On stability criteria for uncertain delay-differential systems of neutral type with time-varying delays

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Abstract

In this paper, we propose a new stability criterion for uncertain neutral systems. The considered system has time-varying structured uncertainties and time-varying delay. Based on the Lyapunov method, a delay-dependent criterion for asymptotic stability is derived in terms of LMI (linear matrix inequality). Numerical examples are given to show the effectiveness of our results.

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1. Introduction

The problem of stability analysis of time-delay systems has been a focused topic of theoretical and practical importance since time-delay occurs in many area such as various mechanics, physics, biology, economy, AIDS epidemics, population dynamic models, large-scale systems, automatic control systems, neural networks, chaotic systems, and so on. It is well known that its existence is frequently a source of oscillation and instability. Therefore, stability analysis of time-delay systems has been extensively studied by many researchers. For more details, see [1–19] and references therein.

Recently, the stability analysis of neutral differential systems, which have delays in both its state and the derivatives of its states, has been widely investigated by many researchers [2–17]. The developed stability criteria of dynamic systems with delays in the literature are often classified into two categories according to the dependence of the size of time-delays, namely, delay-independent stability [7–10] and delay-dependent ones [11–17]. While the delay-independent stability criteria guarantee the asymptotic stability irrespective to the size of time-delay, delay-dependent stability criteria give the maximum delay bounds for guaranteeing the
asymptotic stability of system. In general, delay-dependent stability criteria are less conservative than delay-dependent ones when the size of time-delay is small [1].

In practice, uncertainty in mathematical modelling is unavoidable because it is very difficult to obtain an exact mathematical model due to environmental noise, uncertain or slowly varying parameters, etc. Therefore, considerable amounts of efforts have been done to the robust stability for uncertain neutral systems [14–17]. To derive a less conservative stability criterion, a parameterized neutral model transformation was utilized in [13]. More recently, Zhao et al. [14] proposed a delay-dependent stability criterion with no use of model transformation.

In this paper, we propose a new delay-dependent stability criterion for uncertain neutral systems with time-varying delays. Based on the Lyapunov function method, a novel robust delay-dependent criterion is derived in terms of LMIs which can be solved efficiently by using the interior-point algorithms [20]. In order to derive a less conservative results, a new integral inequality lemma is proposed. Also, the model transformation technique, which leads an additional dynamics, is not used in this work. Finally, two numerical examples are included to show that our results are less conservative than those of the existing ones.

Notation: \( \mathbb{R}^n \) is the \( n \)-dimensional Euclidean space, \( \mathbb{R}^{m \times n} \) denotes the set of \( m \times n \) real matrix. \( \| \cdot \| \) refers to the Euclidean vector norm and the induced matrix norm. \( \text{diag}\{ \cdots \} \) denotes the block diagonal matrix. \( L_2[a, b] \) is the space of the square integral function on the interval \( [a, b] \). \( C([0, \infty), \mathbb{R}^n) \) denotes the Banach space of continuous vector functions from \( [0, \infty) \) to \( \mathbb{R}^n \). \( \star \) represents the elements below the main diagonal of a symmetric matrix.

2. Problem statements

Consider the time-delay system:

\[
\begin{aligned}
\dot{x}(t) - C\dot{x}(t - \tau) &= (A + \Delta A(t))x(t) + (A_1 + \Delta A_1(t))x(t - h(t)), \\
x(s) &= \phi(s), \quad s \in [-\bar{h}, \tau], 0],
\end{aligned}
\]  

(1)

where \( x(t) \in \mathbb{R}^n \) is the state, \( A, A_1, \) and \( C \) are known real parameter matrices of appropriate dimensions, and \( \tau \) and \( h(t) \) are time-delays in the system.

The delay, \( h(t) \), is a time-varying continuous function that satisfies

\[
0 \leq h(t) \leq \bar{h}, \quad \dot{h}(t) \leq \mu,
\]

(2)

where \( \bar{h} > 0 \) and \( \mu \) are constants. The initial condition function, \( \phi(s) \in L_2[-\max(\bar{h}, \tau), 0] \), is a given continuous vector valued function. The parameter uncertainties \( \Delta A(t) \) and \( \Delta A_1(t) \) are assumed to be in the form of

\[
\begin{bmatrix}
\Delta A(t) & \Delta A_1(t)
\end{bmatrix} = DF(t)[E \quad E_1],
\]

(3)

where \( D, E, \) and \( E_1 \) are known real constant matrices of appropriate dimensions, and \( F(t) \) are unknown matrices, which satisfy

\[
F^T(t)F(t) \leq I.
\]

(4)

By utilizing the following zero equation:

\[
G\dot{x}(t) - Gx(t - h(t)) - G \int_{t - h(t)}^{t} \dot{x}(s) \, ds = 0,
\]

we can represent the system (1) as

\[
\begin{aligned}
\dot{x}(t) &= (A + G)x(t) + (A_1 - G)x(t - h(t)) - G \int_{t - h(t)}^{t} \dot{x}(s) \, ds + C\dot{x}(t - \tau) + Dp(t), \\
p(t) &= F(t)q(t), \\
q(t) &= Ex(t) + E_1x(t - h(t)),
\end{aligned}
\]

(5)

where \( G \in \mathbb{R}^{m \times n} \) will be chosen.

We will need the following facts and lemma to obtain the main results.
Fact 1 (Schur complement). Given constant symmetric matrices $\Sigma_1$, $\Sigma_2$, $\Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_1^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if
$$
\begin{bmatrix}
\Sigma_1 & \Sigma_3 \\
\Sigma_3 & -\Sigma_2
\end{bmatrix} < 0 \quad \text{or} \quad 
\begin{bmatrix}
-\Sigma_2 & \Sigma_3 \\
\Sigma_3 & \Sigma_1^T
\end{bmatrix} < 0.
$$

Fact 2. For any real vectors $a$, $b$ and any matrix $Q > 0$ with appropriate dimensions, it follows that:
$$
2a^Tb \leq a^TQa + b^TQ^{-1}b.
$$

Lemma 1. For a positive matrix $Q > 0$, any matrices $G$, $F_1$, $F_2$, $F_3$, $F_4$, $F_5$, $F_6$, $F_7$, and scalar $\bar{h} \geq 0$, the following inequality holds:
$$
-\int_{t-h(t)}^{t} \dot{x}(s)Q\dot{x}(s) \leq \zeta^T(t)F\zeta(t) + \bar{h}\zeta^T(t)F^TQ^{-1}F\zeta(t),
$$
where
$$
F = \begin{bmatrix}
F_1 & F_2 & F_3 & F_4 & F_5 & F_6 & F_7 \\
0 & 0 & F_1^T & 0 & 0 & 0 & 0 \\
\star & 0 & F_2^T & 0 & 0 & 0 & 0 \\
\star & \star & F_3^T + F_3 & F_4 & F_5 & F_6 & F_7 \\
\star & \star & \star & 0 & 0 & 0 & 0 \\
\star & \star & \star & \star & 0 & 0 & 0 \\
\star & \star & \star & \star & \star & 0 & 0 \\
\star & \star & \star & \star & \star & \star & 0
\end{bmatrix},
$$
$$
\zeta = \begin{bmatrix}
x^T(t) & x^T(t-h(t)) & \int_{t-h(t)}^{t} \dot{x}(s)ds & \dot{x}^T(t) & x^T(t-h(t)) & \dot{x}^T(t-h(t)) & p^T(t)
\end{bmatrix}^T.
$$

Proof. Utilizing Fact 2, we have
$$
-\int_{t-h(t)}^{t} \dot{x}(s)Q\dot{x}(s) ds \leq 2 \left( \int_{t-h(t)}^{t} \dot{x}(s)ds \right)^T F\zeta(t) + \int_{t-h(t)}^{t} \zeta^T(t)F^TQ^{-1}F\zeta(t) ds
\leq 2 \zeta^T(t) F\zeta(t) \leq 2 \zeta^T(t) F\zeta(t) + \bar{h}\zeta^T(t)F^TQ^{-1}F\zeta(t).
$$

3. Main results

In this section, we propose a new delay-dependent stability criterion for neutral delay systems (1). Now, we have the following main results.

Theorem 1. For a given scalar value $\bar{h}$ and $\mu$, the system (1) is asymptotically stable for $0 \leq h(t) \leq \bar{h}$ if $\|C\| < 1$ and there exist positive definite matrices $P > 0$, $Q > 0$, $W > 0$, $R_1 > 0$, $R_2 > 0$, $H > 0$, and any matrices $Y_i$, $F_i$, $M_i$, $N_i$ ($i = 1, \ldots, 7$) satisfying the following LMI:

\[ \text{[LMI]} \]
where

\[
\begin{bmatrix}
\Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} & \Sigma_{17} & \tilde{h}F_1^T & E^T H \\
\Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{25} & \Sigma_{26} & \Sigma_{27} & \tilde{h}F_2^T & E_1^T H \\
\Sigma_{33} & \Sigma_{34} & \Sigma_{35} & \Sigma_{36} & \Sigma_{37} & \tilde{h}F_3^T & 0 & 0 \\
\Sigma_{44} & \Sigma_{45} & \Sigma_{46} & \Sigma_{47} & \tilde{h}F_4^T & 0 & 0 & 0 \\
\Sigma_{55} & \Sigma_{56} & \Sigma_{57} & \Sigma_{66} & \tilde{h}F_6^T & 0 & 0 & 0 \\
\Sigma_{77} & \tilde{h}F_7^T & 0 & 0 & 0 & -hQ & 0 & 0 \\
\end{bmatrix} < 0,
\]

(6)

\[
\Sigma_{11} = PA + A^T P + Y_1 + Y_1^T + W + R_1 + N_1 A + A^T N_1^T + M_1 + M_1^T, \\
\Sigma_{12} = PA_1 - Y_1 + N_1 A_1 + A^T N_2^T - M_1 + M_2^T, \\
\Sigma_{13} = -Y_1 + F_1^T + A^T N_3^T - M_1 + M_3^T, \\
\Sigma_{14} = -N_1 + A^T N_4^T + M_4^T, \\
\Sigma_{15} = A^T N_5^T + M_5^T, \\
\Sigma_{16} = PC + N_1 C + A^T N_6^T + M_6^T, \\
\Sigma_{17} = PD + M_7^T + N_1 D + A^T N_7^T, \\
\Sigma_{22} = -(1 - \mu) W + N_2 A_1 + A_1^T N_2^T - M_2 - M_2^T, \\
\Sigma_{23} = F_2^T + A_1^T N_3^T - M_2 - M_3^T, \\
\Sigma_{24} = -N_2 + A_1^T N_4^T - M_4^T, \\
\Sigma_{25} = A_1^T N_5^T - M_5^T, \\
\Sigma_{26} = N_2 C + A_1^T N_6^T - M_6^T, \\
\Sigma_{27} = -M_7^T + N_2 D + A_1^T N_7^T, \\
\Sigma_{33} = F_3^T + F_3 - M_3 - M_3^T, \\
\Sigma_{34} = F_4 - N_3 - M_4^T, \\
\Sigma_{35} = F_5 - M_5^T, \\
\Sigma_{36} = F_6 + N_3 C - M_6^T, \\
\Sigma_{37} = F_7 - M_7^T + N_3 D, \\
\Sigma_{44} = \tilde{h}Q + R_2 - N_4 - N_4^T, \\
\Sigma_{45} = -N_5^T, \\
\Sigma_{46} = N_4 C - N_6^T, \\
\Sigma_{47} = N_4 D - N_7^T, \\
\Sigma_{55} = -R_1, \\
\Sigma_{56} = N_5 C + N_6^T, \\
\Sigma_{57} = N_5 D, \\
\Sigma_{66} = -R_2 + N_6 + N_8^T, \\
\Sigma_{67} = N_6 D + N_7 C, \\
\Sigma_{77} = -H + N_7 D + D^T N_8^T.
\]
**Proof.** For positive definite matrices $P$, $W$, $Q$, $R_1$, and $R_2$ and let us consider the Lyapunov functional candidate:

$$V = V_1 + V_2 + V_3 + V_4 + V_5,$$

where

$$V_1 = x^T(t)P\dot{x}(t),$$

$$V_2 = \int_{t-h(t)}^t x^T(s)W\dot{x}(s) \, ds,$$

$$V_3 = \int_{t-h}^t \int_s^t \dot{x}^T(u)Q\dot{x}(u) \, du \, ds,$$

$$V_4 = \int_t^{t^*} x^T(s)R_1x(s) \, ds,$$

$$V_5 = \int_{t^*}^t \dot{x}^T(s)R_2\dot{x}(s) \, ds.$$

From Eq. (5), differentiating $V_1$ leads to

$$\dot{V}_1 = 2x^T(t)P\dot{x}(t)$$

$$= 2x^T(t)P(A + G)x(t) + 2x^T(t)P(A_1 - G)x(t - h(t)) - 2x^T(t)PG$$

$$\times \int_{t-h(t)}^t \dot{x}(s) \, ds + 2x^T(t)PC\dot{x}(t - \tau) + 2x^T(t)PDp(t).$$

By differentiating $V_2$ and $V_3$, we have

$$\dot{V}_2 = x^T(t)W\dot{x}(t) - (1 - \hat{h}(t))x^T(t - h(t))W\dot{x}(t - h(t))$$

$$\leq x^T(t)Wx(t) - (1 - \mu)x^T(t - h(t))Wx(t - h(t)),

$$\dot{V}_3 = \dot{h}\dot{x}^T(t)Q\dot{x}(t) - \int_{t-h}^t \dot{x}^T(s)Q\dot{x}(s) \, ds.$$

Here, by utilizing Lemma 1, we obtain

$$\int_{t-h}^t \dot{x}^T(s)Q\dot{x}(s) \, ds \leq -\int_{t-h(t)}^t \dot{x}^T(s)Q\dot{x}(s) \, ds \leq \zeta^T(t)\tilde{F}\zeta(t) + \tilde{h}\zeta^T(t)F^TQ^{-1}F\zeta(t).$$

Thus, we have a new upper bound of $V_3$ as follows:

$$\dot{V}_3 \leq \dot{h}\dot{x}^T(t)Q\dot{x}(t) + \zeta^T(t)\tilde{F}\zeta(t) + \tilde{h}\zeta^T(t)F^TQ^{-1}F\zeta(t).$$

The time-derivatives of $V_4$ and $V_5$ are obtained as

$$V_4 = x^T(t)R_1x(t) - x^T(t - \tau)R_1x(t - \tau),$$

$$V_5 = \dot{x}^T(t)R_2\dot{x}(t) - \dot{x}^T(t - \tau)R_2\dot{x}(t - \tau).$$

As a tool of deriving a less conservative stability criterion, we add the following two zero equations with any matrices $N_i(i = 1, \ldots, 7)$ and $M_i(i = 1, \ldots, 7)$ to be chosen as

$$2 \begin{bmatrix} x^T(t)N_1 + x^T(t - h(t))N_2 + \int_{t-h(t)}^t \dot{x}(s) \, ds \end{bmatrix}^T N_3 + \dot{x}^T(t)N_4 + x^T(t - \tau)N_5 + \dot{x}^T(t - \tau)N_6 + p^T(t)N_7$$

$$\times [-\dot{x}(t) + Ax(t) + A_1x(t - h(t)) + C\dot{x}(t - \tau) + Dp(t)] = 0,$$
Since the following inequality holds from (4 and 5):
\[p^T(t)p(t) \leq (Ex(t) + E_1x(t - h(t)))^T(Ex(t) + E_1x(t - h(t))),\]
there exist a positive scalars \(\varepsilon\), and a positive matrix \(T\) satisfying the following inequality:
\[
\varepsilon[(Ex(t) + E_1x(t - h(t))\right)^T(Ex(t) + E_1x(t - h(t))) - p(t)^Tfp(t)] \geq 0.
\] (17)
Let \(H = \varepsilon T\). From (7)-(17), and applying S-procedure [20], the time derivative of \(V\) has a new upper bound as
\[
\dot{V}(t) \leq 2x^T(t)P(A + G)x(t) + 2x^T(t)P(A_1 - G)x(t - h(t)) + 2x^T(t)PC\dot{x}(t - \tau) - 2x^T(t)PG
\]
\[
\times \int_{t-h(t)}^t \dot{x}(s) ds + 2x^T(t)PD\dot{p}(t) + x^T(t)W\dot{x}(t) - (1 - \mu)x^T(t - h(t))W\dot{x}(t - h(t)) + \dot{h}\dot{x}(t)Q\dot{x}(t)
\]
\[
+ \zeta^T(t)(F + F^TQ^{-1}F)\zeta(t) + x^T(t)R_1x(t) - x^T(t - \tau)R_1x(t - \tau) + \dot{x}^T(t)R_2x(t) - \dot{x}^T(t - \tau)R_2\dot{x}(t - \tau)
\]
\[
+ \zeta^T(t)(\Xi_1 + \Xi_2)\zeta(t) + (Ex(t) + E_1x(t - h(t)))^TH(Ex(t) + E_1x(t - h(t))) - p(t)^THp(t)
\]
\[
= \zeta^T(t)(\Omega + \dot{H}^TQ^{-1}F + \dot{E}^TH\dot{E})\zeta(t),
\] (18)
where

\[
\Omega = \begin{bmatrix}
\Omega_{11} & \Omega_{12} & \Omega_{13} & \Omega_{14} & \Omega_{15} & \Omega_{16} & \Omega_{17} \\
\star & \Omega_{22} & \Omega_{23} & \Omega_{24} & \Omega_{25} & \Omega_{26} & \Omega_{27} \\
\star & \star & \Omega_{33} & \Omega_{34} & \Omega_{35} & \Omega_{36} & \Omega_{37} \\
\star & \star & \star & \star & \Omega_{44} & \Omega_{45} & \Omega_{46} \\
\star & \star & \star & \star & \star & \Omega_{55} & \Omega_{56} \\
\star & \star & \star & \star & \star & \star & \Omega_{66} \\
\star & \star & \star & \star & \star & \star & \star & \Omega_{77}
\end{bmatrix},
\]

\[
\bar{E} = [E \ E_1 \ 0 \ 0 \ 0 \ 0 \ 0],
\]

\[
\Omega_{11} = P(A + G) + (A + G)^T P + W + R_1 + N_1 A + A^T N_1^T + M_1 + M_1^T,
\]

\[
\Omega_{12} = P(A_1 - G) + N_1 A_1 + A^T N_2^T - M_1 + M_2^T,
\]

\[
\Omega_{13} = -P G + F_1^T + A^T N_3^T - M_1 + M_3^T,
\]

\[
\Omega_{14} = -N_1 + A^T N_4^T + M_4^T,
\]

\[
\Omega_{15} = A^T N_5^T + M_5^T,
\]

\[
\Omega_{16} = P C + N_1 C + A^T N_6^T + M_6^T,
\]

\[
\Omega_{17} = P D^T + N_1 D + A^T N_7^T,
\]

\[
\Omega_{22} = -(1 - \mu) W + N_2 A_1 + A_1^T N_2^T - M_2 - M_2^T,
\]

\[
\Omega_{23} = F_2^T + A_1^T N_3^T - M_2 - M_3^T,
\]

\[
\Omega_{24} = -N_2 + A_1^T N_4^T - M_4^T,
\]

\[
\Omega_{25} = A_1^T N_5^T - M_5^T,
\]

\[
\Omega_{26} = N_2 C + A_1^T N_6^T - M_6^T,
\]

\[
\Omega_{27} = -M_7^T + N_2 D + A_1^T N_7^T,
\]

\[
\Omega_{33} = F_3 + F_3^T - M_3 - M_3^T,
\]

\[
\Omega_{34} = F_4 - N_3 - M_4^T,
\]

\[
\Omega_{35} = F_5 - M_5^T,
\]

\[
\Omega_{36} = F_6 + N_3 C - M_6^T,
\]

\[
\Omega_{37} = F_7 - M_7^T + N_3 D,
\]

\[
\Omega_{44} = h Q + R_2 - N_4 - N_4^T,
\]

\[
\Omega_{45} = -N_5^T,
\]

\[
\Omega_{46} = N_4 C + N_6^T,
\]

\[
\Omega_{47} = N_4 D - N_7^T,
\]

\[
\Omega_{55} = -R_1,
\]

\[
\Omega_{56} = N_5 C + N_6^T,
\]

\[
\Omega_{57} = N_5 D,
\]

\[
\Omega_{66} = -R_2 + N_6 + N_6^T,
\]

\[
\Omega_{67} = N_6 D + N_7 C,
\]

\[
\Omega_{77} = -H + N_7 D + D^T N_7^T,
\]
By defining \( Y_1 = PG \) and using Fact 1, the inequality \( \Omega + \bar{h}F^TQ^{-1}F + E^T\bar{H}E < 0 \), which guarantees the stability of the system (1) by the Lyapunov stability theory, is equivalent to the LMI (6). Therefore, from Theorem 9.8.1 in [2] if \( \|C\| < 1 \) and the LMI (6) holds, then the system (1) is asymptotically stable. This completes our proof. \( \square \)

**Remark 2.** Since the LMIs (6) in Theorem 1 can be easily solved by various efficient convex algorithms. In this paper, we utilize Matlab's LMI Control Toolbox [21] which implements the interior-point algorithm. This algorithm is significantly faster than classical convex optimization algorithms [20].

**Remark 3.** By iteratively solving the LMIs given in Theorem 1 with respect to \( \bar{C}_2h \), one can find the maximum upper bound of time-delay \( \bar{h} \) for guaranteeing asymptotic stability of system (1).

**Remark 4.** When \( h(t) = \tau = h \) is constant, we can obtain a delay-dependent stability criterion using similar method of Theorem 1 with \( R_1 = 0 \). We will show the obtained stability criterion for this case in Corollary 1.

**Corollary 1.** When \( h(t) = \tau = h \) is constant, for a given scalar value \( h > 0 \) and \( \mu \), the system (1) is asymptotically stable for \( 0 \leq h \leq \bar{h} \) if \( \|C\| < 1 \) and there exist positive definite matrices \( P > 0 \), \( Q > 0 \), \( W > 0 \), \( R_2 > 0 \), \( H > 0 \), and any matrices \( Y_1, F_i, M_i, N_i (i = 1, \ldots, 4, 6, 7) \) satisfying the following LMI:

\[
\begin{bmatrix}
\Sigma_{11} - R_1 & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{16} & \Sigma_{17} & \bar{h}F_1^T & E^T H \\
\star & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{26} & \Sigma_{27} & \bar{h}F_2^T & E_1^T H \\
\star & \star & \Sigma_{33} & \Sigma_{34} & \Sigma_{36} & \Sigma_{37} & \bar{h}F_3^T & 0 \\
\star & \star & \star & \Sigma_{44} & \Sigma_{46} & \Sigma_{47} & \bar{h}F_4^T & 0 \\
\star & \star & \star & \star & \Sigma_{66} & \Sigma_{67} & \bar{h}F_5^T & 0 \\
\star & \star & \star & \star & \star & \Sigma_{77} & \bar{h}F_7^T & 0 \\
\star & \star & \star & \star & \star & \star & \star & -\bar{h}Q \\
\end{bmatrix} < 0,
\]

where \( \Sigma_{ij} \) is defined in Theorem 1.

**Remark 5.** When \( h(t) = \tau = h \) is constant and \( \Delta A(t) = \Delta A_1(t) = 0 \), we have a simplified stability criterion by deleting 6th and 8th columns and rows in (19) as shown below:

\[
\begin{bmatrix}
\Sigma_{11} - R_1 & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{16} & \bar{h}F_1^T \\
\star & \Sigma_{22} & \Sigma_{23} & \Sigma_{24} & \Sigma_{26} & \bar{h}F_2^T \\
\star & \star & \Sigma_{33} & \Sigma_{34} & \Sigma_{36} & \bar{h}F_3^T \\
\star & \star & \star & \Sigma_{44} & \Sigma_{46} & \bar{h}F_4^T \\
\star & \star & \star & \star & \Sigma_{66} & \bar{h}F_6^T \\
\star & \star & \star & \star & \star & -\bar{h}Q \\
\end{bmatrix} < 0,
\]

where \( \Sigma_{ij} \) is defined in Theorem 1.

**4. Numerical example**

**Example 1.** Consider the nominal neutral system (1) with

\[
A = \begin{bmatrix}
-0.9 & 0 \\
0.1 & -0.9
\end{bmatrix}, \quad A_1 = \begin{bmatrix}
-1.1 & -0.2 \\
-0.1 & -1.1
\end{bmatrix}, \quad C = \begin{bmatrix}
-0.2 & 0 \\
0.2 & -0.1
\end{bmatrix}, \quad \mu = 0.
\]

By applying Corollary 1 and Remark 2 to the above system, we can obtain the maximum upper bound of time-delay as 1.8266. In Table 1, we compare our result with those of others. From Table 1, we see that...
our maximum allowable bound of $h$, which guarantees the asymptotic stability of the above system, is larger than those by the results given in other works.

**Example 2.** Consider the following uncertain neutral system in [14]:

$$
\dot{x}(t) - Cx(t - \tau) = (A + \Delta A(t))x(t) + (A_1 + \Delta A_1(t))x(t - h(t)),
$$

where

$$
A = \begin{bmatrix} -2 & 0 \\ 0 & -1 \end{bmatrix}, \quad A_1 = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},
$$

$$
\Delta A(t) = \begin{bmatrix} \delta_1 & 0 \\ 0 & \delta_2 \end{bmatrix}, \quad \Delta A_1(t) = \begin{bmatrix} \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}, \quad C = \begin{bmatrix} c & 0 \\ 0 & c \end{bmatrix},
$$

where $0 \leq |c| < 1$, and $\delta_1$, $\delta_2$, $\gamma_1$ and $\gamma_2$ are unknown parameter satisfying $|\delta_1| \leq 1.6$, $|\delta_2| \leq 0.05$, $|\gamma_1| < 0.1$ and $|\gamma_2| < 0.3$. Tables 2 and 3 show the comparison of our results with other ones for different $\mu$ and $c$.

**5. Conclusion**

In this paper, a new delay-dependent stability criterion for the uncertain neutral system with time-varying delays is proposed. The derived criterion is of the form of LMIs, which can be easily solved by various optimizing algorithms. Also, to lead the less conservative results, a new lemma which gives an integral inequality bound is proposed. Examples show that our proposed method provides a larger time-delay bound than other results.

**References**


