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Guaranteed cost LPV controller design for a class of chaos synchronization

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Abstract— **This paper considers the guaranteed cost synchronization problem for a class of chaotic systems. Using a convex representation of error dynamics, state and output feedback LPV (Linear Parameter Varying) controllers are proposed. The criteria for the existence of the controllers are derived in terms of LMI (Linear Matrix Inequality). Numerical examples show the effectiveness of the proposed method.**

I. INTRODUCTION

Chaotic systems have attracted much attention because many fundamental characteristics appear in chaotic systems, such as excessive sensitivity to initial conditions and fractal properties of the motion in phase space. Since Pecora and Carroll [1] presented the pioneer work for synchronization of two identical chaotic systems, chaos synchronization has received great attention because of its various applications (e.g., biology, economics, signal generator design, secure communications and so on). In the literature, there are various methods which achieve chaotic synchronization, for example, observer based control [2][3], back-stepping design technique [4][5], variable structure control [6][7] and so on.

With the development of convex optimization technology, linear feedback control methods based on LMIs are also proposed [8][9]. Linear feedback control scheme is widely used in real world because it has simple configuration and its implementation is easy. Recently, guaranteed cost control for chaotic synchronization with input constraint is presented in [10]. Considering boundedness of nonlinearity as polytopic uncertainties, the paper [10] derived stability criterion for chaotic synchronization in terms of LMIs. However, gain-scheduling approach based on LPV system representation can give more flexibility to design controllers [11][12][13]. When the parameters are measurable with known bounds, improvement of performance, (e.g., lower cost, less conservative condition), can be obtained by introducing a parameter dependent controller.

This paper proposes guaranteed cost control methods for chaotic synchronization, which give an advantage of providing an upper bound on a given performance index. Using a convex representation for error dynamics, parameter independent and dependent state feedback LPV controllers with input constraint are proposed. Moreover, an output feedback dynamic LPV controller is proposed as well. The criteria for existence of the controllers are given in terms of LMIs, which can be easily solved by solving a convex optimization problem. Numerical examples are given to show the effectiveness of the proposed methods.

II. PROBLEM STATEMENT

Consider the following master-slave synchronization scheme of chaotic systems with subscripts m for master and s for slave :

$$Master \quad \begin{cases} \dot{x}_m(t) &= Ax_m(t) + f(x_m), \\ y_m(t) &= Hx_m(t), \end{cases} \tag{1}$$

$$Slave \quad \begin{cases} \dot{x}_s(t) &= Ax_s(t) + f(x_s) - Bu(t), \\ y_s(t) &= Hx_s(t), \end{cases} \tag{2}$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^q$ is the output vector, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times p}$ and $H \in \mathbb{R}^{q\times n}$ are constant matrices, $u \in \mathbb{R}^p$ is a control input vector, $f(x)$ is a nonlinear function satisfying Lipschitz condition.
Define synchronization error $e(t) = x_m(t) - x_s(t)$ to get error dynamic system :

$$\dot{e} = Ae(t) + f(x_m) - f(x_s) + Bu(t). \tag{3}$$

For a class of chaotic system (e.g., the Chua's circuit), the nonlinearity $f(x)$ has following properties [10] :

$$f(x_m) - f(x_s) = L(w)(x_m - x_s), \tag{4}$$

where $L(w)$ is a bounded matrix which depends on a time-varying parameter w_i as

$$L(w) = \sum_{i=1}^{r} w_i L_i, \tag{5}$$

$$\alpha_i \le w_i \le \beta_i, \quad w_m \le \sum_{i=1}^{r} w_i \le w_M. \tag{6}$$

Using the nonlinear property (4), the error dynamics (3) can be rewritten as following :

$$\begin{cases} \dot{e}(t) &= A(w)e(t) + Bu(t), \\ y_e(t) &= He(t), \end{cases} \tag{7}$$

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where $A(w) = A + L(w)$ and $y_e = y_m - y_s$.
We consider the controller design for both cases, state feedback and output feedback.

- Case I) State feedback
  The state feedback controller is of the form

$$u(t) = K(w)e(t), \tag{8}$$

where

$$K(w) = K_0 + \sum_{i=1}^{r} w_i K_i. \tag{9}$$

To consider input constraints which inevitably exist in practice, the designed controller have to satisfy following condition :

$$|u_i(t)| < \bar{u}_i, \quad i = 1, \ldots, p. \tag{10}$$

- Case II) Output feedback
  The output feedback controller is of the form

$$\begin{cases} \dot{\zeta}(t) &= A_c(w)\zeta(t) + B_c(w)y_e(t), \\ u(t) &= C_c\zeta(t) + D_c y_e(t), \end{cases} \tag{11}$$

where

$$A_c(w) = A_{c0} + \sum_{i=1}^{r} w_i A_{ci}, \tag{12}$$

$$B_c(w) = B_{c0} + \sum_{i=1}^{r} w_i B_{ci}, \tag{13}$$

and $\zeta(t)$ is the controller state vector.

Finally, for the performance criterion of the proposed controller, we introduce a performance index as following :

$$J(t) = \int_t^{\infty} e^T(\tau)Qe(\tau) + u^T(\tau)Ru(\tau)d\tau. \tag{14}$$

III. GUARANTEED COST CONTROLLER DESIGN

In this section, we present guaranteed cost controllers for chaotic synchronization.

A. State feedback controller

With the state feedback controller (8), the resulting closed-loop error dynamic system has the following form :

$$\dot{e}(t) = (A(w)e(t) + BK(w))e(t). \tag{15}$$

Theorem 1: The closed-loop error dynamic system (15) is globally asymptotically stable and the performance index (14) is bounded by γ for any initial states $e(0)$ if there exist positive symmetric matrices X, S, S_i, T, T_i and matrices $\bar{K}_0$, $\bar{K}_i$ with appropriate dimension satisfying the following LMIs

$$\begin{bmatrix} \Psi & \Psi_1^T & \Psi_2^T & \cdots & \Psi_r^T & \bar{K}_0^T & X \\ \Psi_1 & \Lambda_1 & \Delta_{12}^T & \cdots & \Delta_{1r}^T & \bar{K}_1^T & 0 \\ \Psi_2 & \Delta_{12} & \Lambda_2 & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots \\ \Psi_r & \Delta_{1r} & \cdots & \cdots & \Lambda_r & \bar{K}_r^T & \vdots \\ \bar{K}_0 & \bar{K}_1 & \cdots & \cdots & \bar{K}_r & -\gamma R^{-1} & \vdots \\ X & 0 & \cdots & \cdots & \cdots & \cdots & -\gamma Q^{-1} \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} I & e^T(0) \\ \star & X \end{bmatrix} > 0, \tag{17}$$

$$\begin{bmatrix} \Theta & \Theta_1^T & \Theta_2^T & \cdots & \Theta_r^T & \bar{K}_0^T \\ \Theta_1 & \bar{\Lambda}_1 & \bar{\Delta}_{12}^T & \cdots & \bar{\Delta}_{1r}^T & \bar{K}_1^T \\ \Theta_2 & \bar{\Delta}_{12} & \bar{\Lambda}_2 & \ddots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ \Theta_r & \bar{\Delta}_{1r} & \cdots & \cdots & \bar{\Lambda}_r & \bar{K}_1^T \\ \bar{K}_0 & \bar{K}_1 & \cdots & \cdots & \bar{K}_r & I \end{bmatrix} > 0, \tag{18}$$

where

$$\begin{aligned} \Psi &= A_0X + XA_0^T + B\bar{K}_0 + \bar{K}_0^T B^T \\ &\quad -2\sum_{i=1}^{r} \alpha_i S_i \beta_i - 2w_M S w_m, \end{aligned} \tag{19}$$

$$\begin{aligned} \Psi_i &= XA_i^T + \bar{K}_i^T B^T \\ &\quad +(\alpha_i + \beta_i)S_i + (w_M + w_m)S, \end{aligned} \tag{20}$$

$$\Theta = \bar{u}^2 X + 2\sum_{i=1}^{r} \alpha_i T_i \beta_i + 2w_M T w_m, \tag{21}$$

$$\Theta_i = -(\alpha_i + \beta_i)T_i - (w_M + w_m)T \tag{22}$$

$$\Lambda_i = -2S_i - 2S, \tag{23}$$

$$\bar{\Lambda}_i = 2T_i + 2T, \tag{24}$$

$$\Delta_{ij} = -2S. \tag{25}$$

$$\bar{\Delta}_{ij} = 2T. \tag{26}$$

Proof : Define a Lyapunov function as

$$V(t) = e^T(t)Pe(t), \tag{27}$$

and the time derivative of $V(t)$ is given by

$$\begin{aligned} \dot{V}(t) &= e^T(t)((A(w) + BK(w))^T P \\ &\quad + P(A(w) + BK(w)))e(t). \end{aligned} \tag{28}$$

Considering the performance index (14), the closed-loop (15) is said to be guaranteed cost by the proposed controller (8) if there exists a positive symmetric matrix P satisfying the following condition :

$$\begin{aligned} &(A(w) + BK(w))^T P + P(A(w) + BK(w)) \\ &\quad +Q + K(w)^T RK(w) < 0. \end{aligned} \tag{29}$$

Let $P/\gamma \triangleq X^{-1}$, then the inequality (29) is equivalent to

$$X(A(w)+BK(w))^T+(A(w)+BK(w))X +X\frac{Q}{\gamma}X+XK(w)^TRK(w)X<0. \quad (30)$$

Defining $K(w)X \triangleq \bar{K}(w)$, (30) can be rewritten as

$$A_0X+B\bar{K}_0+XA_0^T+\bar{K}_0^TB^T+XQX +\sum_{i=1}^{r} w_i\{A_iX+B\bar{K}_i+XA_i^T+\bar{K}_i^TB^T\} +(\bar{K}_0+\sum_{i=1}^{r} w_i\bar{K}_i)^TR(\bar{K}_0+\sum_{i=1}^{r} w_i\bar{K}_i)<0. \quad (31)$$

The constraints of the parameters (6) mean that there exist positive symmetric matrices S_i and S such that

$$(w_i-\alpha_i)S_i(w_i-\beta_i)<0, \quad (32)$$

$$(\sum_{i=1}^{r} w_i-w_m)S(\sum_{i=1}^{r} w_i-w_M)<0. \quad (33)$$

Considering (32)-(33) by applying S-procedure lemma, and using Schur complement, the condition (31) can be considered as a quadratic form (16) with the vector $\begin{bmatrix} I & w_1I & \cdots & w_rI\end{bmatrix}^T$.
If the inequality (16)-(17) holds, the performance index will be bounded by γ, because

$$\begin{bmatrix} I & e^T(0) \\ \star & X\end{bmatrix}>0 \quad \Longleftrightarrow \quad \gamma>e^T(0)Pe(0), \quad (34)$$

$$J(t)\le J(0)<V(0)=e^T(0)Pe(0)<\gamma, \quad (35)$$

and the guaranteed cost is obtained by minimizing γ.
Next, we consider input constraints (10) as following :

$$\bar{u}_i^2>u_i^2(t)=e^T(t)K^T(w)E_iE_iK(w)e(t), \quad (36)$$

where E_i is the ith row of the identity matrix $I_{p\times p}$.
Consider the invariant ellipsoid (34), then

$$\bar{u}_i^2-e^T(t)K^T(w)E_iE_iK(w)e(t) +\lambda(e^T(t)Pe(t)-\gamma)>0, \quad (37)$$

where $\lambda>0$ is a Lagrange multiplier. By substituting $\lambda=\bar{u}_i^2/\gamma$, the inequality (37) is equivalent to

$$\lambda P-K^T(w)K(w)>0. \quad (38)$$

Pre and post multiplying X, (39) becomes

$$\bar{u}_i^2X-\bar{K}^T(w)\bar{K}(w)>0. \quad (39)$$

Considering (32)-(33) and using Schur complement, the inequality (39) is equivalent to (18) with the vector $\begin{bmatrix} I & w_1I & \cdots & w_rI\end{bmatrix}^T$. This completes the proof. ■

B. Output feedback controller

With the output feedback controller (11), the resulting closed-loop error dynamic system has the following form :

$$\dot{\bar{x}}(t)=A_{cl}\bar{x}(t), \quad (40)$$

where

$$\bar{x}(t)=\begin{bmatrix} e^T(t) & \zeta^T(t)\end{bmatrix}^T, \quad (41)$$

$$A_{cl}=\begin{bmatrix} A(w)+BD_cH & BC_c \\ B_c(w) & A_c(w)\end{bmatrix}. \quad (42)$$

Theorem 2: The closed-loop error dynamic system (40) is globally asymptotically stable and the performance index (14) is bounded by γ for any initial states $e(0)$ if there exist matrices X, $\bar{X}$, M, N_0, N_i, G_0, G_i, D_c, S and S_i with appropriate dimension satisfying the following LMIs

$$\begin{bmatrix} \Phi & \Phi_1^T & \Phi_2^T & \cdots & \Phi_r^T & \Gamma_R^T & \Gamma_Q^T \\ \Phi_1 & \Lambda_1 & \Delta_{12} & \cdots & \Delta_{2r} & 0 & 0 \\ \Phi_2 & \Delta_{12}^T & \Lambda_2 & \ddots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \vdots & \vdots & \vdots \\ \Phi_r & \Delta_{1r}^T & \cdots & \cdots & \Lambda_r & \vdots & \vdots \\ \Gamma_R & 0 & \cdots & \cdots & \cdots & -R^{-1} & \vdots \\ \Gamma_Q & 0 & \cdots & \cdots & \cdots & \cdots & -Q^{-1}\end{bmatrix}<0, \quad (43)$$

$$\begin{bmatrix} \gamma & e^T(0)X \\ \star & X\end{bmatrix}>0, \quad (44)$$

$$\begin{bmatrix} \bar{X} & I \\ \star & X\end{bmatrix}>0, \quad (45)$$

where

$$\Phi = \begin{bmatrix} A_0\bar{X}+\bar{X}^TA_0^T+BM+M^TB^T & A_0+BD_cH+N_0^T \\ \star & XA_0+A_0^TX+G_0H+H^TG_0^T\end{bmatrix} -2\sum_{i=1}^{r}\alpha_iS_i\beta_i-2w_MSw_m, \quad (46)$$

$$\Phi_i = \begin{bmatrix} \bar{X}A_i^T & N_i^T \\ A_i^T & A_i^TX+H^TG_i^T\end{bmatrix} \quad (47)$$

$$+(\alpha_i+\beta_i)S_i+(w_M+w_m)S, \quad (48)$$

$$\Gamma_R = \begin{bmatrix} M & DcH\end{bmatrix}, \quad (49)$$

$$\Gamma_Q = \begin{bmatrix} \bar{X} & I\end{bmatrix}, \quad (50)$$

$$\Lambda_i = -2S_i-2S, \quad (51)$$

$$\Delta_{ij} = -2S. \quad (52)$$

Proof : Define a Lyapunov function

$$V(t)=\bar{x}^TP\bar{x}, \quad (53)$$

and consider the performance index (14). The closed-loop (40) is said to be guaranteed cost by the proposed controller

(11) if there exists a positive symmetric matrix P satisfying the following condition :

$$\begin{aligned} &A_{cl}^T P + P A_{cl} + \begin{bmatrix} I \\ 0 \end{bmatrix} Q \begin{bmatrix} I & 0 \end{bmatrix} \\ &+ \begin{bmatrix} H^T D_c^T \\ C_c^T \end{bmatrix} R \begin{bmatrix} D_c H & C_c \end{bmatrix} < 0. \end{aligned} \tag{54}$$

Let $P = \begin{bmatrix} X & Y \\ Y^T & Z \end{bmatrix}$ and $P^{-1} = \begin{bmatrix} \bar{X} & \bar{Y} \\ \bar{Y}^T & \bar{Z} \end{bmatrix}$, then the equality $P^{-1}P = I$ yields

$$\bar{Y} Y^T = I - \bar{X} X. \tag{55}$$

Define

$$\Upsilon_1 = \begin{bmatrix} \bar{X} & I \\ \bar{Y}^T & 0 \end{bmatrix}, \quad \Upsilon_2 = \begin{bmatrix} I & X \\ 0 & Y^T \end{bmatrix}, \tag{56}$$

then it follows that

$$P\Upsilon_1 = \Upsilon_2, \quad \Upsilon_1^T P \Upsilon_1 = \Upsilon_1^T \Upsilon_2 = \begin{bmatrix} \bar{X} & I \\ I & X \end{bmatrix}. \tag{57}$$

Therefore, if the inequality (45) is satisfied, $P > 0$ is guaranteed.

Using congruence transform with the matrix $\begin{bmatrix} \Upsilon_1 & 0 \\ 0 & I \end{bmatrix}$, the inequality (54) can be rewritten as

$$\begin{aligned} &\begin{bmatrix} A(w)\bar{X} + BM + \bar{X}A^T(w) + M^T B^T & A(w) + BD_c H + N^T(w) \\ \star & XA(w) + G(w)H + A(w)^T X + H^T G(w)^T \end{bmatrix} \\ &+ \begin{bmatrix} \bar{X} \\ I \end{bmatrix} Q \begin{bmatrix} \bar{X} & I \end{bmatrix} + \begin{bmatrix} M^T \\ H^T D_c^T \end{bmatrix} R \begin{bmatrix} M & D_c H \end{bmatrix} < 0, \end{aligned} \tag{58}$$

where

$$\begin{aligned} M &= D_c H \bar{X} + C_c \bar{Y}^T, &(59)\\ N(w) &= XA(w)\bar{X} + XBD_c H\bar{X} + YB_c(w)H\bar{X} \\ &\quad + XBC_c\bar{Y}^T + YA_c(w)\bar{Y}^T, &(60)\\ G(w) &= XBD_c + YB_c(w). &(61) \end{aligned}$$

Analogously to Theorem 1, considering (32)-(33) by applying S-procedure lemma, and using Schur complement, the condition (58) can be considered as a quadratic form (43) with the vector $\begin{bmatrix} I & w_1 I & \cdots & w_r I \end{bmatrix}^T$.

If the inequality (43) holds, the performance index will be bounded by

$$J(t) < V(0) = \begin{bmatrix} e^T(0) & 0 \end{bmatrix} P \begin{bmatrix} e(0) \\ 0 \end{bmatrix} < \gamma. \tag{62}$$

Using Schur complement, (62) can be rewritten as (44) and the guaranteed cost is obtained by minimizing γ. This completes the proof. ■

IV. NUMERICAL EXAMPLES

In this section, we present two examples to show the effectiveness of our results.

Example 1. Consider the following Chua's circuits in [10] :

$$\begin{cases} \dot{x} = a(y - x - f(x)), \\ \dot{y} = x - y + z, \\ \dot{z} = -by, \end{cases} \tag{63}$$

with the nonlinear characteristic

$$f(x) = m_1 x + \frac{1}{2}(m_0 - m_1)(|x + c| - |x - c|), \tag{64}$$

where $a = 9.78$, $b = 14.97$, $m_0 = -1.31$, $m_1 = -0.75$ and $c = 1$. The nonlinearity $f(x)$ has the property of

$$f(x_m) - f(x_s) = w \begin{bmatrix} -a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_m - x_s \\ y_m - y_s \\ z_m - z_s \end{bmatrix} \tag{65}$$

where $m_0 < w < m_1$. Then, the closed-loop error dynamic system (8) is described by

$$\begin{aligned} A(w) &= \begin{bmatrix} -a & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix} + w \begin{bmatrix} -a & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, &(66)\\ B &= \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T. &(67) \end{aligned}$$

The performance index is given by $Q = I$ and $R = I$. The initial conditions are $x_m(t) = \begin{bmatrix} 0.001 & 0.001 & 0.001 \end{bmatrix}^T$, $x_s(t) = \begin{bmatrix} 1.001 & 1.001 & 1.001 \end{bmatrix}^T$.

Results obtained from Theorem 1 are compared with ones in [10] and listed in Table I. The performance index from Theorem 1 is much lower than one in [10] and the stability condition corresponding to input constraint $\bar{u}$ is less conservative. Fig. 1 and Fig. 2 show the stable response of error $e^T(t) = \begin{bmatrix} e_1^T(t) & e_2^T(t) & e_3^T(t) \end{bmatrix}^T$ and the input which is limited in $\bar{u}$.

Example 2. Consider the hyper-chaotic system which combines two Chua's circuits shown in [14][15] :

$$\begin{cases} \dot{x}_1 = a(x_2 - f(x_1)) + u_1(t), \\ \dot{x}_2 = x_1 - x_2 + x_3, \\ \dot{x}_3 = -bx_2, \\ \dot{x}_4 = a(x_5 - f(x_4)) + K(x_4 - x_1)) + u_2(t), \\ \dot{x}_5 = x_4 - x_5 + x_6, \\ \dot{x}_6 = -bx_5, \end{cases}$$
$$y = \begin{bmatrix} x_1^T(t) & x_4^T(t) \end{bmatrix}^T$$

with the nonlinear characteristic

$$f(x) = m_1 x + \frac{1}{2}(m_0 - m_1)(|x + c| - |x - c|),$$

where $m_0 = -1/7$, $m_1 = 2/7$, $a = 9$, $b = 14.28$ and

$K = 0.01$. In this case, we define

$$A_0 = \begin{bmatrix} -am_1 & a & 0 & 0 & 0 & 0 \\ 1 & -1 & 1 & 0 & 0 & 0 \\ 0 & -b & 0 & 0 & 0 & 0 \\ -K & 0 & 0 & K-am_1 & a & 0 \\ 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 & -b & 0 \end{bmatrix},$$

$$A_1 = \begin{bmatrix} -a(m_0-m_1) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -a(m_0-m_1) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix},$$

then the parameters are bounded as $0 < w_i < 1$ and $0 < \sum_{i=1}^{2} w_i < 2$. The performance index is also given by $Q = I$ and $R = I$. The initial conditions are $x_m(t) = \begin{bmatrix} 0.001 & 0.001 & 0.001 & 1.001 & 1.001 & 1.001 \end{bmatrix}^T$, $x_s(t) = \begin{bmatrix} 1.001 & 1.001 & 1.001 & 0.001 & 0.001 & 0.001 \end{bmatrix}^T$. Using the output signal, the designed output feedback controller (11) is derived by Theorem 2. Due to the limitation of the length of this paper, we provide a part of the feasible solution.

$$A_{c0} = \left[\begin{matrix} -6.6551 & -2.8298 & -2.3307 \\ 61.3014 & -11.0199 & 0.0610 \\ -133.0630 & 16.4107 & -5.8295 \\ -186.2993 & 23.3737 & 1.3759 \\ -686.7579 & 84.5922 & -212.9004 \\ 260.2748 & -39.7076 & -848.7878 \end{matrix} \right.$$

$$\left. \begin{matrix} 0.8879 & -1.3623 & 0.5288 \\ -3.7500 & 4.0690 & -1.0896 \\ 9.2998 & -12.2051 & 3.7332 \\ 10.2628 & -17.9354 & 7.2634 \\ 232.3817 & -76.2339 & 15.3359 \\ 686.9336 & -0.9964 & -83.1643 \end{matrix} \right].$$

Fig. 3 and Fig. 4 show the stable response of error and the input with the performance index $\gamma = 201.6208$. The performance index can be reduced by $\gamma^* = 141.5886$.

V. Conclusion

This paper proposed guaranteed cost controllers for a class of chaos synchronization. By a convex representation of error dynamics, we regarded the synchronization problem as a LPV problem. This approach gives more flexibility to design controllers when it comes to reducing performance index or conservatism. We designed state and output feedback LPV controllers and the results were given in numerical examples.

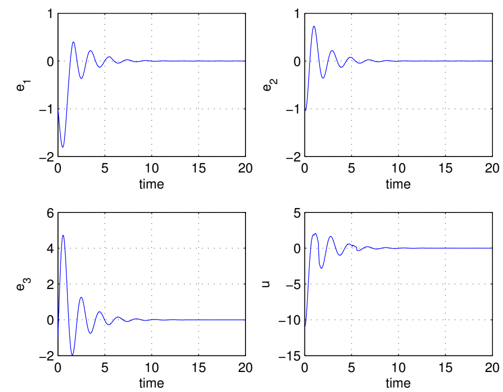

Fig. 1. The response of the error and the constrained input ($\bar{u} = 12$) in Example 1.

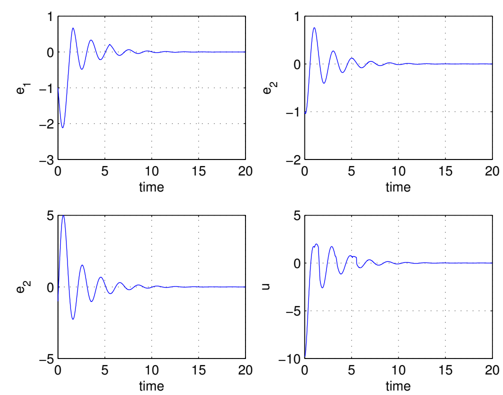

Fig. 2. The response of the error and the constrained input ($\bar{u} = 10.516$) in Example 1.

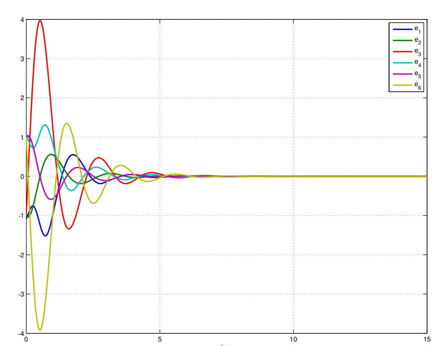

Fig. 3. The response of the error in Example 2.

TABLE I

COMPARED RESULTS IN EXAMPLE 1.

Control gain when $\bar{u} = 12$	
Theorem 1	$K_0 = \begin{bmatrix} 0.5305 & -4.9998 & -2.3911 \end{bmatrix}$ $K_1 = \begin{bmatrix} 3.6029 & -0.8766 & -1.3714 \end{bmatrix}$
[10]	$K = \begin{bmatrix} 3.6956 & -7.5851 & -0.3453 \end{bmatrix}$
Performance index γ^*	
Theorem 1	160.9349
[10]	365.5516
Minimum allowable $\bar{u}$	
Theorem 1	10.516
[10]	11.293

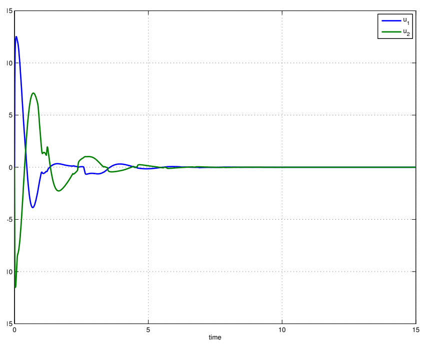

Fig. 4. The response of the inputs in Example 2.

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