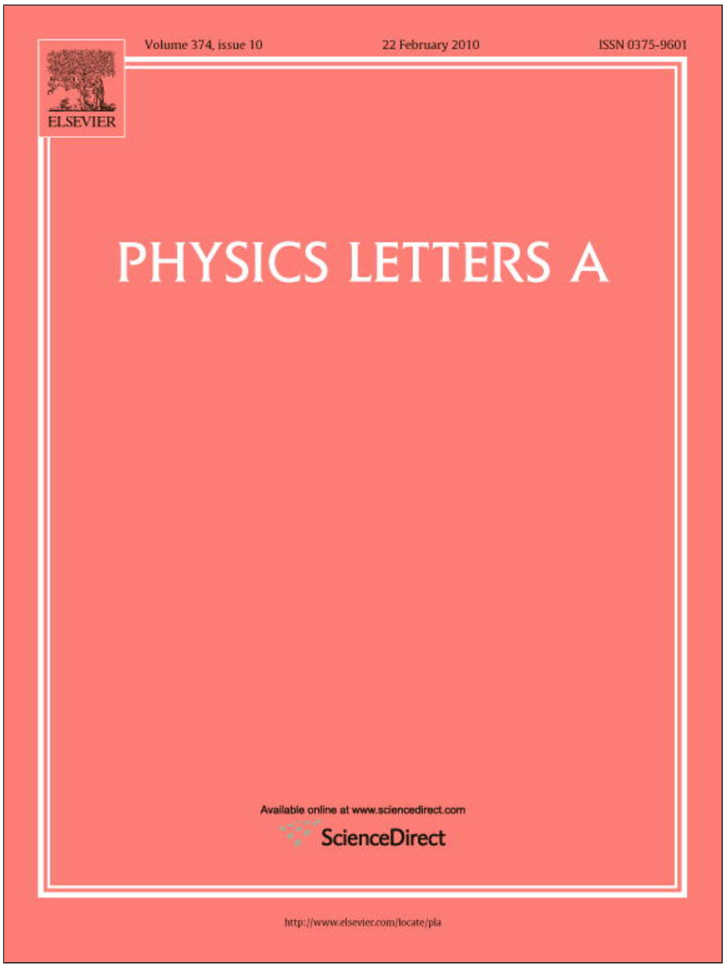
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Improved delay-dependent exponential stability for uncertain stochastic neural networks with time-varying delays

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ABSTRACT

This Letter investigates the problem of delay-dependent exponential stability analysis for uncertain stochastic neural networks with time-varying delay. Based on the Lyapunov stability theory, improved delay-dependent exponential stability criteria for the networks are established in terms of linear matrix inequalities (LMIs).

1. Introduction

During last two decades, the dynamics of neural networks have been extensively studied due to their extensive applications in the fields of pattern recognition, optimization problems, associative memories, signal processing, fixed-point computation, and so on. Therefore, considerable efforts have been done to the stability analysis of various types of neural networks since these applications rely on the dynamic behaviors of the equilibrium point of the networks. For example, see the works of [1,2] and references therein. On the other hand, in the processing of storage and transmission of information, time-delays often occur due to the finite switching speed of amplifiers in electronic networks or finite speed for signal propagation in biological networks. The important factor is that the delays may cause instability and oscillation of neural networks. Hence, many researchers have focused on the stability analysis of delayed cellular neural networks in recent years [3–13].

Recently, increasing attentions for the stability analysis of the stochastic neural networks have been paid by some researchers [14–16] since the synaptic transmission in real nervous systems is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes [17]. In this regard, delay-dependent stability analysis of uncertain stochastic neural networks with time-varying delays has been studied in [14–16] since it is well known that delay-dependent stability analysis is in general less conservative than delay-independent stability ones when the size of time-delays are small. In the literature [14–16], by constructing some suitable Lyapunov–Krasovskii's functionals, several improved stability criteria are derived in terms of LMIs with the recently developed free-weighting matrices techniques. However, there are still rooms for further improvement to the stability criteria for uncertain stochastic neural networks with time-varying delays.

In this Letter, we propose new improved delay-dependent exponential stability criteria for uncertain stochastic neural networks with time-varying delays. Additional stochastic perturbations are considered as two cases: 1) trace bounded and 2) linear function and norm-bounded. By constructing suitable Lyapunov–Krasovskii's functionals, new delay-dependent stability criteria for uncertain stochastic neural networks with time-varying delays are proposed. The derived stability criteria have the form of LMIs which can be solved efficiently by using the interior-point algorithm [18]. As a tradeoff between the computational burden and the reduction of the conservatism of stability criteria, delay fraction number is chosen as two. And no free-weighting matrices are employed in obtaining upper bounds of integral terms

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obtained by calculating the stochastic differential of Lyapunov–Krasovskii's functionals. Three numerical examples are given and compared with the very recent results to show the improved results with less decision variables.

Notation: $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \geqslant Y$) means that the matrix $X - Y$ is positive definite (respectively, nonnegative). $diag\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix. I is the identity matrix with appropriate dimension. For $h > 0$, $\mathcal{C}([-h, 0], \mathcal{R}^n)$ means the family of continuous functions ϕ from $[-h, 0]$ to $\mathcal{R}^n$ with the norm $\|\phi\| = \sup_{-h \leqslant s \leqslant 0} |\phi(s)|$. Let $(\Omega, \mathcal{F}, \{F_t\}_{t \geqslant 0}, P)$ be a complete probability space with a filtration $\{F_t\}_{t \geqslant 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains all P-pull sets). $\mathcal{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P.

2. Problem statement

Consider the following uncertain neural networks with discrete time-varying delays:

$$\dot{v}(t) = -\big(A + \Delta A(t)\big)v(t) + \big(W_0 + \Delta W_0(t)\big)g\big(v(t)\big) + \big(W_1 + \Delta W_1(t)\big)g\big(v\big(t - h(t)\big)\big) + b \tag{1}$$

where $v(t) = [v_1(t), \ldots, v_n(t)]^T \in \mathcal{R}^n$ is the neuron state vector, n denotes the number of neurons in a neural network, $g(v(t)) = [g_1(v_1(t)), \ldots, g_n(v_n(t))]^T \in \mathcal{R}^n$ represents the neuron activation function, $g(v(t - h(t))) = [g_1(v_1(t - h(t))), \ldots, g_n(v_n(t - h(t)))]^T \in \mathcal{R}^n$, $A = diag\{a_i\}$ is a positive diagonal matrix, $W_0 = (w_{ij}^0)_{n \times n}$, and $W_1 = (w_{ij}^1)_{n \times n}$ are the interconnection matrices representing the weight coefficients of the neurons, $b = [b_1, b_2, \ldots, b_n]^T$ means a constant input vector, and $\Delta A(t)$, $\Delta W_0(t)$, and $\Delta W_1(t)$, are the uncertainties of system matrices of the form

$$\begin{bmatrix} \Delta A(t) & \Delta W_0(t) & \Delta W_1(t) \end{bmatrix} = DF(t)\begin{bmatrix} E_1 & E_2 & E_3 \end{bmatrix}, \quad F^T(t)F(t) \leqslant I, \ \forall t \geqslant 0, \tag{2}$$

where D and E_i are constant matrices and $F(t)$ is the time-varying nonlinear function.

The delays, $h(t)$, are time-varying continuous functions that satisfies $0 \leqslant h(t) \leqslant h_U$, $\dot{h}(t) \leqslant h_D$ where h_U is positive constant and h_D is any constant one.

The activation functions, $g_i(v_i(t))$, $i = 1, \ldots, n$, are assumed to posses the following properties:

(A1) g_i is bounded on $\mathcal{R}^n$, and $g_i(0) = 0$ $(i = 1, \ldots, n)$.
(A2) There exist real numbers k_i^- and k_i^+ $(i = 1, \ldots, n)$ such that

$$k_i^- \leqslant \frac{g_i(\xi_i) - g_i(\xi_j)}{\xi_i - \xi_j} \leqslant k_i^+, \quad \xi_i, \xi_j \in \mathcal{R}, \ \xi_i \neq \xi_j, \ i, j = 1, \ldots, n. \tag{3}$$

For simplicity, in stability analysis of the system (1), the equilibrium point $v^* = [v_1^*, \ldots, v_n^*]^T$ is shifted to the origin by utilizing the transformation $x(\cdot) = v(\cdot) - v^*$, which leads the system (1) to the following form:

$$\dot{x}(t) = -\big(A + \Delta A(t)\big)x(t) + \big(W_0 + \Delta W_0(t)\big)f\big(x(t)\big) + \big(W_1 + \Delta W_1(t)\big)f\big(x\big(t - h(t)\big)\big) \tag{4}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathcal{R}^n$ is the state vector of the transformed system, $f(x(t)) = [f_1(x(t)), \ldots, f_n(x(t))]^T$ and $f_i(x_i(t)) = g_i(x_i(t) + v_i^*) - g_i(v_i^*)$.

Here, the activation functions f_i satisfy the following properties:

(B1) f_i is bounded on $\mathcal{R}^n$, and $f_i(0) = 0$ $(i = 1, \ldots, n)$.
(B2) There exist real numbers k_i^- and k_i^+ $(i = 1, \ldots, n)$ such that

$$k_i^- \leqslant \frac{f_i(\xi_i) - f_i(\xi_j)}{\xi_i - \xi_j} \leqslant k_i^+, \quad \xi_i, \xi_j \in \mathcal{R}, \ \xi_i \neq \xi_j, \ i, j = 1, \ldots, n. \tag{5}$$

In this Letter, we consider the following uncertain stochastic neural networks with time-varying delays

$$dx(t) = \big[-\big(A + \Delta A(t)\big)x(t) + \big(W_0 + \Delta W_0(t)\big)f\big(x(t)\big) + \big(W_1 + \Delta W_1(t)\big)f\big(x\big(t - h(t)\big)\big)\big]dt + \sigma\big(t, x(t), x\big(t - h(t)\big)\big)d\omega(t), \tag{6}$$

where $\omega(t)$ is m-dimensional Wiener Process (Brownian Motion) on $(\Omega, \mathcal{F}, \{F_t\}_{t \geqslant 0}, P)$, $\sigma(t, x(t), x(t - h(t)))$ is assumed to satisfy the following assumptions:

(C1) $\sigma(t, 0, 0) = 0$ and $\sigma(t, x(t), x(t - h(t)))$ is locally Lipschitz continuous and satisfies the linear growth condition.
(C2) There exist constant real matrices G_1 and G_2 such that

$$Trace(\sigma^T\big(t, x(t), x\big(t - h(t)\big)\big)\sigma\big(t, x(t), x\big(t - h(t)\big)\big) \leqslant \big\|G_1 x(t)\big\|^2 + \big\|G_2 x\big(t - h(t)\big)\big\|^2. \tag{7}$$

Now, the system (6) can be written as:

$$\begin{aligned} dx(t) &= \big[-Ax(t) + W_0 f\big(x(t)\big) + W_1 f\big(x\big(t - h(t)\big)\big) + Dp_1(t)\big]dt + \sigma\big(t, x(t), x\big(t - h(t)\big)\big)d\omega(t), \\ p_1(t) &= F(t)q_1(t), \\ q_1(t) &= -E_1 x(t) + E_2 f\big(x(t)\big) + E_3 f\big(x\big(t - h(t)\big)\big). \end{aligned} \tag{8}$$

The objective of this Letter is to find delay-dependent exponential stability criteria for system (6).

Before deriving our main results, we state the following lemma.

Lemma 1. *(See [19].) For any constant matrix $M \in \mathcal{R}^{n\times n}$, $M = M^T > 0$, scalar $\gamma > 0$, vector function $x : [0,\gamma] \to \mathcal{R}^n$ such that the integrations concerned are well defined, then*

$$\left(\int_0^{\gamma} x(s)\,ds\right)^T M\left(\int_0^{\gamma} x(s)\,ds\right) \leqslant \gamma\int_0^{\gamma} x^T(s)Mx(s)\,ds. \tag{9}$$

In deriving our main results, Itô's formula plays a key role in stability analysis of stochastic systems (see [20] for details).

3. Main results

In this section, we propose new delay-dependent stability criteria for uncertain stochastic neural networks with time-varying delays described by Eq. (8). Before introducing the main result, the following notations are defined for simplicity:

$$\begin{aligned}
&y(t) = Ax(t) + W_0 f\big(x(t)\big) + W_1 f\big(x\big(t-h(t)\big)\big) + Dp_1(t), \qquad g(t) = \sigma\big(t, x(t), x\big(t-h(t)\big)\big),\\
&K_p = diag\{k_1^+,\dots,k_n^+\}, \qquad K_m = diag\{k_1^-,\dots,k_n^-\},\\
&\zeta_1^T(t) = \begin{bmatrix} x^T(t) & x^T(t-h(t)) & x^T(t-h_U/2) & x^T(t-h_U) & y^T(t) & f^T(x(t)) & f^T(x(t-h(t))) & p_1^T(t)\end{bmatrix},\\
&\Sigma_1 = [\Sigma_{1(i,j)}], \quad i,j = 1,\dots,8,\\
&\Sigma_{1(1,1)} = N_{11} + M_{11} + \varepsilon_1 E_1^T E_1 - P_1 A - A^T P_1^T - 2K_m S_1 K_p + \sum_{i=1}^{3}\rho_i G_1^T G_1 - A^T P_3^T, \qquad \Sigma_{1(1,2)} = 0,\\
&\Sigma_{1(1,3)} = N_{12}, \qquad \Sigma_{1(1,4)} = 0, \qquad \Sigma_{1(1,5)} = R_1 - P_1 - A^T P_2^T - K_m\Lambda + K_p\Delta,\\
&\Sigma_{1(1,6)} = M_{12} - \varepsilon_1 E_1^T E_2 + P_1 W_0 + S_1(K_m + K_p), \qquad \Sigma_{1(1,7)} = -\varepsilon_1 E_1^T E_3 + P_1 W_1, \qquad \Sigma_{1(1,8)} = P_1 D,\\
&\Sigma_{1(2,2)} = -(1-h_D)M_{11} - 2K_m S_2 K_p + \sum_{i=1}^{3}\rho_i G_2^T G_2,\\
&\Sigma_{1(2,3)} = 0, \qquad \Sigma_{1(2,4)} = 0, \qquad \Sigma_{1(2,5)} = 0, \qquad \Sigma_{1(2,6)} = 0, \qquad \Sigma_{1(2,7)} = -(1-h_D)M_{12} + S_2(K_m + K_p),\\
&\Sigma_{1(2,8)} = 0, \qquad \Sigma_{1(3,3)} = N_{22} - N_{11}, \qquad \Sigma_{1(3,4)} = -N_{12}, \qquad \Sigma_{1(3,5)} = 0, \qquad \Sigma_{1(3,6)} = 0, \qquad \Sigma_{1(3,7)} = 0,\\
&\Sigma_{1(3,8)} = 0, \qquad \Sigma_{1(4,4)} = -N_{22}, \qquad \Sigma_{1(4,5)} = 0, \qquad \Sigma_{1(4,6)} = 0, \qquad \Sigma_{1(4,7)} = 0, \qquad \Sigma_{1(4,8)} = 0,\\
&\Sigma_{1(5,5)} = -P_2 - P_2^T + (h_U/2)(Q_1 + Q_2), \qquad \Sigma_{1(5,6)} = P_2 W_0 + \Lambda - \Delta - P_3, \qquad \Sigma_{1(5,7)} = P_2 W_1,\\
&\Sigma_{1(5,8)} = P_2 D, \qquad \Sigma_{1(6,6)} = M_{22} + P_3 W_0 + W_0^T P_3^T, \qquad \Sigma_{1(6,7)} = \varepsilon_1 E_2^T E_3 + P_3 W_1,\\
&\Sigma_{1(6,8)} = -2S_1 + P_3 D, \qquad \Sigma_{1(7,7)} = -(1-h_D)M_{22} + \varepsilon_1 E_3^T E_3 - 2S_2, \qquad \Sigma_{1(7,8)} = 0, \qquad \Sigma_{1(8,8)} = -\varepsilon_1 I,\\
&\Gamma_1 = \begin{bmatrix} 0 & 0 & I & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0\end{bmatrix}, \qquad \Gamma_2 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0\end{bmatrix},\\
&\Xi_1 = \Sigma_1 + (h_U/2)^{-1}\Gamma_1^T\begin{bmatrix} -Q_2 & Q_2\\ \star & -Q_2\end{bmatrix}\Gamma_1, \qquad \Xi_2 = \Sigma_1 + (h_U/2)^{-1}\Gamma_2^T\begin{bmatrix} -Q_1 & Q_1\\ \star & -Q_1\end{bmatrix}\Gamma_2,\\
&\Pi_1 = \begin{bmatrix} I & -I & 0 & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \qquad \Pi_2 = \begin{bmatrix} 0 & I & -I & 0 & 0 & 0 & 0 & 0\end{bmatrix},\\
&\Pi_3 = \begin{bmatrix} 0 & -I & I & 0 & 0 & 0 & 0 & 0\end{bmatrix}, \qquad \Pi_4 = \begin{bmatrix} 0 & I & 0 & -I & 0 & 0 & 0 & 0\end{bmatrix}.
\end{aligned} \tag{10}$$

Now, we have the following theorem.

Theorem 1. *For given $h_U > 0$, and h_D, system (8) is exponentially stable in the mean square if there exist positive scalars ε_1, ρ_i $(i = 1,2,3)$, positive definite matrices R_1, Q_1, Q_2, $\mathcal{N} = \left[\begin{smallmatrix} N_{11} & N_{12}\\ \star & N_{22}\end{smallmatrix}\right]$, $\mathcal{M} = \left[\begin{smallmatrix} M_{11} & M_{12}\\ \star & M_{22}\end{smallmatrix}\right]$, positive diagonal matrices S_i $(i = 1,2)$ $= diag\{s_{i1},\dots,s_{in}\}$, $\Lambda = diag\{\lambda_1,\dots,\lambda_n\}$, $\Delta = diag\{\delta_1,\dots,\delta_n\}$ and any matrices P_i $(i = 1,2,3)$ satisfying the following LMIs:*

$$R_1 \leqslant \rho_1 I, \tag{11}$$

$$\Lambda(K_p - K_m) \leqslant \rho_2 I, \tag{12}$$

$$\Delta(K_p - K_m) \leqslant \rho_3 I, \tag{13}$$

$$\Xi_1 + \Pi_1^T\Phi_1(0)\Pi_1 + \Pi_2^T\Phi_2(0)\Pi_2 < 0, \tag{14}$$

$$\Xi_1 + \Pi_1^T\Phi_1(h_U/2)\Pi_1 + \Pi_2^T\Phi_2(h_U/2)\Pi_2 < 0, \tag{15}$$

$$\Xi_2 + \Pi_3^T\Phi_3(h_U/2)\Pi_3 + \Pi_4^T\Phi_4(h_U/2)\Pi_4 < 0, \tag{16}$$

$$\Xi_2 + \Pi_3^T\Phi_3(h_U)\Pi_3 + \Pi_4^T\Phi_4(h_U)\Pi_4 < 0, \tag{17}$$

where $\Phi_1(h(t)) = (-2(h_U/2)^{-1} + (h_U/2)^{-2}h(t))Q_1$, $\Phi_2(h(t)) = (-(h_U/2)^{-2}h(t) - (h_U/2)^{-1})Q_1$, $\Phi_3(h(t)) = (-(h_U/2)^{-2}(h_U - h(t)) - (h_U/2)^{-1})Q_2$, $\Phi_4(h(t)) = (-(h_U/2)^{-1} - (h_U/2)^{-2}(h(t) - h_U/2))Q_2$, *and other notations are defined in (10).*

Proof. From the definition $y(t)$ and $g(t)$ in (10), system (8) can be rewritten as:

$$dx(t) = y(t)\,dt + g(t)\,d\omega(t). \tag{18}$$

For positive definite matrices R_1, $\mathcal{N}$, $\mathcal{M}$, Q_i $(i = 1, 2)$, and positive diagonal matrices Λ and Δ, let us consider the Lyapunov–Krasovskii functional candidate:

$$V = \sum_{i=1}^{3} V_i, \tag{19}$$

where

$$\begin{aligned}
V_1 &= x^T(t)R_1x(t) + 2\sum_{i=1}^{n}\left(\lambda_i\int_0^{x_i(t)}\big(f_i(s) - k_i^- s\big)\,ds + \delta_i\int_0^{x_i(t)}\big(k_i^+ s - f_i(s)\big)\,ds\right) + \int_{t-h_U/2}^{t}\begin{bmatrix} x(s)\\ x(s-h_U/2)\end{bmatrix}^T\mathcal{N}\begin{bmatrix} x(s)\\ x(s-h_U/2)\end{bmatrix}ds,\\
V_2 &= \int_{t-h(t)}^{t}\begin{bmatrix} x(s)\\ f(x(s))\end{bmatrix}^T\mathcal{M}\begin{bmatrix} x(s)\\ f(x(s))\end{bmatrix}ds,\\
V_3 &= \int_{t-h_U/2}^{t}\int_s^t y^T(u)Q_1y(u)\,du\,ds + \int_{t-h_U}^{t-h_U/2}\int_s^t y^T(u)Q_2y(u)\,du\,ds.
\end{aligned} \tag{20}$$

With the mentioned V, the calculation of $\mathcal{L}V_1$ can be obtained as

$$\begin{aligned}
\mathcal{L}V_1 &= 2x^T(t)R_1y(t) + trace\big(g^T(t)R_1g(t)\big) + 2\big[f\big(x(t)\big) - K_mx(t)\big]^T\Lambda y(t) + 2\big[K_px(t) - f\big(x(t)\big)\big]^T\Delta y(t)\\
&\quad + trace\left(g^T(t)\Lambda\, diag\left(\frac{\partial(f(x_1) - k_1^-x_1)}{\partial x_1},\ldots,\frac{\partial(f(x_n) - k_n^-x_n)}{\partial x_n}\right)g(t)\right)\\
&\quad + trace\left(g^T(t)\Delta\, diag\left(\frac{\partial(k_1^+x_1 - f(x_1))}{\partial x_1},\ldots,\frac{\partial(k_n^+x_n - f(x_n))}{\partial x_n}\right)g(t)\right)\\
&\quad + \begin{bmatrix} x(t)\\ x(t-h_U/2)\end{bmatrix}^T\mathcal{N}\begin{bmatrix} x(t)\\ x(t-h_U/2)\end{bmatrix} - \begin{bmatrix} x(t-h_U/2)\\ x(t-h_U)\end{bmatrix}^T\mathcal{N}\begin{bmatrix} x(t-h_U/2)\\ x(t-h_U)\end{bmatrix},
\end{aligned} \tag{21}$$

where $\mathcal{L}$ means the weak infinitesimal operator [20]. If the inequalities (11)–(13) hold, then

$$trace\big(g^T(t)R_1g(t)\big) \leqslant \rho_1\big(x^T(t)G_1^TG_1x(t) + x^T\big(t-h(t)\big)G_2^TG_2x\big(t-h(t)\big)\big), \tag{22}$$

$$\begin{aligned}
&trace\left(g^T(t)\Lambda\, diag\left(\frac{\partial(f(x_1) - k_1^-x_1)}{\partial x_1},\ldots,\frac{\partial(f(x_n) - k_n^-x_n)}{\partial x_n}\right)g(t)\right)\\
&\quad \leqslant trace\big(g^T(t)\Lambda(K_p - K_m)g(t)\big) \leqslant \rho_2\big(x^T(t)G_1^TG_1x(t) + x^T\big(t-h(t)\big)G_2^TG_2x\big(t-h(t)\big)\big),
\end{aligned} \tag{23}$$

$$\begin{aligned}
&trace\left(g^T(t)\Delta\, diag\left(\frac{\partial(k_1^+x_1 - f(x_1))}{\partial x_1},\ldots,\frac{\partial(k_n^+x_n - f(x_n))}{\partial x_n}\right)g(t)\right)\\
&\quad \leqslant trace\big(g^T(t)\Delta(K_p - K_m)g(t)\big) \leqslant \rho_3\big(x^T(t)G_1^TG_1x(t) + x^T\big(t-h(t)\big)G_2^TG_2x\big(t-h(t)\big)\big).
\end{aligned} \tag{24}$$

From the inequalities (22)–(24), an upper bound of $\mathcal{L}V_1$ can be obtained as

$$\begin{aligned}
\mathcal{L}V_1 &\leqslant 2x^T(t)R_1y(t) + \sum_{i=1}^{3}\rho_i\big(x^T(t)G_1^TG_1x(t)x(t) + x^T\big(t-h(t)\big)G_2^TG_2x\big(t-h(t)\big)\big)\\
&\quad + 2\big[f\big(x(t)\big) - K_mx(t)\big]^T\Lambda y(t) + 2\big[K_px(t) - f\big(x(t)\big)\big]^T\Delta y(t)\\
&\quad + \begin{bmatrix} x(t)\\ x(t-h_U/2)\end{bmatrix}^T\mathcal{N}\begin{bmatrix} x(t)\\ x(t-h_U/2)\end{bmatrix} - \begin{bmatrix} x(t-h_U/2)\\ x(t-h_U)\end{bmatrix}^T\mathcal{N}\begin{bmatrix} x(t-h_U/2)\\ x(t-h_U)\end{bmatrix}.
\end{aligned} \tag{25}$$

Also, an upper bound of $\mathcal{L}V_2$ can be obtained as

$$\mathcal{L}V_2 \leqslant \begin{bmatrix} x(t)\\ f(x(t))\end{bmatrix}^T\mathcal{M}\begin{bmatrix} x(t)\\ f(x(t))\end{bmatrix} - (1-h_D)\begin{bmatrix} x(t-h(t))\\ f(x(t-h(t)))\end{bmatrix}^T\mathcal{M}\begin{bmatrix} x(t-h(t))\\ f(x(t-h(t)))\end{bmatrix}. \tag{26}$$

By calculating $\mathcal{L}V_3$, we have

$$\mathcal{L}V_3 = (h_U/2)y^T(t)(Q_1+Q_2)y(t) - \int_{t-h_U/2}^{t} y^T(s)Q_1y(s)\,ds - \int_{t-h_U}^{t-h_U/2} y^T(s)Q_2y(s)\,ds. \tag{27}$$

Depending on whether the time-varying delay $h(t)$ belongs the interval $0 \leqslant h(t) \leqslant h_U/2$ or $h_U/2 \leqslant h(t) \leqslant h_U$, different upper bounds of the integral terms of (27) can be obtained as two cases:

Case I: $0 \leqslant h(t) \leqslant h_U/2$.
For this condition, the term $-\int_{t-h_U/2}^{t} y^T(s)Q_1y(s)\,ds$ can be divided into two ones as:

$$-\int_{t-h_U/2}^{t} y^T(s)Q_1y(s)\,ds = -\int_{t-h(t)}^{t} y^T(s)Q_1y(s)\,ds - \int_{t-h_U/2}^{t-h(t)} y^T(s)Q_1y(s)\,ds. \tag{28}$$

When time delay belongs to $0 \leqslant h(t) \leqslant h_U/2$, from [20], the following two equations hold

$$x(t) - x\big(t-h(t)\big) - \int_{t-h(t)}^{t} y(s)\,ds - \int_{t-h(t)}^{t} g(s)\,d\omega(s) = 0, \tag{29}$$

$$x\big(t-h(t)\big) - x(t-h_U/2) - \int_{t-h_U/2}^{t-h(t)} y(s)\,ds - \int_{t-h_U/2}^{t-h(t)} g(s)\,d\omega(s) = 0. \tag{30}$$

Note that

$$-1 = -(h_U/2)^{-1}h(t) - \big(1-(h_U/2)^{-1}h(t)\big). \tag{31}$$

By utilizing Eqs. (29) and (31), the inequality $-h_U/2 \leqslant -h(t)$, and Lemma 1, an upper bound of the integral term $-\int_{t-h(t)}^{t} y^T(s)Q_1y(s)\,ds$ can be obtained as

$$\begin{aligned}
-\int_{t-h(t)}^{t} y^T(s)Q_1y(s)\,ds &= -(h_U/2)^{-1}h(t)\int_{t-h(t)}^{t} y^T(s)Q_1y(s)\,ds - (h_U/2)^{-1}\big(1-(h_U/2)^{-1}h(t)\big)(h_U/2)\int_{t-h(t)}^{t} y^T(s)Q_1y(s)\,ds \\
&\leqslant -(h_U/2)^{-1}h(t)\int_{t-h(t)}^{t} y^T(s)Q_1y(s)\,ds - (h_U/2)^{-1}\big(1-(h_U/2)^{-1}h(t)\big)\big(h(t)\big)\int_{t-h(t)}^{t} y^T(s)Q_1y(s)\,ds \\
&\leqslant \left(\int_{t-h(t)}^{t} y(s)\,ds\right)^T \Phi_1\big(h(t)\big)\left(\int_{t-h(t)}^{t} y(s)\,ds\right) \\
&= \big(x(t)-x\big(t-h(t)\big)\big)^T\Phi_1\big(h(t)\big)\big(x(t)-x\big(t-h(t)\big)\big) - 2\big(x(t)-x\big(t-h(t)\big)\big)^T\Phi_1\big(h(t)\big)\int_{t-h(t)}^{t} g(s)\,d\omega(s) \\
&\quad + \left(\int_{t-h(t)}^{t} g(s)\,d\omega(s)\right)^T\Phi_1\big(h(t)\big)\left(\int_{t-h(t)}^{t} g(s)\,d\omega(s)\right),
\end{aligned} \tag{32}$$

where $\Phi_1(h(t))$ are defined in Theorem 1.

By using the similar method introduced above, an upper bound of the term $-\int_{t-h_U/2}^{t-h(t)} y^T(s)Q_1y(s)\,ds$ can be estimated

$$\begin{aligned}
-\int_{t-h_U/2}^{t-h(t)} y^T(s)Q_1y(s)\,ds &\leqslant \big(x\big(t-h(t)\big)-x(t-h_U/2)\big)^T\Phi_2\big(h(t)\big)\big(x\big(t-h(t)\big)-x(t-h_U/2)\big) \\
&\quad - 2\big(x\big(t-h(t)\big)-x(t-h_U/2)\big)^T\Phi_2\big(h(t)\big)\int_{t-h_U/2}^{t-h(t)} g(s)\,d\omega(s) \\
&\quad + \left(\int_{t-h_U/2}^{t-h(t)} g(s)\,d\omega(s)\right)^T\Phi_2\big(h(t)\big)\left(\int_{t-h_U/2}^{t-h(t)} g(s)\,d\omega(s)\right),
\end{aligned} \tag{33}$$

where $\Phi_2(h(t))$ are defined in Theorem 1.

The other integral term $-\int_{t-h_U}^{t-h_U/2} y^T(s)Q_2y(s)\,ds$ can be estimated as

$$-\int_{t-h_U}^{t-h_U/2} y^T(s)Q_2y(s)\,ds \leqslant -(h_U/2)^{-1}\big(x(t-h_U/2)-x(t-h_U)\big)^T Q_2\big(x(t-h_U/2)-x(t-h_U)\big)$$
$$+2(h_U/2)^{-1}\big(x(t-h_U/2)-x(t-h_U)\big)^T Q_2\left(\int_{t-h_U}^{t-h_U/2} g(s)\,d\omega(s)\right)$$
$$-(h_U/2)^{-1}\left(\int_{t-h_U}^{t-h_U/2} g(s)\,d\omega(s)\right)^T Q_2\left(\int_{t-h_U}^{t-h_U/2} g(s)\,d\omega(s)\right), \tag{34}$$

where the equation $x(t-h_U/2)-x(t-h_U)-\int_{t-h_U}^{t-h_U/2} y(s)\,ds-\int_{t-h_U}^{t-h_U/2} g(s)\,d\omega(s)=0$ is utilized.

In order to derive less conservative results, we add the following zero equation with free-weighting matrices P_i $(i=1,2,3)$ to be chosen as

$$0=2\big[x^T(t)P_1+y^T(t)P_2+f^T\big(x(t)\big)P_3\big]\big[-y(t)+Ax(t)+A_dx\big(t-h(t)\big)+Dp_1(t)\big]. \tag{35}$$

Note that Eq. (5) means

$$\big[f_j\big(x_j(t)\big)-k_j^-x_j(t)\big]\big[f_j\big(x_j(t)\big)-k_j^+x_j(t)\big]\leqslant 0 \quad (j=1,\dots,n), \tag{36}$$
$$\big[f_j\big(x_j\big(t-h(t)\big)\big)-k_j^-x_j\big(t-h(t)\big)\big]\big[f_j\big(x_j\big(t-h(t)\big)\big)-k_j^+x_j\big(t-h(t)\big)\big]\leqslant 0 \quad (j=1,\dots,n). \tag{37}$$

From two inequalities (36) and (37), for any positive diagonal matrices $S_1=diag\{s_{1i},\dots,s_{1n}\}$, and $S_2=diag\{s_{2i},\dots,s_{2n}\}$, the following inequality holds

$$0\leqslant -2\sum_{j=1}^n s_{1j}\big[f_j\big(x_j(t)\big)-k_j^-x_j(t)\big]\big[f_j\big(x_j(t)\big)-k_j^+x_j(t)\big]$$
$$-2\sum_{j=1}^n s_{2j}\big[f_j\big(x_j\big(t-h(t)\big)\big)-k_j^-x_j\big(t-h(t)\big)\big]\big[f_j\big(x_j\big(t-h(t)\big)\big)-k_j^+x_j\big(t-h(t)\big)\big]$$
$$=-2x^T(t)K_mS_1K_px(t)+2x^T(t)(K_m+K_p)S_1f\big(x(t)\big)-2f^T\big(x(t)\big)S_1f\big(x(t)\big)-2x^T\big(t-h(t)\big)K_mS_2K_px\big(t-h(t)\big)$$
$$+2x^T\big(t-h(t)\big)(K_m+K_p)S_2f\big(x\big(t-h(t)\big)\big)-2f^T\big(x\big(t-h(t)\big)\big)S_2f\big(x\big(t-h(t)\big)\big). \tag{38}$$

From (2) and (8), we have $p_1^T(t)p_1(t)\leqslant q_1^T(t)q_1(t)$. Then, there exists a positive scalar ε_1 satisfying the following inequality

$$\varepsilon_1\big[q_1^T(t)q_1(t)-p_1^T(t)p_1(t)\big]\geqslant 0. \tag{39}$$

From the condition $0\leqslant h(t)\leqslant h_U/2$, note that

$$\left(\int_{t-h(t)}^{t} g(s)\,d\omega(s)\right)^T \Phi_1\big(h(t)\big)\left(\int_{t-h(t)}^{t} g(s)\,d\omega(s)\right)<0,\qquad \left(\int_{t-h_U/2}^{t-h(t)} g(s)\,d\omega(s)\right)^T \Phi_2\big(h(t)\big)\left(\int_{t-h_U/2}^{t-h(t)} g(s)\,d\omega(s)\right)<0. \tag{40}$$

From (18)–(40) and by applying S-procedure [18], the $\mathcal{L}V=\sum_{i=1}^3\mathcal{L}V_i$ has a new upper bound as

$$\mathcal{L}V\leqslant \zeta_1^T(t)\Omega_1\big(h(t)\big)\zeta_1(t)+\xi_1\big(d\omega(t)\big), \tag{41}$$

where $\zeta_1(t)$, Ξ_1, Π_1, and Π_2 are defined in (10), $\Omega_1(h(t))=\Xi_1+\Pi_1^T\Phi_1(h(t))\Pi_1+\Pi_2^T\Phi_2(h(t))\Pi_2$, and

$$\xi_1\big(d\omega(t)\big)=-2\big(x(t)-x\big(t-h(t)\big)\big)^T\Phi_1\big(h(t)\big)\int_{t-h(t)}^{t} g(s)\,d\omega(s)-2\big(x\big(t-h(t)\big)-x(t-h_U/2)\big)^T\Phi_2\big(h(t)\big)\int_{t-h_U/2}^{t-h(t)} g(s)\,d\omega(s)$$
$$+2(h_U/2)^{-1}\big(x(t-h_U/2)-x(t-h_U)\big)^T Q_2\left(\int_{t-h_U}^{t-h_U/2} g(s)\,d\omega(s)\right). \tag{42}$$

Since $\Phi_1(h(t))$ and $\Phi_2(h(t))$ are convex combination of the matrix Q_1 on $h(t)$, $\Omega_1(h(t))<0$ for $0\leqslant h(t)\leqslant h_U/2$ can be handled by the two LMIs $\Omega_1(0)<0$ and $\Omega_1(h_U/2)<0$, which are equivalent to the LMIs (14) and (15). Note that the mathematical expectation of $\xi_1(d\omega(t))$ is zero [21]. Therefore, if the LMIs (11)–(15) hold, there exists a positive scalar $\gamma_1>0$ satisfying

$$\mathcal{E}\{\mathcal{L}V\}\leqslant -\gamma_1\mathcal{E}\big|x(t)\big|^2. \tag{43}$$

Case II: $h_U/2 \leqslant h(t) \leqslant h_U$.
For this condition, an upper bound of the integral term $-\int_{t-h_U/2}^{t} y^T(s)Q_1 y(s)\,ds$ can be obtained as

$$-\int_{t-h_U/2}^{t} y^T(s)Q_1 y(s)\,ds \leqslant -(h_U/2)^{-1}\big(x(t)-x(t-h_U/2)\big)^T Q_1\big(x(t)-x(t-h_U/2)\big) + 2(h_U/2)^{-1}\big(x(t)-x(t-h_U/2)\big)^T Q_1\left(\int_{t-h_U/2}^{t} g(s)\,d\omega(s)\right) - (h_U/2)^{-1}\left(\int_{t-h_U/2}^{t} g(s)\,d\omega(s)\right)^T Q_1\left(\int_{t-h_U/2}^{t} g(s)\,d\omega(s)\right), \tag{44}$$

where $x(t)-x(t-h_U/2)-\int_{t-h_U/2}^{t} y(s)\,ds-\int_{t-h_U/2}^{t} g(s)\,d\omega(s)=0$ is utilized in (44).

Note that

$$-1 = -(h_U/2)^{-1}(h_U-h_U/2) = -(h_U/2)^{-1}\big(h_U-h(t)\big)-(h_U/2)^{-1}\big(h(t)-h_U/2\big). \tag{45}$$

When $h(t)$ belongs to the interval $h_U/2 \leqslant h(t) \leqslant h_U$, the following equations hold:

$$x(t-h_U/2)-x\big(t-h(t)\big)-\int_{t-h(t)}^{t-h_U/2} y(s)\,ds-\int_{t-h(t)}^{t-h_U/2} g(s)\,d\omega(s)=0, \tag{46}$$

$$x\big(t-h(t)\big)-x(t-h_U)-\int_{t-h_U}^{t-h(t)} y(s)\,ds-\int_{t-h_U}^{t-h(t)} g(s)\,d\omega(s)=0. \tag{47}$$

Using Eqs. (45)–(47), inequality $h_U/2 \leqslant h(t) \leqslant h_U$, and Lemma 1, an upper bound of the integral term $-\int_{t-h_U}^{t-h_U/2} y^T(s)Q_2 y(s)\,ds$ can be obtained

$$\begin{aligned}
-\int_{t-h_U}^{t-h_U/2} y^T(s)Q_2 y(s)\,ds &= -\int_{t-h(t)}^{t-h_U/2} y^T(s)Q_2 y(s)\,ds-\int_{t-h_U}^{t-h(t)} y^T(s)Q_2 y(s)\,ds\\
&\leqslant \big[x(t-h_U/2)-x(t-h(t))\big]^T \Phi_3\big(h(t)\big)\big[x(t-h_U/2)-x(t-h(t))\big]\\
&\quad+\big[x\big(t-h(t)\big)-x(t-h_U)\big]^T \Phi_4\big(h(t)\big)\big[x\big(t-h(t)\big)-x(t-h_U)\big]+\hat{\xi}_2\big(d\omega(t)\big)\\
&\quad+\left(\int_{t-h(t)}^{t-h_U/2} g(s)\,d\omega(s)\right)^T \Phi_3\big(h(t)\big)\left(\int_{t-h(t)}^{t-h_U/2} g(s)\,d\omega(s)\right)\\
&\quad+\left(\int_{t-h_U}^{t-h(t)} g(s)\,d\omega(s)\right)^T \Phi_4\big(h(t)\big)\left(\int_{t-h_U}^{t-h(t)} g(s)\,d\omega(s)\right),
\end{aligned} \tag{48}$$

where

$$\hat{\xi}_2\big(d\omega(t)\big) = -2\big(x(t-h_U/2)-x\big(t-h(t)\big)\big)^T \Phi_3\big(h(t)\big)\int_{t-h(t)}^{t-h_U/2} g(s)\,d\omega(s) - 2\big(x\big(t-h(t)\big)-x(t-h_U)\big)^T \Phi_4\big(h(t)\big)\int_{t-h_U}^{t-h(t)} g(s)\,d\omega(s). \tag{49}$$

From (18)–(27), (35)–(39), (44)–(49), and by applying S-procedure [18], the $\mathcal{L}V=\sum_{i=1}^{3}\mathcal{L}V_i$ has a new upper bound as

$$\mathcal{L}V \leqslant \zeta_1^T(t)\Omega_2\big(h(t)\big)\zeta_1(t)+\xi_2\big(d\omega(t)\big), \tag{50}$$

where $\xi_2(d\omega(t))=\hat{\xi}_2(d\omega(t))+2(h_U/2)^{-1}(x(t)-x(t-h_U/2))^T Q_1(\int_{t-h_U/2}^{t} g(s)\,d\omega(s))$, $\Omega_2(h(t))=\Xi_2+\Pi_3^T\Phi_3(h(t))\Pi_3+\Pi_4^T\Phi_4(h(t))\Pi_4$, and Ξ_2, Π_3, and Π_4 are defined in (10).

Since $\Phi_3(h(t))$ and $\Phi_4(h(t))$ are convex combination of the matrix Q_2 on $h(t)$, $\Omega_2(h(t))<0$ for $h_U/2 \leqslant h(t) \leqslant h_U$ can be handled by the two LMIs $\Omega_2(h_U/2)<0$ and $\Omega_2(h_U)<0$, which are equivalent to the LMIs (16) and (17). Therefore, if the LMIs (11)–(13) and

(16)–(17) hold, there exists a positive scalar γ_2 such that

$$\mathcal{E}\{\mathcal{L}V\} \leqslant -\gamma_2 \mathcal{E}\big|x(t)\big|^2. \tag{51}$$

From (43) and (51), if the LMIs (11)–(17) hold, then we have

$$\mathcal{E}\{\mathcal{L}V\} \leqslant -\min(\gamma_1, \gamma_2)\mathcal{E}\big|x(t)\big|^2. \tag{52}$$

By using similar method in the proof of Theorem 1 in [14], we can prove that the obtained inequality (52) indicates that the uncertain stochastic neural network (8) with $0 \leqslant h(t) \leqslant h_U$ is exponentially stable in the mean square. This completes our proof. □

For a special case, if $\sigma(t, x(t), x(t-h(t)))$ is a linear function, that is,

$$\sigma\big(t, x(t), x\big(t-h(t)\big)\big) = \big(Hx(t) + \Delta H(t)\big)x(t) + \big(H_d + \Delta H_d(t)\big)x\big(t-h(t)\big), \tag{53}$$

where $[\, \Delta H(t) \;\; \Delta H_d(t) \,] = DF_2(t)[\, E_4 \;\; E_5 \,]$, and $F_2^T(t)F_2(t) \leqslant I$, then system (1) can be written as:

$$\begin{aligned}
&dx(t) = \big[-Ax(t) + W_0 f\big(x(t)\big) + W_1 f\big(x\big(t-h(t)\big)\big) + Dp_1(t)\big]dt + \big[Hx(t) + H_d x\big(t-h(t)\big) + Dp_2(t)\big]d\omega(t),\\
&p_1(t) = DF_1(t)q_1(t), \qquad p_2(t) = DF_2(t)q_2(t),\\
&q_1(t) = -E_1x(t) + E_2 f\big(x(t)\big) + E_3 f\big(x\big(t-h(t)\big)\big), \qquad q_2(t) = E_4x(t) + E_5x\big(t-h(t)\big).
\end{aligned} \tag{54}$$

For the above system, we can also obtain a delay-dependent stability criterion using similar methods presented in the proof of Theorem 1, which can be obtained as Corollary 1. We also introduce the following notations for simplicity:

$$\begin{aligned}
&y(t) = Ax(t) + W_0 f\big(x(t)\big) + W_1 f\big(x\big(t-h(t)\big)\big) + Dp_1(t), \qquad g(t) = Hx(t) + H_d x\big(t-h(t)\big) + Dp_2(t),\\
&\zeta_2^T(t) = \big[\, x^T(t) \;\; x^T(t-h(t)) \;\; x^T(t-h_U/2) \;\; x^T(t-h_U) \;\; y^T(t) \;\; g^T(t) \;\; f^T(x(t)) \;\; f^T(x(t-h(t))) \;\; p_1^T(t) \;\; p_2^T(t) \,\big],\\
&\Sigma_2 = [\Sigma_{2(i,j)}], \quad i, j = 1, \ldots, 10,\\
&\Sigma_{2(1,1)} = N_{11} + M_{11} + \varepsilon_1 E_1^T E_1 + \varepsilon_2 E_4^T E_4 - P_1A - A^T P_1^T + P_3H + H^T P_3^T - 2K_m S_1 K_p,\\
&\Sigma_{2(1,2)} = \varepsilon_2 E_4^T E_5 + P_3 H_d, \qquad \Sigma_{2(1,3)} = N_{12}, \qquad \Sigma_{2(1,4)} = 0, \qquad \Sigma_{2(1,5)} = R_1 - P_1 - A^T P_2^T - K_m\Lambda + K_p\Delta,\\
&\Sigma_{2(1,6)} = -P_3 + H^T P_4^T, \qquad \Sigma_{2(1,7)} = M_{12} - \varepsilon_1 E_1^T E_2 + P_1W_0 + S_1(K_m + K_p) - A^T P_5^T - H^T P_6^T,\\
&\Sigma_{2(1,8)} = -\varepsilon_1 E_1^T E_3 + P_1 W_1, \qquad \Sigma_{2(1,9)} = P_1D, \qquad \Sigma_{2(1,10)} = P_3D,\\
&\Sigma_{2(2,2)} = -(1-h_D)M_{11} + \varepsilon_2 E_5^T E_5 - 2K_m S_2 K_p, \qquad \Sigma_{2(2,3)} = 0, \qquad \Sigma_{2(2,4)} = 0, \qquad \Sigma_{2(2,5)} = 0,\\
&\Sigma_{2(2,6)} = H_d^T P_4^T, \qquad \Sigma_{2(2,7)} = -H_d^T P_6^T, \qquad \Sigma_{2(2,8)} = -(1-h_D)M_{12} + S_2(K_m + K_p), \qquad \Sigma_{2(2,9)} = 0,\\
&\Sigma_{2(2,10)} = 0, \qquad \Sigma_{2(3,3)} = N_{22} - N_{11}, \qquad \Sigma_{2(3,4)} = -N_{12}, \qquad \Sigma_{2(3,5)} = 0, \qquad \Sigma_{2(3,6)} = 0, \qquad \Sigma_{2(3,7)} = 0,\\
&\Sigma_{2(3,8)} = 0, \qquad \Sigma_{2(3,9)} = 0, \qquad \Sigma_{2(3,10)} = 0, \qquad \Sigma_{2(4,4)} = -N_{22}, \qquad \Sigma_{2(4,5)} = 0, \qquad \Sigma_{2(4,6)} = 0, \qquad \Sigma_{2(4,7)} = 0,\\
&\Sigma_{2(4,8)} = 0, \qquad \Sigma_{2(4,9)} = 0, \qquad \Sigma_{2(4,10)} = 0, \qquad \Sigma_{2(5,5)} = -P_2 - P_2^T + (h_U/2)(Q_1 + Q_2), \qquad \Sigma_{2(5,6)} = 0,\\
&\Sigma_{2(5,7)} = P_2W_0 + \Lambda - \Delta - P_5^T, \qquad \Sigma_{2(5,8)} = P_2W_1, \qquad \Sigma_{2(5,9)} = P_2D, \qquad \Sigma_{2(5,10)} = 0,\\
&\Sigma_{2(6,6)} = R_1 - P_4 - P_4^T + (\Lambda + \Delta)(K_p - K_m), \qquad \Sigma_{2(6,7)} = -P_6^T, \qquad \Sigma_{2(6,8)} = 0, \qquad \Sigma_{2(6,9)} = 0,\\
&\Sigma_{2(6,10)} = P_4D, \qquad \Sigma_{2(7,7)} = M_{22} - 2S_1 + P_5W_0 + W_0^T P_5^T, \qquad \Sigma_{2(7,8)} = \varepsilon_1 E_2^T E_3 + P_5W_1,\\
&\Sigma_{2(7,9)} = P_5D, \qquad \Sigma_{2(7,10)} = P_6D, \qquad \Sigma_{2(8,8)} = -(1-h_D)M_{22} + \varepsilon_1 E_3^T E_3 - 2S_2,\\
&\Sigma_{2(8,9)} = 0, \qquad \Sigma_{2(8,10)} = 0, \qquad \Sigma_{2(9,9)} = -\varepsilon_1 I, \qquad \Sigma_{2(9,10)} = 0, \qquad \Sigma_{2(10,10)} = -\varepsilon_2 I,\\
&\Gamma_3 = \begin{bmatrix} 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \qquad \Gamma_4 = \begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ 0 & 0 & I & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},\\
&\Xi_3 = \Sigma_2 + (h_U/2)^{-1}\Gamma_3^T \begin{bmatrix} -Q_2 & Q_2\\ \star & -Q_2 \end{bmatrix}\Gamma_3, \qquad \Xi_4 = \Sigma_2 + (h_U/2)^{-1}\Gamma_4^T \begin{bmatrix} -Q_1 & Q_1\\ \star & -Q_1 \end{bmatrix}\Gamma_4,\\
&\Pi_5 = \big[\, I \;\; -I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\big], \qquad \Pi_6 = \big[\, 0 \;\; I \;\; -I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\big],\\
&\Pi_7 = \big[\, 0 \;\; -I \;\; I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\big], \qquad \Pi_8 = \big[\, 0 \;\; I \;\; 0 \;\; -I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,\big].
\end{aligned} \tag{55}$$

Now, we have the following corollary.

Corollary 1. *For given $h_U > 0$, and h_D, system (54) is exponentially stable in the mean square if there exist positive scalars ε_1 and ε_2, positive definite matrices R_1, Q_1, Q_2, $\mathcal{N}$, $\mathcal{M}$, positive diagonal matrices S_i $(i = 1, 2)$ $= diag\{s_{i1}, \ldots, s_{in}\}$, $\Lambda = diag\{\lambda_1, \ldots, \lambda_n\}$, $\Delta = diag\{\delta_1, \ldots, \delta_n\}$ and any matrices P_i $(i = 1, \ldots, 6)$ satisfying the following LMIs:*

$$\Xi_3 + \Pi_5^T \Phi_1(0)\Pi_5 + \Pi_6^T \Phi_2(0)\Pi_6 < 0, \quad (56)$$
$$\Xi_3 + \Pi_5^T \Phi_1(h_U/2)\Pi_5 + \Pi_6^T \Phi_2(h_U/2)\Pi_6 < 0, \quad (57)$$
$$\Xi_4 + \Pi_7^T \Phi_3(h_U/2)\Pi_7 + \Pi_8^T \Phi_4(h_U/2)\Pi_8 < 0, \quad (58)$$
$$\Xi_4 + \Pi_7^T \Phi_3(h_U)\Pi_7 + \Pi_8^T \Phi_4(h_U)\Pi_8 < 0. \quad (59)$$

Proof. Let us consider the same Lyapunov–Krasovskii's functional (19). With the following additional zero equation

$$0 = 2\big[x^T(t)P_4 + g^T(t)P_5 + f^T\big(x(t)\big)P_6\big]\big[Hx(t) + H_d x\big(t - h(t)\big) + Dp_2(t)\big], \quad (60)$$

augmentation vector $\zeta_2(t)$, and the similar method in the proof of Theorem 1, we can easily show that if the LMIs (56)–(59) hold, then system (54) is exponentially stable in the mean square. This completes our proof. □

Remark 1. Since $d\omega(t)$ in (54) is one-dimensional Wiener Process, the following equation holds:

$$trace\big(g^T(t)R_1 g(t)\big) = g^T(t)R_1 g(t). \quad (61)$$

In Corollary 1, $g(t)$ was chosen as an augmentation variable, which may improve the feasible region of stability for the system (54).

Remark 2. When $M_{11} = M_{12} = M_{22} = 0$, Theorem 1 and Corollary 1 do not include the information of time-derivative of $h(t)$.

Remark 3. In [14–16], free-weighting matrices technique are utilized when obtaining upper bounds of integral terms obtained by calculating the stochastic differential of Lyapunov–Krasovskii's functionals to reduce the conservatism or increase the feasible of stability criteria. However, the large employing of decision variables makes the stability criteria more complex and the computational burden and time-consuming of stability criteria will be increased. Recently, in the field of delay-dependent stability analysis of dynamic system with time delays, a discretization scheme [22] of the delay was proposed to improve the feasible region of stability criteria. This method utilizes a Lyapunov–Krasovskii's functional which employs redundant state of differential equations shifted delay in time by a fraction of the time delay. However, if delay fraction number increases, then the computational burden is large and the solving of the concerned LMIs much time-consuming. In this Letter, the delay fraction number is chosen as two for the tradeoff between time-consuming and improved results. To reduce LMI variables, we do not use any free-weighting matrix in obtaining upper bounds of integral terms such as $-\int_{t-h_U/2}^{t} y^T(s)Q_1 y(s)\,ds$ and $-\int_{t-h_U}^{t-h_U/2} y^T(s)Q_2 y(s)\,ds$ at each subintervals $0 \leqslant h(t) < h_U/2$ and $h_U/2 \leqslant h(t) \leqslant h_U$, which is different from the method of [14–16]. Through numerical examples, we will show that Theorem 1 and Corollary 1 with less number of LMI variables can provide an improved result compared with the recent ones in [14–16].

Remark 4. To treat the integral term which include $g(t)$ as shown at Eq. (20) in [14] and Eq. (26)–(27) in [15], the double integral form of Lyapunov–Krasovskii's functional which include $g(t)$ were used. However, the proposed Theorem 1 and Corollary 1 do not include this form of Lyapunov–Krasovskii's functional. This consideration may lead to an improved results of the proposed methods.

4. Numerical examples

Example 1. Consider the uncertain stochastic neural networks (17) studied in [16]:

$$A = diag\{1.5, 0.5, 2.3\}, \qquad W_0 = \begin{bmatrix} 0.3 & -0.19 & 0.3 \\ -0.15 & 0.2 & 0.36 \\ -0.17 & 0.29 & -0.3 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} 0.19 & -0.13 & 0.2 \\ 0.26 & 0.09 & 0.1 \\ 0.02 & -0.15 & 0.07 \end{bmatrix},$$
$$G_1 = G_2 = 0.1I, \qquad D = 0.1I, \qquad E_1 = E_2 = E_3 = I. \quad (62)$$

When $K_p = diag\{1, 1, 1\}$, $K_m = diag\{-0.5, -0.5, -0.5\}$ and $h_D = 0.5$, the maximum delay bound by Theorem 1 in [16] was $h_U = 2.2471$, However, by applying Theorem 1 to the above system with the same K_p, K_m and h_D, one can see that the system (62) is exponentially stable in the mean square for any $h_U > 0$. When h_D is unknown, our maximum delay bound is obtained as $h_U = 3.5304$ by Theorem 1. For the case of $K_p = diag\{1.2, 0.5, 1.3\}$ and $K_m = diag\{0, 0, 0\}$, when $h_D = 0.85$ and unknown, the obtained delay bounds were $h_U = 9.6876$ and 2.3879, respectively in [16]. However, our results when $h_D = 0.85$ and unknown are $h_U = 19.9261$ and 4.6364, respectively. Furthermore, note that the number of decision variable of Theorem 1 is 103 while the one of [16, Theorem 1] was 422. From these results, our criterion with less decision variables than those of [16] provides larger delay bounds than the results of [16].

Example 2. Consider the following uncertain stochastic neural networks with time-varying delays:

$$dx(t) = \big[-\big(A + \Delta A(t)\big)x(t) + \big(W_0 + \Delta W_0(t)\big)f\big(x(t)\big) + \big(W_1 + \Delta W_1(t)\big)f\big(x\big(t - h(t)\big)\big)\big]dt + \big[Hx(t) + H_d x\big(t - h(t)\big)\big]d\omega(t) \quad (63)$$

where $A = diag\{4, 5\}$, $K_p = diag\{0.5, 0.5\}$, $K_m = diag\{0, 0\}$ and

$$W_0 = \begin{bmatrix} 0.4 & -0.7 \\ 0.1 & 0 \end{bmatrix}, \qquad W_1 = \begin{bmatrix} -0.2 & 0.6 \\ 0.5 & -0.1 \end{bmatrix}, \qquad H_0 = 0.5I, \qquad H_1 = \begin{bmatrix} 0 & -0.5 \\ -0.5 & 0 \end{bmatrix},$$
$$D = \begin{bmatrix} 0.1 & -0.1 \end{bmatrix}^T, \qquad E_1 = \begin{bmatrix} 0.2 & 0.3 \end{bmatrix}, \qquad E_2 = \begin{bmatrix} 0.2 & -0.3 \end{bmatrix}, \qquad E_3 = -E_1.$$

Table 1
Delay bounds h_U with different conditions h_D (Example 2).

	h_D is known		h_D is unknown
	h_D	h_U	h_U
Zhang et al. [16]	$\leqslant 0.90$	∞	0.6520
Corollary 1	$\leqslant 0.94$	∞	1.8693

Table 2
Delay bounds h_U with different conditions h_D (Example 3).

	$h_D = 0.5$		h_D is unknown	
	h_U	Num. of variables	h_U	Num. of variables
Chen et al. [14]	0.264	95	0.1962	89
Yu et al. [15]	0.273	122	0.209	116
Corollary 1	0.284	63	0.2101	53

When the information of time-derivative is unknown, Corollary 1 with the 53 decision variables provides that the maximum delay bound for stability in the mean square is $h_U = 1.8788$. However, in [15], the delay bound by Theorem 2 with 116 decision variables was $h_U = 0.8269$. This means that Corollary 1 provides improved delay bounds in spite of using less number of decision variables than those of [15]. For the case of $K_p = diag\{0.5, 0.5\}$, $K_m = diag\{-0.5, -0.5\}$, the comparison of our results with the ones of [16] are listed in Table 1, which shows Corollary 1 improves the feasible region of stability criterion.

Example 3. Consider the following stochastic system [14]:

$$dx(t) = \left[-Ax(t) + W_0 f\left(x(t)\right) + W_1 f\left(x\left(t - h(t)\right)\right)\right]dt + \left[\Delta H(t)x(t) + \Delta H_d(t)\right]d\omega(t) \tag{64}$$

where $A = I$, $D = I$, $E_1 = 0$, $E_2 = 0$, $E_3 = 0$, $E_4 = 0.01I$, $E_5 = 0.02I$ and

$$W_0 = \begin{bmatrix} -1 & 2 \\ 1 & -2 \end{bmatrix}, \qquad W_1 = 2W_0, \qquad H_0 = 0, \qquad H_1 = 0.$$

When $K_p = diag\{1, 1\}$, $K_m = diag\{0, 0\}$, the obtained delay bounds by Corollary 1 are listed in Table 2 with the recent ones in [14,15]. From Table 2, one can see that Corollary 1 with less number of decision variable than the ones of [14,15] provides larger delay bounds for different condition of h_D.

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