

Decentralized Guaranteed Cost Dynamic Control for Synchronization of a Complex Dynamical Network

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Abstract: This paper considers synchronization problem of a complex dynamical network. For this problem, a decentralized guaranteed cost dynamic feedback controller is designed to achieve the synchronization of the network. Based on Lyapunov stability theory and linear matrix inequality framework, the existence condition for feasible controllers is derived in terms of linear matrix inequalities. Finally, the proposed method is applied to two numerical examples in order to show the effectiveness of our result.

Keywords: Complex network, Synchronization, Guaranteed cost controller, Dynamic controller, Decentralized controller.

1. INTRODUCTION

Complex dynamical networks have received a great deal of attention since they are shown to widely exist in various fields of real world, such as Internet, World Wide Web, food webs, electric power grids, relationship networks, disease transmission networks, scientific citation networks, social networks and so on, which have become a focal research topic and have drawn much attention from researchers working in different fields [1]-[2]. For these reasons, the complex dynamical network has already been one of the most popular topics in the areas of scientific research in recent years. In particular, one significant and interesting phenomenon is the synchronization of all its dynamics. Until now, in order to treat the synchronization problem for a complex network, researchers have been apply to various control schemes [3]- [8]. However, the investigation of the synchronization problem via dynamic feedback controller for complex networks have been received little attention to date. The dynamic controller will provide more flexibility compared to the static controller and the apparent advantage of this type of controller is that it provides more free parameters for selection [9]. So it is very worth to consider the design problem of dynamic controller for synchronization in a complex network.

On the other hand, designing a stabilizing controller for a complex network with N nodes is often difficult if

not impossible, because of their complexity and strong interconnection. Therefore, it is more efficient to construct decentralized controllers to achieve control objective for the network. The strategy of decentralized control scheme is individually to design the controller for each node of the network. In addition, when controlling a real plant, it is also desirable to design a control system which is not only stable but also guarantees an adequate level of performance. There is a solution so called guaranteed cost control approach [10]- [11]. The approach has the advantage of providing an upper bound on a given linear quadratic cost function. However, to the best of authors' knowledge, the decentralized and guaranteed cost control problem for complex dynamical network is still an open area and worth considering.

From above motivation, we consider the problem of decentralized control via dynamic feedback controllers for asymptotic synchronization of a complex network. Furthermore, in order to consider both stability and performance of the network, the guaranteed cost control scheme for the synchronization problem is also applied. The existence condition of such controller is derived in terms of LMIs which can be easily solved by standard convex optimization algorithms [12].

Notation: $\mathbb{R}^n$ is the n -dimensional Euclidean space, $\mathbb{R}^{m \times n}$ denotes the set of $m \times n$ real matrix. $X > 0$ (respectively, $X \geq 0$) means that the matrix X is a real symmetric positive definite matrix (respectively, positive semi-definite). I_n denotes the n -dimensional identity matrix. $\|\cdot\|$ refers to the Euclidean vector norm and induced matrix norm.

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2. PROBLEM FORMULATION

Consider a complex dynamical network consisting of N linearly coupled identical nodes as follows:

$$\begin{aligned} \dot{x}_i(t) &= f(x_i(t)) + \sum_{j=1}^N c_{ij}x_j(t) + u_i(t), \\ i &= 1, \dots, N \end{aligned} \quad (1)$$

where $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in \mathbb{R}^n$ is the state vector of the i th node, $f : \mathbb{R}^n \rightarrow \mathbb{R}^n$ is a smooth nonlinear vector field, $u_i(t)$ is the control input of i th node, and $C = (c_{ij})_{N \times N}$ is the coupling matrix of the network, where the coupling configuration parameter, c_{ij} , is defined as follows: If there is a connection from node i to node j ($i \neq j$) then c_{ij} is nonzero; otherwise $c_{ij} = 0$ ($i \neq j$), and the diagonal elements of matrix C are assumed $c_{ii} = -\sum_{j=1, j \neq i}^N c_{ij}$, ($i = 1, \dots, N$).

Assumption 1. The smooth nonlinear function $f(\cdot)$ is satisfied following Lipschitz condition:

$$\|f(a) - f(b)\| \leq l\|a - b\|. \quad (2)$$

Our objective of the paper is to design stabilizing controllers $u_i(t)$ for each nodes which renders asymptotical synchronization between all nodes of the complex network and a target node, i.e.

$$x_1(t) = x_2(t) = \dots = x_N(t) = s(t) \quad \text{as } t \rightarrow \infty,$$

where $s(t) \in \mathbb{R}^n$ is a solution of a target node, satisfying $\dot{s}(t) = f(s(t))$. (3)

For our synchronization scheme, let us define error vectors as follows :

$$e_i(t) = s(t) - x_i(t). \quad (4)$$

From Eq. (4), the error dynamics is given to

$$\begin{aligned} \dot{e}_i(t) &= f(s(t)) - f(x_i(t)) - \sum_{j=1}^N c_{ij}e_j(t) - u_i(t) \\ &= \bar{f}_i(e_i(t)) - \sum_{j=1}^N c_{ij}e_j(t) - u_i(t), \\ i &= 1, \dots, N \end{aligned} \quad (5)$$

where $\bar{f}_i(t) = f(s(t)) - f(x_i(t))$.

In order to stabilize the error system given in Eq. (5), the following dynamic feedback controllers is proposed:

$$\begin{aligned} \dot{\zeta}_i(t) &= A_{ci}\zeta_i(t) + B_{ci}e_i(t), \\ u_i(t) &= C_{ci}\zeta_i(t), \quad \zeta_i(0) = 0, \end{aligned} \quad (6)$$

where $\zeta_i(t) \in \mathbb{R}^n$ is the state vector of controller, and A_{ci}, B_{ci} and C_{ci} are constant gain matrices of i th node with $n \times n$ dimensions.

Applying this controller (6) to error system (5) results in the following closed-loop system

$$\dot{z}_i(t) = \bar{H}_i z_i(t) - \sum_{j=1}^N \bar{C}_{ij} z_j(t) + \begin{bmatrix} \bar{f}_i(t) \\ 0 \end{bmatrix}, \quad (7)$$

where

$$\begin{aligned} z_i(t) &= \begin{bmatrix} e_i(t) \\ \zeta_i(t) \end{bmatrix} \in \mathbb{R}^{2n}, \\ \bar{C}_{ij} &= \begin{bmatrix} c_{ij}I_n & 0 \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{2n \times 2n} \\ \bar{H}_i &= \begin{bmatrix} 0 & -C_{ci} \\ B_{ci} & A_{ci} \end{bmatrix} \in \mathbb{R}^{2n \times 2n}. \end{aligned}$$

As a tool of measuring performance, the following quadratic cost function is defined:

$$J_i = \int_0^\infty (e_i^T(t)Q_i e_i(t) + u_i^T(t)R_i u_i(t))dt, \quad (8)$$

where Q_i and $R_i \in \mathbb{R}^{n \times n}$ are given positive-definite matrices.

The corresponding closed-loop cost function can be written

$$\begin{aligned} J_i &= \int_0^\infty z_i^T(t) \begin{bmatrix} Q_i & 0 \\ 0 & C_{ci}^T R_i C_{ci} \end{bmatrix} z_i(t) dt \\ &\equiv \int_0^\infty z_i^T(t) \bar{Q}_i z_i(t) dt. \end{aligned} \quad (9)$$

Definition 1. Consider system (5). If there exist a controller $u_i(t)$ given in (6) and a positive constant J^* such that the closed-loop system (7) is asymptotically stable and the closed-loop value of the cost function (8) satisfies $J \leq J^*$, then J^* is said to be a guaranteed cost and $u_i(t)$ is said to be decentralized guaranteed cost synchronization controller.

Before proceeding further, a well-known fact is given, below.

Fact 1. (Schur complements) Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

3. CONTROLLER DESIGN

In this section, the existence criterion for a decentralized guaranteed cost dynamic controller (6) for error system (5) will be derived by use of Lyapunov theory and LMI convex optimization framework.

The following is a main result of this paper.

Theorem 1. For given $Q_i > 0$ and $R_i > 0$ and a known

Lipschitz constant $L = lI_n$, the controller (6) is the guaranteed cost synchronization dynamic controller for the complex network (1) if there exist positive-definite matrices $S_i, Y_i, X_{i4} \in \mathbb{R}^{n \times n}$ and matrices $X_{i1}, X_{i2}, X_{i3} \in \mathbb{R}^{n \times n}$ satisfying the following LMIs :

$$\Gamma_i < 0, \quad (10)$$

$$\begin{bmatrix} Y_i & I \\ I & S_i \end{bmatrix} > 0, \quad i = 1, \dots, N \quad (11)$$

where

$$\begin{aligned} \alpha &= \sqrt{N}, \quad \bar{c}_i = \left(\sum_{j=1}^N c_{ij}^2 \right)^{\frac{1}{2}}, \\ \Gamma_i(1,1) &= Y_i L + L Y_i - X_{i1} - X_{i1}^T + \alpha^2 X_{i4}, \\ \Gamma_i(1,5) &= L + X_{i3} + \alpha^2 Y_i + Y_i Q_i, \\ \Gamma_i(5,5) &= L S_i + S_i L + X_{i2} + X_{i2}^T + \alpha^2 I + Q_i, \\ \Gamma_i(1,2) &= \alpha Y_i, \quad \Gamma_i(1,3) = Y_i Q_i, \\ \Gamma_i(1,4) &= X_{i1} R_i, \quad \Gamma_i(1,6) = \bar{c}_i I, \\ \Gamma_i(2,2) &= -I, \quad \Gamma_i(3,3) = -Q_i, \\ \Gamma_i(4,4) &= -R_i, \quad \Gamma_i(5,6) = S_i \bar{c}_i, \quad \Gamma_i(6,6) = -I, \end{aligned}$$

and otherwise entries of the matrix Γ_i are zero.

Then, the upper bound of cost function is given by

$$J_i \leq e_i^T(0) S_i e_i(0) \triangleq J_i^*. \quad (12)$$

Proof. Consider a Lyapunov function for system (5)

$$V = \sum_{i=1}^N V_i(t) = \sum_{i=1}^N (z_i^T(t) P_i z_i(t)), \quad (13)$$

where $P_i \in \mathbb{R}^{2n \times 2n} > 0$, $i = 1, \dots, N$.

By Assumption 1, the time derivative of V is

$$\begin{aligned} \dot{V} &= \sum_{i=1}^N (z_i^T (P_i \bar{H}_i + \bar{H}_i^T P_i) z_i(t) + 2z_i^T(t) P_i \\ &\quad \times \begin{bmatrix} \bar{f}_i(t) \\ 0 \end{bmatrix} - 2z_i^T(t) P_i \sum_{j=1}^N \bar{C}_{ij} z_j(t)) \\ &\leq \sum_{i=1}^N (z_i^T (P_i H_i + H_i^T P_i) z_i(t) \\ &\quad - 2z_i^T(t) P_i \sum_{j=1}^N \bar{C}_{ij} z_j(t)), \end{aligned} \quad (14)$$

where

$$H = \begin{bmatrix} L & -C_{ci} \\ B_{ci} & A_{ci} \end{bmatrix}.$$

By a well-known fact that

$$-2a^T b \leq \epsilon a a^T + \epsilon^{-1} b b^T, \quad \epsilon > 0, \quad (15)$$

we have

$$\begin{aligned} &\sum_{i=1}^N \left(-2z_i^T(t) P_i \sum_{j=1}^N \bar{C}_{ij} z_j(t) \right) \\ &\leq \sum_{i=1}^N \left(z_i^T(t) P_i C_{di} C_{di}^T P_i z_i(t) + \sum_{j=1}^N z_i(t)^T z_i(t) \right) \\ &= \sum_{i=1}^N (z_i^T(t) P_i C_{di} C_{di}^T P_i z_i(t) \\ &\quad + \alpha^2 z_i(t)^T z_i(t)), \end{aligned} \quad (16)$$

$$\text{where } C_{di} = \begin{bmatrix} \bar{c}_i I_n & 0 \\ 0 & 0 \end{bmatrix}.$$

Thus, from Eq. (16), Eq. (14) can be rewritten as follows:

$$\begin{aligned} \dot{V} &\leq \sum_{i=1}^N (z_i^T (P_i H_i + H_i^T P_i + P_i C_{di} C_{di}^T P_i \\ &\quad + \alpha^2 I) z_i(t) \\ &= \sum_{i=1}^N z_i^T \Sigma_i z_i(t) - \sum_{i=1}^N z_i^T(t) \bar{Q}_i z_i(t), \end{aligned} \quad (17)$$

where $\Sigma_i = P_i H_i + H_i^T P_i + P_i C_{di} C_{di}^T P_i + \alpha^2 I_n + \bar{Q}_i$.

Therefore, if $\Sigma_i < 0$, there exist positive scalars γ_i such that

$$\dot{V} \leq - \sum_{i=1}^N z_i^T(t) \bar{Q}_i z_i(t) \leq - \sum_{i=1}^N \gamma_i \|e_i(t)\|^2, \quad (18)$$

which guarantees the asymptotic stability of error system (5) by Lyapunov stability theory.

It should be pointed out that the matrix $P_i > 0$ and the controller parameters A_{ci}, B_{ci} and C_{ci} in the matrix Σ_i are unknown and occur in nonlinear fashion. Hence, the inequality $\Sigma_i < 0$ cannot be considered as an LMI problem. So, we will use a method of changing variables such that the inequality can be solved as convex optimization algorithm [13].

First, partition the matrix P_i and its inverse as

$$P_i = \begin{bmatrix} S_i & D_i \\ D_i^T & T_i \end{bmatrix}, \quad P_i^{-1} = \begin{bmatrix} Y_i & M_i \\ M_i^T & W_i \end{bmatrix}, \quad (19)$$

where S_i, Y_i are positive-definite matrices, and $M_i, D_i \in \mathbb{R}^{n \times n}$ are invertible matrices. It should be noted that the equality $P_i^{-1} P_i = I$ gives that

$$M_i D_i^T = I - Y_i S_i. \quad (20)$$

Define two matrices as

$$E_i = \begin{bmatrix} Y_i & I \\ M_i^T & 0 \end{bmatrix}, \quad F_i = \begin{bmatrix} I & S_i \\ 0 & D_i^T \end{bmatrix}. \quad (21)$$

Then, it follows that

$$P_i E_i = F_i, \quad E_i^T P_i E_i = E_i^T F_i = \begin{bmatrix} Y_i & I \\ I & S_i \end{bmatrix} > 0. \quad (22)$$

Now, postmultiplying and premultiplying the matrix inequality, $\Sigma_i < 0$, by the matrix E_i^T and by its transpose, respectively, gives

$$F_i^T H_i E_i + E_i^T H_i^T F_i + E_i^T (\alpha^2 I_n + \bar{Q}_i) E_i + F_i C_{di} C_{di}^T F_i < 0. \quad (23)$$

By Fact 1, the inequality (23) is modified to

$$\begin{bmatrix} (1,1) & (1,2) & \bar{c}_i I & 0 \\ \star & (2,2) & S_i \bar{c}_i & 0 \\ \star & \star & -I & 0 \\ \star & \star & \star & -I \end{bmatrix} < 0, \quad (24)$$

where

$$\begin{aligned} (1,1) &= Y_i L + L Y_i - M_i C_{ci}^T - C_{ci} M_i^T + \alpha^2 Y_i Y_i \\ &\quad + Y_i Q_i Y_i + \alpha^2 M_i M_i^T + M_i C_{ci}^T R_i C_{ci} M_i^T, \\ (1,2) &= L + Y_i L S_i + Y_i B_{ci}^T D_i^T - M_i C_{ci}^T S \\ &\quad + M_i A_{ci}^T D_i^T + \alpha^2 Y_i + Y_i Q_i, \\ (2,2) &= L S_i + S_i L + D_i B_{ci} + B_{ci}^T D_i^T + \alpha^2 I + Q_i. \end{aligned}$$

By defining a new set of variables as follows:

$$\begin{aligned} X_{i1} &= M_i C_{ci}^T, \\ X_{i2} &= D_i B_{ci}, \\ X_{i3} &= Y_i L S_i + Y_i X_{i2}^T - X_{i1} S_i + M_i A_{ci}^T D_i^T, \\ X_{i4} &= M_i M_i^T, \end{aligned} \quad (25)$$

then, by Fact 1, inequality (24) is equivalent to the LMI (10).

On the other hand, from Eq. (18), we get

$$\dot{V} \leq -z_i^T(t) \bar{Q}_i z_i(t). \quad (26)$$

Integrating both sides of the above inequality from 0 to T_f leads to

$$\int_0^{T_f} z_i^T(t) \bar{Q}_i z_i(t) dt < V(0) - V(T_f). \quad (27)$$

Since the asymptotic stability of the system has already been established, we conclude that $V(T_f) \rightarrow 0$ as $t \rightarrow \infty$. Hence we have

$$J_i \leq z_i^T(0) \bar{Q}_i z_i(0) = e_i^T(0) S_i e_i(0) = J_i^*. \quad (28)$$

This completes the proof. ■

Theorem 1 presents a method of designing a dynamic feedback controller for synchronization of a complex network (1). In the following, we will present a method of selecting the optimal controller minimizing the upper bound of the guaranteed cost (12).

Theorem 2. Consider error system (5) and cost function (8). For all i , if the following LMI optimization problem,

$$\min_{\beta_i, S_i, Y_i, X_{i1}, X_{i2}, X_{i3}} \beta_i \quad (29)$$

subject to

$$(i) \text{ LMIs (10) and (11)} \quad (30)$$

$$(ii) \begin{bmatrix} -\beta_i & e_i^T(0) S_i \\ \star & -S_i \end{bmatrix} < 0 \quad (31)$$

has the solution set $(\beta_i, S_i, Y_i, X_{i1}, X_{i2}, X_{i3})$, the controller (6) is the optimal guaranteed cost dynamic controller which ensures the minimization of the guaranteed cost (12) of the system. The optimal cost is $\sum_{i=1}^N J_i^* = \sum_{i=1}^N \beta_i \equiv \beta$.

Proof. By Theorem 1, Eq. (30) is clear, and from Fact 1, Eq. (31) is equivalent to $e_i^T(0) S_i e_i(0) < \beta_i$. So, it follows from (12). Thus, the minimization of β_i implies the minimization of the guaranteed cost (8). It is well-known that the convexity of the LMI optimization problem ensures that a global optimum, when it exists, is reachable. This completes the proof. ■

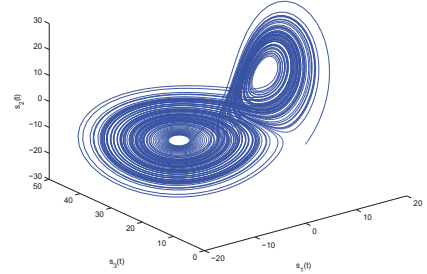


Fig. 1 The chaotic behavior of Lorenz system

4. NUMERICAL EXAMPLES

In this section, a simulation result is presented to show the effectiveness of the proposed control scheme, which synchronize all nodes of a complex network to a target node.

Consider a complex dynamical network with five linearly coupled identical nodes which are Lorenz chaotic systems [14]. Fig. 1 depicts its chaotic behavior. The dynamic equation of a complex network of this example is described by:

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^N c_{ij} x_j(t) + u_i(t), \quad (32)$$

where

$$f(x_i(t)) = \begin{bmatrix} a(x_{i2} - x_{i1}) \\ cx_{i1} - x_{i2} - x_{i1}x_{i3} \\ x_{i1}x_{i2} - bx_{i3} \end{bmatrix}, \quad i = 1, \dots, 5$$

with the parameters $a = 10, b = 8/3, c = 28$.

It is noted that the Lipschitz constant of Lorenz system is $l = 50$ and initial conditions of each nodes are chosen: $x_1(0) = [1, -0.5, -1.7]$, $x_2(0) = [-0.5, 0.4, 0.3]$, $x_3(0) = [0.6, 0.5, 0]$, $x_4(0) = [1.1, -1.5, 1.3]$, $x_5(0) = [0.1, -0.5, 1.7]$, $s(0) = [0.1, 0.1, 0.1]$. The parameters associated with cost function are chosen as $Q = I$ and $R = I$ and the coupling matrix, C , is given by

$$C = 0.2 \times \begin{bmatrix} -3 & 1 & 1 & 0 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 0 & 0 & 1 & -2 \end{bmatrix}. \quad (33)$$

In order to show original behavior of the complex network (32), the trajectories of the error signals of each nodes without controller is depicted in Fig 2. Now, by consideration of LMI problem given in (10), (11) in Theorem 1, we can calculate feasible solution set from Eqs. (25) by utilization of MATLAB LMI Toolbox (see Appendix).

The simulation result with control inputs which are calculated by Theorem 1 are presented in Fig. 3. As seen in Fig. 3, the trajectories of error systems are indeed well stabilized and also the state orbits of controller approach to zero. It can be concluded that our proposed dynamic controller guarantees asymptotic synchronization of the complex network (32) under some value of performance index.

Finally, by solving the optimization problem (29) given in Theorem 2, the optimal guaranteed cost of closed-loop system are obtained and listed in Table 1.

Table 1 The cost value of each method

		β
Example 1	Theorem 1	5.1008×10^6
	Theorem 2	2.5718×10^3

5. CONCLUSIONS

In this paper, the synchronization problem of a complex dynamical network has been studied. The decentralization control method was considered in the sense of reality. Unlike other works, a guaranteed cost dynamic feedback controller was designed for the synchronization

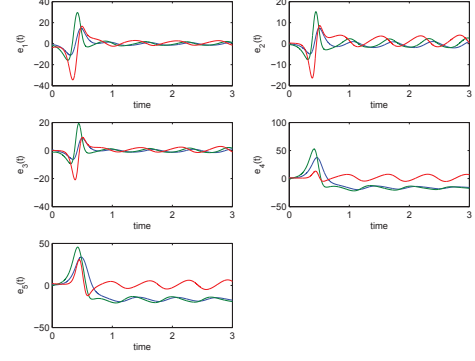


Fig. 2 Error signals of the example without controller

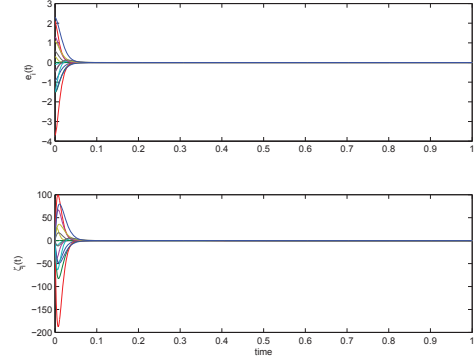


Fig. 3 Error signals and control states of the example by applying Theorem 1

problem based on the Lyapunov method and LMI framework. Then, two criteria expressed by LMIs for achieving both stability and performance of error dynamics was derived. Finally, a numerical example was illustrated to show the effectiveness of the designed controller.

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APPENDIX

• The parameters of controller of an example by using Theorem 1.

$$\begin{aligned}
 A_{c1} &= 10^3 \times \begin{bmatrix} -1.2122 & -0.0027 & -0.0080 \\ -0.0027 & -1.2088 & -0.0043 \\ -0.0080 & -0.0043 & -1.2204 \end{bmatrix}, \\
 B_{c1} &= 10^4 \times \begin{bmatrix} 6.9704 & 0.0296 & 0.0888 \\ 0.0296 & 6.9323 & 0.0485 \\ 0.0888 & 0.0485 & 7.0615 \end{bmatrix}, \\
 C_{c1} &= \begin{bmatrix} 5.9369 & 0.0014 & 0.0041 \\ 0.0014 & 5.9351 & 0.0022 \\ 0.0041 & 0.0022 & 5.9411 \end{bmatrix}, \\
 A_{c2} &= 10^3 \times \begin{bmatrix} -2.1216 & 0.0076 & 0.0051 \\ 0.0076 & -2.1102 & -0.0025 \\ 0.0051 & -0.0025 & -2.1080 \end{bmatrix}, \\
 B_{c2} &= 10^5 \times \begin{bmatrix} 1.0828 & -0.0007 & -0.0005 \\ -0.0007 & 1.0818 & 0.0002 \\ -0.0005 & 0.0002 & 1.0816 \end{bmatrix}, \\
 C_{c2} &= \begin{bmatrix} 12.0894 & -0.0917 & -0.0611 \\ -0.0917 & 11.9519 & 0.0306 \\ -0.0611 & 0.0306 & 11.9264 \end{bmatrix}, \\
 A_{c3} &= \begin{bmatrix} -850.2763 & -0.6060 & 0.1515 \\ -0.6060 & -850.0036 & 0.1212 \\ 0.1515 & 0.1212 & -849.5491 \end{bmatrix}, \\
 B_{c3} &= 10^4 \times \begin{bmatrix} 4.3137 & 0.0054 & -0.0014 \\ 0.0054 & 4.3113 & -0.0011 \\ -0.0014 & -0.0011 & 4.3072 \end{bmatrix}, \\
 C_{c3} &= \begin{bmatrix} 4.7031 & -0.0000 & 0.0000 \\ -0.0000 & 4.7031 & 0.0000 \\ 0.0000 & 0.0000 & 4.7031 \end{bmatrix}, \\
 A_{c4} &= 10^3 \times \begin{bmatrix} -1.2114 & 0.0064 & -0.0048 \\ 0.0064 & -1.2176 & 0.0077 \\ -0.0048 & 0.0077 & -1.2131 \end{bmatrix}, \\
 B_{c4} &= 10^4 \times \begin{bmatrix} 6.9606 & -0.0710 & 0.0533 \\ -0.0710 & 7.0298 & -0.0852 \\ 0.0533 & -0.0852 & 6.9801 \end{bmatrix}, \\
 C_{c4} &= \begin{bmatrix} 5.9363 & -0.0031 & 0.0023 \\ -0.0031 & 5.9393 & -0.0037 \\ 0.0023 & -0.0037 & 5.9372 \end{bmatrix}, \\
 A_{c5} &= \begin{bmatrix} -849.5188 & 0 & 0 \\ 0 & -850.6680 & 3.0645 \\ 0 & 3.0645 & -857.6908 \end{bmatrix}, \\
 B_{c5} &= 10^4 \times \begin{bmatrix} 4.3070 & 0 & 0 \\ 0 & 4.3164 & -0.0252 \\ 0 & -0.0252 & 4.3741 \end{bmatrix}, \\
 C_{c5} &= \begin{bmatrix} 4.7031 & 0 & 0 \\ 0 & 4.7027 & 0.0009 \\ 0 & 0.0009 & 4.7006 \end{bmatrix}
 \end{aligned}$$