Robust guaranteed cost control for uncertain linear differential systems of neutral type

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Abstract

In this paper, the robust guaranteed cost control problem for a class of uncertain linear differential systems of neutral type with a given quadratic cost functions is investigated. The uncertainty is assumed to be norm-bounded and time-varying nonlinear. The problem is to design a state feedback control laws such that the closed-loop system is robustly stable and the closed-loop cost function value is not more than a specified upper bound for all admissible uncertainty and time delay. A criterion for the existence of such controllers is derived based on the matrix inequality approach combined with the Lyapunov method. A parameterized characterization of the robust guaranteed cost controllers is given in terms of the feasible solutions to the certain matrix inequalities. A numerical example is given to illustrate the proposed method.

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1. Introduction

During the last decades, the robust stability and stabilization problem for uncertain dynamic systems with delay has received much attention and many
papers dealing with this problem have appeared because of the existence of delays in various practical control problems and also because of the fact that the delay is frequently a source of instability and performance degradation of systems. Especially, in recent years, considerable attention has been focused on the stability analysis of various neutral differential systems [1–3]. The theory of neutral delay-differential systems is of both theoretical and practical interest. For example, functional differential equations of neutral type are the natural models of fluctuations of voltage and current in problems arising in transmission lines. In Salamon [4] a four-dimensional linear neutral system was derived for the current and voltage in a lossless transmission line. Also, the neutral systems often appear in the study of automatic control, population dynamics, and vibrating masses attached to an elastic bar. In the literature, various analysis techniques such as Lyapunov technique, characteristic equation method, or state solution approach have been utilized to derive stability criteria for asymptotic stability of the systems by many researchers [5–11]. However, the stabilization problem of the neutral systems has been explored by only a few researcher [12–15]. A periodic output feedback controller to stabilize the neutral system is proposed by Tarn et al. [13]. To treat the same problem, a transformation technique is presented by Fiagbedzi [14]. Also, Ma et al. [15] developed a control method utilizing a new differential-difference inequality and the transformation technique, and derived a delay-independent sufficient condition for stabilization of the system. However, their method have some constraint on the structure of system matrices.

On the other hand, when controlling a real plant, it is also desirable to design a control systems which is not only asymptotically stable but also guarantees an adequate level of performance. One way to address the robust performance problem is to consider a linear quadratic cost function. This approach is the so-called guaranteed cost control [16]. The approach has the advantage of providing an upper bound on a given performance index and thus the system performance degradation incurred by uncertainty and time delay is guaranteed to be less than this bound. Recently, there have been some efforts to tackle the guaranteed cost controller design problem [17,18]. To date, unfortunately, the topic of robust guaranteed cost control for neutral differential systems has been received very little attention.

In this paper, we consider a class of uncertain linear differential systems of neutral type. The nonlinear uncertainty is assumed to be norm-bounded. Using the Lyapunov functional technique combined with the matrix inequality technique, we develop a robust guaranteed cost control for this system via memoryless state feedback, which makes the closed-loop system asymptotically stable and guarantees an adequate level of performance. A stabilization criterion for the existence of the guaranteed cost controller is derived in terms of matrix inequalities, and theirs solutions provide a parameterized representation of the control. The matrix inequality can be easily solved by various efficient
convex optimization algorithms [20]. Finally, a numerical example is given to illustrate the proposed design method.

Through the paper, \(\mathbb{R}^n\) denotes \(n\)-dimensional Euclidean space, \(\mathbb{R}^{n \times m}\) is the set of all \(n \times m\) real matrices, \(I\) denotes identity matrix of appropriate order, and \(*\) represents the elements below the main diagonal of a symmetric block matrix. \(\|\cdot\|\) denotes Euclidean norm of a given vector. \(\text{tr}(\cdot)\) and \(\lambda_{\min}(\cdot)\) denote the trace and the minimum eigenvalue of the matrix \((\cdot)\), respectively. \(\text{diag}\{\cdots\}\) denote the block diagonal matrix. The notation \(X > Y\), where \(X\) and \(Y\) are matrices of same dimensions, means that the matrix \(X - Y\) is positive definite.

2. Main result

Consider a class of uncertain neutral differential system of the form:
\[
\dot{x}(t) = A_0x(t) + A_1x(t - h) + C\dot{x}(t - h) + f(t, x(t), x(t - h)) + Bu(t),
\]
with the initial condition function
\[
x(t_0 + \theta) = \phi(\theta), \quad \forall \theta \in [-h, 0],
\]
where \(x(t) \in \mathbb{R}^n\) is the state vector, \(A_0 \in \mathbb{R}^{n \times n}\), \(A_1 \in \mathbb{R}^{n \times n}\), \(C \in \mathbb{R}^{n \times n}\) and \(B \in \mathbb{R}^{n \times m}\) are constant system matrices, \(u(t) \in \mathbb{R}^m\) is a control variable, \(f(\cdot)\) is unknown and represent the nonlinear parameter uncertainty, \(h\) is a positive constant time-delay, and \(\phi(\cdot)\) is the given continuously differentiable function on \([-h, 0]\).

Throughout this paper, the following assumptions are made on the system (1).

**Assumption 1.** The pair \((A_0, B)\) is completely controllable.

**Assumption 2.** There exist nonnegative constants \(z_0\) and \(z_1\) such that
\[
\|f(t, x(t), x(t - h))\| \leq z_0\|x(t)\| + z_1\|x(t - h)\|.
\]

Now, we are interested in designing a memoryless state feedback controller for the system (1) as
\[
u(t) = -B^TPx(t),
\]
where \(P\) is a positive-definite matrix to be designed.

Associated with the system (1) is the following quadratic cost function
\[
J = \int_0^\infty (x^T(t)Qx(t) + u^T(t)Su(t))\,dt,
\]
where \(Q \in \mathbb{R}^{n \times n}\) and \(S \in \mathbb{R}^{m \times m}\) are given positive-definite matrices.
Here, the objective of this paper is to develop a procedure to design a state feedback controller $u(t)$ for the system (1) and cost function (5), such that the resulting closed-loop subsystem given by

$$
\dot{x}(t) = (A_0 - BB^T P)x(t) + A_1 x(t - h) + Cx(t - h) + f(t, x(t), x(t - h)),
$$

is asymptotically stable and the closed-loop value of the cost function (5) satisfies $J \leq J^*$, where $J^*$ is some specified constant.

**Definition 1.** For the uncertain neutral system (1) and cost function (5), if there exist a control law $u'(t)$ and a positive $J^*$ such that for all admissible uncertainty, the closed-loop system (6) is asymptotically stable and the closed-loop value of the cost function (5) satisfies $J \leq J^*$, then $J^*$ is said to be a guaranteed cost and $u'(t)$ is said to be a guaranteed cost control law of the system (1) and cost function (5).

Before proceeding further, we will state well known lemmas.

**Lemma 1** [19]. For any real vector $D$ and $E$ with appropriate dimension and any positive scalar $\delta$, we have

$$
DE + E^T D^T \leq \delta DD^T + \delta^{-1} E^T E.
$$

**Lemma 2** [20]. The linear matrix inequality

$$
\begin{bmatrix}
Z(x) & Y(x) \\
Y^T(x) & W(x)
\end{bmatrix} > 0,
$$

is equivalent to

$$
W(x) > 0, \quad Z(x) - Y(x)W^{-1}(x)Y^T(x) > 0,
$$

where $Z(x) = Z^T(x)$, $W(x) = W^T(x)$ and $Y(x)$ depend affinely on $x$.

Now, we establish a delay-independent criterion in terms of matrix inequality, for stabilization of the uncertain neutral delay-differential system (1) using the Lyapunov method.

**Theorem 1.** For given $Q > 0$ and $S > 0$, $u(t) = -B^T Px(t)$ is a robust guaranteed cost controller for the system (1) if there exist positive scalars $\varepsilon, \beta_0, \beta_1, \beta_2$ and $\beta_3$, and a positive-definite matrix $X$ satisfying the following matrix inequality:
\[
\Omega(X, \varepsilon, \beta_0, \beta_1, \beta_2, \beta_3)
\]
\[
= \begin{bmatrix}
\Omega_{1,1} & \Omega_{1,2} & (A_1 + XA_0^T A_1 - BB^T A_1) & 0 & C + XA_0^T C - BB^T C \\
0 & \Omega_{2,2} & 0 & 0 \\
* & * & (A_1^T A_1 - \varepsilon I + 4x_0^2 I + \beta_1 A_1^T A_1) & \Omega_{3,4} & A_1^T C \\
* & * & * & \Omega_{4,4} & 0 \\
* & * & * & * & C^T C - I + \beta_2 C^T C
\end{bmatrix}
\]
< 0.

(7)

where \( X = P^{-1} \) and

\[
\Omega_{1,1} = XA_0^T + A_0 X - 2BB^T + BB^T BB^T + \beta_3 BB^T BB^T - XA_0^T BB^T - BB^T A_0 X + BSB^T + \beta_0 I,
\]

\[
\Omega_{1,2} = \left[ \sqrt{2}XA_0^T \varepsilon X \ XQ \ 2x_0 X \ \sqrt{2}x_0 X \ \sqrt{2}x_0 X \ \sqrt{2}x_0 X \right],
\]

\[
\Omega_{2,2} = \text{diag}\{-I, -\varepsilon I, -Q, -I, -\beta_0 I, -\beta_1 I, -\beta_2 I, -\beta_3 I\},
\]

\[
\Omega_{3,4} = \left[ \sqrt{2}x_1 I \ \sqrt{2}x_1 I \ \sqrt{2}x_1 I \ \sqrt{2}x_1 I \right],
\]

\[
\Omega_{4,4} = \text{diag}\{-\beta_0 I, -\beta_1 I, -\beta_2 I, -\beta_3 I\}.
\]

Then, the upper bound of the quadratic cost function \( J \) is

\[
J^* = x^T(0)X^{-1}x(0) + \int_{-h}^{0} \hat{x}^T(s)\hat{x}(s) \, ds + \varepsilon \int_{-h}^{0} x^T(s)x(s) \, ds.
\]

Proof. The functional given by

\[
V = V_1 + V_2 + V_3,
\]

(8)

where

\[
V_1 = x^T(t)Px(t),
\]

(9)

\[
V_2 = \int_{-h}^{t} \hat{x}^T(s)\hat{x}(s) \, ds,
\]

(10)

\[
V_3 = \varepsilon \int_{-h}^{t} x^T(s)x(s) \, ds,
\]

(11)

is a legitimate Lyapunov functional candidate [1].

Taking the time derivative of \( V \) along the solution of (6), we have

\[
\dot{V}_1 = x^T(A_0^T P + PA_0 - 2PPBB^T P)x + 2x^T PA_1 x_h + 2x^T PC\hat{x}_h + 2x^T Pf
\]

(12)
\[ \dot{V}_2 = x^T \dot{x} - x_h^T \dot{x}_h \]
\[ = x^T A_0^T A_0 x + x^T PBB^T BB^T P x + x_h^T A_1^T A_1 x_h + x_h^T C^T C \dot{x}_h - x_h^T \dot{x}_h \]
\[ + f^T f - 2x^T A_0^T BB^T P x + 2x^T A_0^T A_1 x_h + 2x_h^T A_1^T C \dot{x}_h \]
\[ - 2x^T PBB^T A_1 x_h - 2x^T PBB^T C \dot{x}_h + 2x_h^T A_1^T C \dot{x}_h + 2x^T A_0^T f \]
\[ - 2x^T PBB^T f + 2x_h^T A_1^T f + 2x_h^T C^T f, \tag{13} \]
\[ \dot{V}_3 = e^T x - e^T x_h, \tag{14} \]

where \( x, x_h \) and \( \dot{x}_h \) denote \( x(t), x(t-h) \) and \( \dot{x}(t-h) \), respectively.

Using Lemma 1, the term \( f^T f \) on right-hand side of (13) satisfies the following inequality:
\[ f^T f = \| f \|^2 \leq x_0^2 \| x \|^2 + x_1^T \| x_h \|^2 + 2x_0 x_1 \| x \| \cdot \| x_h \| \]
\[ \leq x_0^2 \| x \|^2 + x_1^T \| x_h \|^2 + (x_0^2 \| x \|^2 + x_1^T \| x_h \|^2) \]
\[ = 2x_0^2 x^T x + 2x_1^T x_h, \tag{15} \]

Again, using Lemma 1 and (15), the other terms on right-hand side of (12) and (13) satisfy
\[ 2x^T P f \leq \beta_0 x^T P P x + \beta_0^{-1} f^T f \leq \beta_0 x^T P P x + \beta_0^{-1} 2x_0^2 x^T x + \beta_0^{-1} 2x_1^T x_h, \]
\[ 2x^T A_0^T f \leq x^T A_0^T A_0 x + f^T f \leq x^T A_0^T A_0 x + 2x_0^2 x^T x + 2x_1^T x_h, \]
\[ 2x_h^T A_1^T f \leq \beta_1 x_h^T A_1 x_h + \beta_1^{-1} f^T f \leq \beta_1 x_h^T A_1 x_h + \beta_1^{-1} (2x_0^2 x^T x + 2x_1^T x_h), \]
\[ 2x_h^T C^T f \leq \beta_2 x_h^T C^T C \dot{x}_h + \beta_2^{-1} f^T f \leq \beta_2 x_h^T C^T C \dot{x}_h + \beta_2^{-1} (2x_0^2 x^T x + 2x_1^T x_h), \]
\[ -2x^T PBB^T f \leq \beta_3 x^T PBB^T BB^T P x + \beta_3^{-1} 2x_0^2 x^T x + \beta_3^{-1} 2x_1^T x_h, \tag{16} \]

where \( \beta_0, \beta_1, \beta_2 \) and \( \beta_3 \) are positive scalars to be chosen.

Then, we obtain
\[ \dot{V} = \dot{V}_1 + \dot{V}_2 + \dot{V}_3 \]
\[ \leq \xi^T(t) M(P, e, \beta_0, \beta_1, \beta_2, \beta_3) \xi(t) - x^T(t)(Q + PBB^T P) x(t) \tag{17} \]

where \( \xi(t) = [x^T \ x_h^T \ x_h^T] \) and
\[ M(P, e, \beta_0, \beta_1, \beta_2, \beta_3) = \begin{bmatrix} M_{11} & (PA_1 + A_0^T A_1 - PBB^T A_1) & (PC + A_0^T C - PBB^T C) \\ * & M_{22} & A_1^T C \\ * & * & C^T C - I + \beta_2 C^T C \end{bmatrix} \]
with
\[
M_{11} = A_0^T P + PA_0 - 2PBB^T P + 2A_0^T A_0 + PBB^T BB^T P + \beta_3 PBB^T BB^T P
\]
\[ - A_0^T BB^T P - PBB^T A_0 + \varepsilon I + Q + PBSB^T P + 4x_0^2 + \beta_0 PP + \beta_0^{-1} 2x_0^2 I \]
\[ + \beta_1^{-1} 2x_0^2 I + \beta_2^{-1} 2x_0^2 I + \beta_3^{-1} 2x_0^2 I \]
and
\[
M_{22} = A_1^T A_1 - \varepsilon I + 4x_1^2 + \beta_1 A_1^T A_1 + \beta_0^{-1} 2x_1^2 I + \beta_1^{-1} 2x_1^2 I \\
+ \beta_2^{-1} 2x_1^2 I + \beta_3^{-1} 2x_1^2 I.
\]
Therefore, if \( M(\cdot) < 0 \), there exist a positive scalar \( \gamma \) such that
\[
\dot{V} < -\gamma \| x(t) \|^2, \quad \gamma = \lambda_m(Q + PBSB^T P), \tag{18}
\]
which guarantees the asymptotic stability of the system [1].

Pre- and post-multiplying the matrix \( M(\cdot) \) by \( \mathcal{T}^T \) and \( \mathcal{T} \), where
\[
\mathcal{T} = \begin{bmatrix}
X & 0 & 0 \\
0 & I & 0 \\
0 & 0 & I
\end{bmatrix},
\]
the fact that \( M(\cdot) < 0 \) is equivalent to
\[
\begin{pmatrix}
X A_0^T + A_0 X - 2BB^T \\
+ 2X A_0^T A_0 X + BB^T BB^T \\
+ \beta_3 BB^T BB^T - X A_0^T BB^T \\
- BB^T A_0 X + \varepsilon XX + QX \\
+ BSB^T + 4x_0^2 XX + \beta_0 I \\
+ \beta_0^{-1} 2x_0^2 XX + \beta_1^{-1} 2x_0^2 XX \\
+ \beta_2^{-1} 2x_0^2 XX + \beta_3^{-1} 2x_0^2 XX
\end{pmatrix}
\begin{pmatrix}
A_1 + X A_0^T A_1 \\
- BB^T A_1 \\
C + X A_0^T C \\
- BB^T C
\end{pmatrix}
\]
\[
\begin{pmatrix}
A_1^T A_1 - \varepsilon I \\
+ 4x_1^2 I + \beta_1 A_1^T A_1 \\
+ \beta_0^{-1} 2x_1^2 I + \beta_1^{-1} 2x_1^2 I \\
+ \beta_2^{-1} 2x_1^2 I + \beta_3^{-1} 2x_1^2 I
\end{pmatrix}
\begin{pmatrix}
A_1^T \\
C^T C - I + \beta_2 C^T C
\end{pmatrix}
\]
\[
< 0. \tag{19}
\]
By Lemma 2 (Schur complement), the inequality (19) is equivalent to the matrix inequality (7).

Here, the matrix inequality (7) implies that
\[
\dot{V} < -x^T(t)(Q + PBSB^T P)x(t) < 0. \tag{20}
\]
Noting $Q > 0$ and $S > 0$, this implies that the system (6) is asymptotically stable by Lyapunov stability theory. Furthermore, from (20) we have

$$x^T(t)(Q + PBSB^TP)x(t) < -\dot{V}.$$ 

Integrating both sides of the above inequality from 0 to $T_f$ leads to

$$\int_0^{T_f} x^T(t)(Q + PBSB^TP)x(t) \, dt < V(0) - V(T_f)$$

$$= (x^T(0)Px(0) - x^T(T_f)Px(T_f))$$

$$+ \left( \int_{-h}^{0} \dot{x}^T(s)\dot{x}(s) \, ds - \int_{T_f - h}^{T_f} \dot{x}^T(s)\dot{x}(s) \, ds \right)$$

$$+ \left( \varepsilon \int_{-h}^{0} x^T(s)x(s) \, ds - \varepsilon \int_{T_f - h}^{T_f} x^T(s)x(s) \, ds \right).$$

As the closed-loop system (6) is asymptotically stable, when $T_f \to \infty$,

$$x^T(T_f)Px(T_f) \to 0, \quad \int_{T_f - h}^{T_f} \dot{x}^T(s)\dot{x}(s) \, ds \to 0, \quad \varepsilon \int_{T_f - h}^{T_f} x^T(s)x(s) \, ds \to 0.$$ 

Hence we get

$$\int_0^{\infty} x^T(t)(Q + PBSB^TP)x(t) \, dt$$

$$< V(0) = x^T(0)Px(0) + \int_{-h}^{0} \dot{x}^T(s)\dot{x}(s) \, ds + \varepsilon \int_{-h}^{0} x^T(s)x(s) \, ds \triangleq J^*.$$ 

(21)

**Remark 1.** The problem (7) is to determine whether the problem is feasible or not. It is called the feasibility problem. Also, the solutions of the problem can be found by solving generalized eigenvalue problem in $\beta_0, \beta_1, \beta_2, \beta_3, X$ and $\varepsilon$, which is a quasiconvex optimization problem. Note that a locally optimal point of a quasiconvex optimization problem with strictly quasiconvex objective is globally optimal. For details, see Boyd et al. [20]. Various efficient convex optimization algorithms can be used to check whether the matrix inequality (7) is feasible. In this paper, in order to solve the matrix inequality, we utilize Matlab’s LMI Control Toolbox [21], which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithms [20].
Theorem 1 presents a method of designing a state feedback guaranteed cost controller. The following theorem presents a method of selecting a controller minimizing the upper bound of the guaranteed cost (21).

**Theorem 2.** Consider the system (6) with cost function (5). If the following optimization problem

\[
\min_{X>0,e>0,x>0,G_1>0,G_2>0,\beta_0>0,\beta_1>0,\beta_2>0,\beta_3>0} \{x + \text{tr}(G_1) + \text{tr}(G_2)\} \tag{22}
\]

subject to

(i) matrix inequality (7)

(ii) \[
\begin{bmatrix}
-x & x^T(0) \\
x(0) & -X
\end{bmatrix} < 0,
\]

(iii) \[
\begin{bmatrix}
-G_1 & e.M^T \\
 e.M & -eI
\end{bmatrix} < 0,
\]

(iv) \[
\begin{bmatrix}
-G_2 & \mathcal{N}^T \\
\mathcal{N} & -I
\end{bmatrix} < 0,
\]

has a positive solution set \((X, e, \beta_0, \beta_1, \beta_2, \beta_3, x, G_1, G_2)\), then the control law (4) is an optimal robust guaranteed cost control law which ensures the minimization of the guaranteed cost (21) for the neutral system (6), where \(\int_{-h}^{0} \dot{x}(s)x^T(s)\,ds = \mathcal{N}\mathcal{N}^T\) and \(\int_{-h}^{0} x(s)x^T(s)\,ds = M.M^T\).

**Proof.** By Theorem 1, (i) in (22) is clear. Also, it follows from the Lemma 2 that (ii), (iii), and (iv) in (22) are equivalent to \(x^T(0)X^{-1}x(0) < x, e.M^T.M < G_1, \text{ and } \mathcal{N}^T\mathcal{N} < G_2\), respectively. On the other hand,

\[
\varepsilon \int_{-h}^{0} x^T(s)x(s)\,ds = \varepsilon \int_{-h}^{0} \text{tr}(x^T(s)x(s))\,ds = \text{tr}(e.M^T.M) = \text{tr}(e.M^T.M) = \text{tr}(e.M^T.M) < \text{tr}(G_1),
\]

\[
\int_{-h}^{0} \dot{x}^T(s)\dot{x}(s)\,ds = \int_{-h}^{0} \text{tr}(\dot{x}^T(s)\dot{x}(s))\,ds = \text{tr}(\mathcal{N}\mathcal{N}^T) = \text{tr}(\mathcal{N}\mathcal{N}^T) < \text{tr}(G_2).
\]

Hence, it follows from (21) that

\[
J^* < x + \text{tr}(G_1) + \text{tr}(G_2).
\]

Thus, the minimization of \(x + \text{tr}(G_1) + \text{tr}(G_2)\) implies the minimization of the guaranteed cost for the system (6). In the light of Remark 1, this quasiconvex optimization problem guarantees that a global optimum, when it exists, is reachable. \(\square\)
**Remark 2.** Ma et al. [15] investigated the stabilization problem of a class of neutral systems. However, their method is only applicable to the system with single input and have restrictions on the structure of system matrices $A_1$ and $C$.

**Example 1.** Consider the following linear differential system of neutral type:
\[
\dot{x}(t) = A_0x(t) + A_1x(t - h) + C\dot{x}(t - h) + f(t, x(t), x(t - h)) + Bu(t),
\]  
where
\[
A_0 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} 0 & 0.5 \\ -0.2 & -0.2 \end{bmatrix},
\]
\[
C = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}, \quad h = 0.3
\]
and the initial condition of the system is as follows:
\[
x(t) = \begin{bmatrix} 0.5e^{-t} - 0.5e^{-t} \end{bmatrix}^T, \quad \text{for} \quad -0.3 \leq t \leq 0.
\]
Actually, when the control input is not forced to the system (23), i.e., $u(t) = 0$, the system is unstable since the states of the system go to infinity as $t \to \infty$.

Here, associated with this system is the cost function of (5) with $Q = I$ and $S = 0.1I$.

From the relations, $\int_{-h}^{0} \dot{x}(s)x^T(s)ds = \mathcal{N}\mathcal{N}^T$ and $\int_{-h}^{0} x(s)x^T(s)ds = \mathcal{M}\mathcal{M}^T$, we have
\[
\mathcal{M} = \begin{bmatrix} 0.1614 & -0.1742 \\ -0.1742 & 0.2691 \end{bmatrix}, \quad \mathcal{N} = \begin{bmatrix} 0.1614 & 0.1742 \\ 0.1742 & 0.2691 \end{bmatrix}.
\]

Now, solving the optimization problem of Theorem 2, we find the positive solutions of the matrix inequalities (22) for the system as
\[
X = \begin{bmatrix} 0.0462 & -0.0356 \\ -0.0356 & 0.1187 \end{bmatrix}, \quad \varepsilon = 13.5927, \quad \beta_0 = 0.0145,
\]
\[
\beta_1 = 0.5981, \quad \beta_2 = 0.7361, \quad \beta_3 = 0.1594, \quad \alpha = 5.5577,
\]
\[
\Gamma_1 = \begin{bmatrix} 0.7666 & -1.0195 \\ -1.0195 & 1.3969 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0.0564 & 0.0750 \\ 0.0750 & 0.1028 \end{bmatrix}.
\]

Therefore, the stabilizing optimal guaranteed cost control law, $u(t)$, for the system (23) is
\[
u(t) = -B^TPx(t) = -B^TX^{-1}x(t) = -[4.2312 \ 5.4836]x(t)
\]
and the optimal guaranteed cost of the closed-loop system is as follows:
\[
J^* = \alpha + \text{tr}(\Gamma_1) + \text{tr}(\Gamma_2) = 7.8804.
\]
Fig. 1. State responses of system.

Fig. 2. Control law for system.
For computer simulation, we have employed the following uncertainty:

\[ f(t,x(t),x(t-h)) = \begin{bmatrix} 0.1x_1(t) \sin t \\ 0.1x_2(t-h) \cos t \end{bmatrix}. \]

The simulation results are in Figs. 1 and 2. In the figures, one can see that the system is indeed well stabilized.

3. Concluding remarks

In this paper, we have presented a solution to the optimal robust guaranteed cost control problem via memoryless state feedback control laws for neutral delay-differential systems in a matrix inequality framework. A guaranteed cost control gain was obtained through a optimization problem which can be solved by using software such as Matlab’s LMI Control Toolbox [21]. Finally, a simulation result is illustrated to show that the neutral system is indeed well stabilized irrespective of uncertainty and time delay.

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