{lp}$ 

and thus

$$\Pr\{r(k+1) = j, \sigma(k+1) = p | r(k) = i, \sigma(k) = l\} = \Pr\{r(k+1) = j | r(k) = i, \sigma(k) = l, \sigma(k+1) = p\} \Pr\{\sigma(k+1) = p | r(k) = i, \sigma(k) = l\} = \pi_{ij}^p q_{lp}$$

which means  $\hat{A}_{il} = X_i^l$ . Therefore, we have that  $\mathcal{E}[\Delta V_1(k)] = x(k+1)^{\mathrm{T}} E^{\mathrm{T}} X_i^l E x(k+1) - x(k)^{\mathrm{T}} E^{\mathrm{T}} P_{i,l} E x(k) = x(k+1)^{\mathrm{T}} E^{\mathrm{T}} X_i^l E x(k+1)$  $-\eta(k)^{\mathrm{T}} g_1^{\mathrm{T}} E^{\mathrm{T}} P_{i,l} E g_1 \eta(k) = (A_i g_1 \eta(k) + A_{di} x(k-d(k)))^{\mathrm{T}} X_i^l (A_i g_1 \eta(k) + A_{di} x(k-d(k))) - \eta(k)^{\mathrm{T}} g_1^{\mathrm{T}} E^{\mathrm{T}} P_{i,l} E g_1 \eta(k)$ (16)

On the other hand,

$$\mathcal{E}[\Delta V_2(k)] = \Upsilon(k)^{\mathrm{T}} Q \Upsilon(k) - \Upsilon\left(k - \frac{d_1}{m}\right)^{\mathrm{T}} Q \Upsilon\left(k - \frac{d_1}{m}\right)$$
$$= \eta(k)^{\mathrm{T}} W_1^{\mathrm{T}} Q W_1 \eta(k) - \eta(k)^{\mathrm{T}} W_2^{\mathrm{T}} Q W_2 \eta(k)$$
(17)

$$\mathcal{E}[\Delta V_{3}(k)] = x(k-d_{1})^{\mathrm{T}} Z_{1} x(k-d_{1}) - x(k-d_{2})^{\mathrm{T}} Z_{1} x(k-d_{2}) + (d_{12}+1) x(k-d_{1})^{\mathrm{T}} Z_{2} x(k-d_{1}) - \sum_{s=k-d_{2}}^{k-d_{1}} x(s)^{\mathrm{T}} Z_{2} x(s) \le \eta(k)^{\mathrm{T}} g_{m+1}^{\mathrm{T}} Z_{1} g_{m+1} \eta(k) - x(k-d_{2})^{\mathrm{T}} Z_{1} x(k-d_{2}) + (d_{12}+1) \eta(k)^{\mathrm{T}} g_{m+1}^{\mathrm{T}} Z_{2} g_{m+1} \eta(k) - x(k-d(k))^{\mathrm{T}} Z_{2} x(k-d(k))$$
(18)

$$\mathcal{E}[\Delta V_4(k)] = \left(\frac{d_1}{m}\right)^2 \sum_{i=1}^m \delta(k)^{\mathrm{T}} E^{\mathrm{T}} S_i E \delta(k) - \frac{d_1}{m} \sum_{i=1}^m \sum_{s=k-(i/m)d_1}^{k-((i-1)/m)d_1-1} \delta(s)^{\mathrm{T}} E^{\mathrm{T}} S_i E \delta(s) + d_{12}^2 \delta(k)^{\mathrm{T}} E^{\mathrm{T}} S_{m+1} E \delta(k) - d_{12} \sum_{s=k-d_2}^{k-d_1-1} \delta(s)^{\mathrm{T}} E^{\mathrm{T}} S_{m+1} E \delta(s) \le ((A_i - E)g_1\eta(k) + A_{di} x(k-d(k)))^{\mathrm{T}} \mathcal{D}((A_i - E)g_1\eta(k) + A_{di} x(k-d(k)))$$

$$-\sum_{i=1}^{m} \eta(k)^{\mathrm{T}} (g_{i} - g_{i+1})^{\mathrm{T}} E^{\mathrm{T}} S_{i} E(g_{i} - g_{i+1}) \eta(k) + \begin{bmatrix} \eta(k) \\ x(k - d(k)) \\ x(k - d_{2}) \end{bmatrix}^{\mathrm{T}} \Gamma \begin{bmatrix} \eta(k) \\ x(k - d(k)) \\ x(k - d_{2}) \end{bmatrix}$$
(19)

where Lemmas 1 and 2 are applied, and

$$\Gamma = \begin{bmatrix} -g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} S_{m+1} E g_{m+1} & g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} S_{m+1} E - g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} \mathcal{Y}_{i,l} E & g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} \mathcal{Y}_{i,l} E \\ * & -2E^{\mathrm{T}} S_{m+1} E + E^{\mathrm{T}} \mathcal{Y}_{i,l} E + E^{\mathrm{T}} \mathcal{Y}_{i,l}^{\mathrm{T}} E & -E^{\mathrm{T}} \mathcal{Y}_{i,l} E + E^{\mathrm{T}} S_{m+1} E \\ * & * & -E^{\mathrm{T}} S_{m+1} E \end{bmatrix}$$

Furthermore, it can be easily obtained from Eq. (8) that

$$2\eta(k)^{\mathrm{T}}g_{1}^{\mathrm{T}}W_{i,l}R^{\mathrm{T}}(A_{i}g_{1}\eta(k) + A_{di}x(k-d(k))) = 0$$
<sup>(20)</sup>

Thus, adding the left-hand side of Eq. (20) to  $\mathcal{E}[\Delta V(k)]$ , we can get from Eqs. (16)–(19) that

$$\mathcal{E}[\Delta V(k)] \le \zeta(k)^T \Theta \zeta(k) \tag{21}$$

where

$$\Theta = \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} \\ * & \Xi_{22} & \Xi_{23} \\ * & * & \Xi_{33} \end{bmatrix} + \begin{bmatrix} g_1^{\mathrm{T}} A_i^{\mathrm{T}} \\ A_{di}^{\mathrm{T}} \\ 0 \end{bmatrix} X_i^l \begin{bmatrix} g_1^{\mathrm{T}} A_i^{\mathrm{T}} \\ A_{di}^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}} + \begin{bmatrix} g_1^{\mathrm{T}} (A_i - E)^{\mathrm{T}} \\ A_{di}^{\mathrm{T}} \\ 0 \end{bmatrix} \mathcal{D} \begin{bmatrix} g_1^{\mathrm{T}} (A_i - E)^{\mathrm{T}} \\ A_{di}^{\mathrm{T}} \\ 0 \end{bmatrix}^{\mathrm{T}}$$

Based on Schur complement, we can obtain from Eq. (9) that  $\Theta < 0$ . Therefore, there exists a scalar  $\alpha > 0$  such that

$$\mathcal{E}[\Delta V(k)] \le -\alpha \|x(k)\|^2 \tag{22}$$

Thus, we can conclude that

$$\sum_{i=0}^{\kappa} \mathcal{E}[\|x(i)\|^2] \le \frac{1}{\alpha} \mathcal{E}[V(0)] < \infty$$
(23)

which implies  $\sum_{i=0}^{\infty} \mathcal{E}[||x(i)||^2]$  converge, and thus it can be found that Eq. (5) holds. Based on Definition 1, system (1) is stochastically stable. This completes the proof.  $\Box$ 

**Remark 2.** Theorem 1 gives a criterion guaranteeing the regularity, causality and stochastic stability of system (1), which is formulated by LMIs and can readily be solved by standard numerical software. It should be pointed out that the condition is independent of the choice of R, which is introduced by Eq. (20).

When  $\mathcal{M} = \{1\}$ , the piecewise homogeneous Markov jump system (1) reduces to a homogeneous Markov jump system, and Theorem 1 reduces the following corollary.

**Corollary 1.** Given an integer m > 0, system (1) is regular, causal and stochastically stable, if there exist matrices  $P_i > 0$ , Q > 0,  $Z_1 > 0$ ,  $Z_2 > 0$ , U > 0,  $S_i > 0$  (i = 1, 2, ..., m + 1),  $\mathcal{Y}_i$ , and  $W_i$  such that for any  $i \in \mathcal{N}$ :

$$\begin{bmatrix} \hat{\Xi}_{11} & \hat{\Xi}_{12} & \hat{\Xi}_{13} & \Xi_{14} & g_1^{\mathrm{T}} A_i^{\mathrm{T}} X_i \\ * & \hat{\Xi}_{22} & \hat{\Xi}_{23} & A_{di}^{\mathrm{T}} \mathcal{D} & A_{di}^{\mathrm{T}} X_i \\ * & * & \Xi_{33} & 0 & 0 \\ * & * & * & -\mathcal{D} & 0 \\ * & * & * & * & -\mathcal{X}_i \end{bmatrix} < 0$$

$$(24)$$

$$\begin{bmatrix} S_{m+1} & \mathcal{Y}_i \\ * & S_{m+1} \end{bmatrix} \ge 0 \tag{25}$$

where  $X_i = \sum_{j=1}^N \pi_{ij} P_j$ , and

$$\begin{split} \hat{\Xi}_{11} &= -g_{1}^{\mathrm{T}} E^{\mathrm{T}} P_{i} E g_{1} + W_{1}^{\mathrm{T}} Q W_{1} - W_{2}^{\mathrm{T}} Q W_{2} - \sum_{\iota=1}^{m} (g_{\iota} - g_{\iota+1})^{\mathrm{T}} E^{\mathrm{T}} S_{\iota} E(g_{\iota} - g_{\iota+1}) \\ &+ g_{m+1}^{\mathrm{T}} Z_{1} g_{m+1} + (d_{12} + 1) g_{m+1}^{\mathrm{T}} Z_{2} g_{m+1} - g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} S_{m+1} E g_{m+1} + g_{1}^{\mathrm{T}} W_{i} R^{\mathrm{T}} A_{i} g_{1} \\ &+ g_{1}^{\mathrm{T}} A_{i}^{\mathrm{T}} R W_{i}^{\mathrm{T}} g_{1} \\ \hat{\Xi}_{12} &= g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} S_{m+1} E - g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} \mathcal{Y}_{i} E + g_{1}^{\mathrm{T}} W_{i} R^{\mathrm{T}} A_{di} \\ \hat{\Xi}_{13} &= g_{m+1}^{\mathrm{T}} E^{\mathrm{T}} \mathcal{Y}_{i} E \end{split}$$

$$\hat{\Xi}_{22} = -Z_2 - 2E^{\mathrm{T}}S_{m+1}E + E^{\mathrm{T}}\mathcal{Y}_iE + E^{\mathrm{T}}\mathcal{Y}_i^{\mathrm{T}}E$$
$$\hat{\Xi}_{23} = -E^{\mathrm{T}}\mathcal{Y}_iE + E^{\mathrm{T}}S_{m+1}E$$

the other parameters follow the same definitions as those in Theorem 1.

**Remark 3.** When  $\mathcal{N} = \{1\}$ , which means that Markov jumping parameters disappear, Corollary 1 reduces to Corollary 1 in [41]. Therefore, Theorem 1 and Corollary 1 can be viewed as an extension of the existing result on stability analysis for discrete-time singular time-delay systems to discrete-time singular time-delay systems with Markov jumping parameters.

Generally speaking, for some practical systems, the transition probabilities of the Markov chain we get will never be precise, that is, some elements in transition probability matrix are unknown [42–45]. For instance, when  $\mathcal{M} = \{1, 2, 3, 4\}$ , the transition probability matrix  $\Sigma$  may be

 $\Sigma = \begin{bmatrix} q_{11} & \hat{q}_{12} & q_{13} & \hat{q}_{14} \\ \hat{q}_{21} & \hat{q}_{22} & \hat{q}_{23} & q_{24} \\ q_{31} & \hat{q}_{32} & q_{33} & \hat{q}_{34} \\ \hat{q}_{33} & \hat{q}_{34} & q_{33} & q_{34} \end{bmatrix}$ 

where the unaccessible elements are labeled with a hat "?". For notation clarity, we denote that for each  $l \in \mathcal{M}$ :

$$\mathcal{M}_{\mathcal{K}}^{l} = \{p : q_{lp} \text{ is known}\}$$
$$\mathcal{M}_{\mathcal{U}\mathcal{K}}^{l} = \{p : q_{lp} \text{ is unknown}\}$$

Also, we denote  $q_{\mathcal{K}}^l = \sum_{p \in \mathcal{M}_{\mathcal{K}}^l} q_{lp}$ . The following corollary presents the result on the stability analysis for system (1) with partially unknown transition probabilities.

**Corollary 2.** Given an integer m > 0, system (1) with partially unknown transition probabilities is regular, causal and stochastically stable, if there exist matrices  $P_{i,l} > 0$ ,  $Q > 0, Z_1 > 0, Z_2 > 0, U > 0, S_i > 0$  (i = 1, 2, ..., m + 1),  $\mathcal{Y}_{i,l}$ , and  $W_{i,l}$  such that for any  $i \in \mathcal{N}$ and  $l \in \mathcal{M}$ , Eqs. (11) and (27) hold,

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & g_1^{\mathrm{T}} A_i^{\mathrm{T}} (\hat{X}_i^{l} + \check{X}_i^{lp}) \\ * & \Xi_{22} & \Xi_{23} & A_{di}^{\mathrm{T}} \mathcal{D} & A_{di}^{\mathrm{T}} (\hat{X}_i^{l} + \check{X}_i^{lp}) \\ * & * & \Xi_{33} & 0 & 0 \\ * & * & * & -\mathcal{D} & 0 \\ * & * & * & * & -\hat{X}_i^{l} - \check{X}_i^{lp} \end{bmatrix} < 0$$

$$(27)$$

where  $p \in \mathcal{M}_{U\mathcal{K}}^{l}$  and

$$\hat{X}_{i}^{l} = \sum_{p \in \mathcal{M}_{\mathcal{K}}^{l}} q_{lp} \sum_{j=1}^{N} \pi_{ij}^{p} P_{j,p}$$

(26)

$$\check{X}_{i}^{lp} = (1 - q_{\mathcal{K}}^{l}) \sum_{j=1}^{N} \pi_{ij}^{p} P_{j,p}$$

the other parameters follow the same definitions as those in Theorem1.

**Proof.** It is clear that  $0 \le q_{\mathcal{K}}^l \le 1$ . we exclude the case of  $q_{\mathcal{K}}^l = 1$  due to the fact that in this case all the elements in the *l*th row are known. When  $0 \le q_{\mathcal{K}}^l < 1$ , it can be found from Eq. (9) that

$$\begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & g_{1}^{\mathrm{T}} A_{i}^{\mathrm{T}} \left( \hat{X}_{i}^{l} + \sum_{p \in \mathcal{M}_{ld\mathcal{K}}^{l}} \hat{q}_{lp} \sum_{j=1}^{N} \pi_{ij}^{p} P_{j,p} \right) \\ * & \Xi_{22} & \Xi_{23} & A_{di}^{\mathrm{T}} \mathcal{D} & A_{di}^{\mathrm{T}} \left( \hat{X}_{i}^{l} + \sum_{p \in \mathcal{M}_{ld\mathcal{K}}^{l}} \hat{q}_{lp} \sum_{j=1}^{N} \pi_{ij}^{p} P_{j,p} \right) \\ * & * & \Xi_{33} & 0 & 0 \\ * & * & * & -\mathcal{D} & 0 \\ * & * & * & * & -\mathcal{D} & 0 \\ * & * & * & * & -\hat{X}_{i}^{l} - \sum_{p \in \mathcal{M}_{ld\mathcal{K}}^{l}} \hat{q}_{lp} \sum_{j=1}^{N} \pi_{ij}^{p} P_{j,p} \end{bmatrix} < 0$$

$$(28)$$

According to the fact that  $0 \le \hat{q}_{lp}/(1-q_{\mathcal{K}}^l) \le 1$  and  $\sum_{p \in \mathcal{M}_{\mathcal{U}\mathcal{K}}^l} \hat{q}_{lp}/(1-q_{\mathcal{K}}^l) = 1$ , we have that

$$\Lambda_{il} = \sum_{p \in \mathcal{M}_{ld\mathcal{K}}^{l}} \frac{\hat{q}_{lp}}{1 - q_{\mathcal{K}}^{l}} \begin{bmatrix} \Xi_{11} & \Xi_{12} & \Xi_{13} & \Xi_{14} & g_{1}^{\mathsf{T}} A_{i}^{\mathsf{T}} (\hat{X}_{i}^{l} + \check{X}_{i}^{lp}) \\ * & \Xi_{22} & \Xi_{23} & A_{di}^{\mathsf{T}} \mathcal{D} & A_{di}^{\mathsf{T}} (\hat{X}_{i}^{l} + \check{X}_{i}^{lp}) \\ * & * & \Xi_{33} & 0 & 0 \\ * & * & * & -\mathcal{D} & 0 \\ * & * & * & * & -\hat{X}_{i}^{l} - \check{X}_{i}^{lp} \end{bmatrix}$$
(29)

Thus, Eq. (27) holds implies Eq. (29) holds. This completes the proof.  $\Box$ 

## 4. Numerical examples

This section presents two numerical examples that demonstrate the effectiveness of the methods described in the above section.

Example 1. Consider system (1) with

$$A_{1} = \begin{bmatrix} 6.1 & 10.4 \\ 7.15 & 11.6 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} -1.1 & -2 \\ -1.4 & -2.5 \end{bmatrix}$$
$$A_{2} = \begin{bmatrix} 6.37 & 10.74 \\ 7.48 & 12.01 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.92 & -1.62 \\ -1.13 & -1.93 \end{bmatrix}$$
$$E = \begin{bmatrix} 3 & 6 \\ 2 & 4 \end{bmatrix}$$

$d_1$	2	4	6	8	10	12
[29,31]	12	13	14	15	16	17
[30]	14	14	15	16	18	19
[28]	14	15	16	17	18	19
Corollary 1 ( $m = d_1$ )	18	19	21	23	25	27

Table 1 Comparison of the allowable upper bound  $d_2$ .

and the transition probability matrix

 $\Pi^1 = \begin{bmatrix} 0.45 & 0.55 \\ 0.7 & 0.3 \end{bmatrix}$ 

By using the methods of [28–31] and Corollary 1 in this paper, the allowable maximum values of  $d_2$  for various  $d_1$  that guarantees the regularity, causality and stochastic stability of the considered system are presented in Table 1, from which we can find that our result has less conservatism than those in [28–31].

**Example 2.** Consider the dynamic Leontief model of economic systems, which describes the time pattern of production sectors given by [2]

$$x(k) = Mx(k) + G(x(k+1) - x(k)) + Hu(k)$$
(30)

It is clear that Eq. (30) can be rewritten as

$$Gx(k+1) = (I - M + G)x(k) - Hu(k)$$
(31)

Typically the capital coefficient matrix G has nonzero elements in only a few rows, corresponding to the fact that capital is formed from only a few sectors. Thus, the system (31) is a typical discrete-time singular system, since G is often singular.

In this example, we choose

-

$$u(k) = K_1 x(k) + K_2 x(k - d(k))$$
(32)

In practical control system, actuators may fail during the course of system operation and the faults of the actuators may be random in nature. We make use of the following fault model to represent the stochastic behavior of the actuator faults:

$$u^{F}(k) = F(r(k))u(k)$$
(33)

where  $F(r(k)) = \text{diag}\{f_1(r(k)), f_2(r(k)), \dots, f_p(r(k))\}, 0 \le f_q(r(k)) \le 1 \quad (q = 1, 2, \dots, p), \forall r(k) \in \mathcal{N}.$  Obviously, when  $f_q(r(k)) = 0$ , the fault model (33) corresponds to the *q*-th actuator outage case. When  $0 < f_q(r(k)) < 1$ , it corresponds to the case of partial failure of the *q*-th actuator. When  $f_q(r(k)) = 1$ , it corresponds to the case of no fault in the *q*-th actuator.

Here, we consider a Leontief model described by

$$G = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \quad M = \begin{bmatrix} 2.04 & 1 \\ 0.8 & 1 \end{bmatrix}, \quad H = \begin{bmatrix} -1 \\ 3.05 \end{bmatrix}$$

Then system (31) can be rewritten as

$$\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} x(k+1) = \begin{bmatrix} -0.04 & -1 \\ -0.8 & 0 \end{bmatrix} x(k) - \begin{bmatrix} -1 \\ 3.05 \end{bmatrix} u(k)$$
(34)

On the other hand, choose

$$K_1 = [-0.01 \ 0.6], \quad K_2 = [0.1676 \ 0.1170]$$
 (35)

and  $F_1=0.3$  and  $F_2=0.8$ , and suppose the transition probability matrices

$$\Pi^{1} = \begin{bmatrix} 0.1 & 0.9 \\ 0.3 & 0.7 \end{bmatrix}, \quad \Pi^{2} = \begin{bmatrix} 0.2 & 0.8 \\ 0.45 & 0.55 \end{bmatrix}$$
$$\Pi^{3} = \begin{bmatrix} 0.3 & 0.7 \\ 0.55 & 0.45 \end{bmatrix}, \quad \Pi^{4} = \begin{bmatrix} 0.4 & 0.6 \\ 0.5 & 0.5 \end{bmatrix}$$

and

$$\Sigma = \begin{bmatrix} 0.3 & 0.2 & 0.1 & 0.4 \\ 0.3 & 0.2 & 0.3 & 0.2 \\ 0.1 & 0.1 & 0.5 & 0.3 \\ 0.2 & 0.2 & 0.1 & 0.5 \end{bmatrix}$$

Thus, the resultant closed-loop system can be described by discrete-time singular Markov jump system (1) with

$$A_{1} = \begin{bmatrix} -0.0430 & -0.8200 \\ -0.7909 & -0.5490 \end{bmatrix}, \quad A_{d1} = \begin{bmatrix} 0.0503 & 0.0351 \\ -0.1534 & -0.1071 \end{bmatrix}$$
$$A_{2} = \begin{bmatrix} -0.0480 & -0.5200 \\ -0.7756 & -1.4640 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.1341 & 0.0936 \\ -0.4089 & -0.2855 \end{bmatrix}$$
$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$$

It is assumed that  $d_1 = 4$  and  $d_2 = 6$ . Applying Matlab Toolbox, by Theorem 1 with m=2, it is found that system (1) with given parameters is regular, causal and stochastically stable.

## 5. Conclusions

The problem of stochastic stability analysis has been investigated in this paper for discrete-time singular Markov jump systems with time-varying delay and piecewise-constant transition probabilities. Based on the delay partitioning technique, a Lyapunov functional has been introduced to arrive at the delay-dependent sufficient condition that warrants the regularity, causality, and stochastic stability of the considered systems. The obtained delay-dependent results rely upon the partitioning size. The results on some special cases have also been established. Finally, two numerical examples have been given to show the reduction of conservatism and effectiveness of the developed approaches. We would like to point out that it is possible to extend our main results to more general discrete-time singular Markov jump systems with parameter uncertainties, mixed time-delays, and nonlinear disturbances. The results will appear in the near future.

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