Guaranteed cost control of uncertain nonlinear neutral systems via memory state feedback

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Abstract

This article studies a guaranteed cost control problem for a class of uncertain nonlinear neutral systems. Attention is focused on the design of memory state feedback controllers such that the resulting closed-loop system is asymptotically stable and an adequate level of performance is also guaranteed. Using the Lyapunov method and LMI (linear matrix inequality) technique, two criteria for the existence of the controller are derived in terms of LMIs. A numerical example is given to illustrate the proposed method.

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1. Introduction

During the last three decades, the analysis and synthesis of robust controller for stabilizing dynamic systems with delays have been of intense interest [1,2]. Recently, more attention is focused on the stability and stabilization problem of neutral delay differential systems, which are the general form of delay systems and contain delay on the derivatives of some system variables. The neutral delay systems often appear in the study of population dynamics, biology models, automatic control, models of voltage and current fluctuations in transmission lines, vibrating mass attached to an elastic bar, and so on. Therefore, many papers dealing with this problem have appeared because of the fact that the delay is frequently a source of instability and performance degradation of such physical systems. In the literature, various stability analysis and stabilization techniques have been utilized to derive stability/stabilization criteria for asymptotic stability of the systems by many researchers [3–7].

In addition to the simple stabilization, there have been various efforts to design a controller which not only stabilize the uncertain system but also guarantees an adequate level of performance. One way to address this problem is so-called guaranteed cost control [8]. The approach has the advantage of providing an upper bound on a given performance index and thus the system performance degradation incurred by time delays is guaranteed to be less than this bound. Based on this idea, some results have been proposed for discrete-delay systems [9] and for neutral delay differential systems [10–12].
system [10] using memoryless feedback controller. However, if we design a memory state feedback controller with feedback provisions on current state and the past history of the state, we may expect to achieve an improved performance.

With this motivation, we consider the problem of guaranteed cost stabilization of a class of nonlinear neutral delay differential systems. Using the Lyapunov functional technique combined with LMI technique, we develop a guaranteed cost control for the system via retarded integral state feedback controller, which makes the closed-loop system asymptotically stable and guarantees an adequate level of performance. Two stabilization criteria for the existence of the guaranteed cost controller are derived in terms of LMIs, and their solutions provide a parameterized representation of the control. The LMIs can be easily solved by various efficient convex optimization algorithms [11].

Through the article, $\mathbb{R}^n$ denotes $n$-dimensional real space, $\mathbb{R}^{m \times n}$ is the set of all real $m$ by $n$ matrices, and $x^T$ (or $A^T$) denotes transpose of vector $x$ (or matrix $A$). $P > 0$ (respectively $P < 0$) means that matrix $P$ is symmetric positive (respectively negative) definite. $I$ denotes identity matrix of appropriate dimension. $*$ denotes the elements below the main diagonal of a symmetric block matrix, $\mathcal{G}_0$ is a set of all continuous differentiable function on given interval, and diag $\{\cdots\}$ denotes block diagonal matrix.

2. Problem statements and main results

Consider uncertain neutral delay-differential system of the form:

$$\frac{dx(t)}{dt} = A_0x(t) + A_1x(t-h) + f_0(t,x(t)) + f_1(t,x(t-h)) + Bu(t),$$

$$x(t_0 + \theta) = \phi(\theta), \quad \forall \theta \in [-H, 0],$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $A_0, A_1, A_2$, and $B$ are known constant real matrices of appropriate dimensions, $u(t) \in \mathbb{R}^m$ is the control input vector, $h$ and $\tau$ are the positive constant time delays, $H = \max\{h, \tau\}$, $\phi(\cdot) \in \mathcal{G}_0: [-H, 0] \rightarrow \mathbb{R}^n$ is the initial vector, and $f_0$ and $f_1$ are nonlinear perturbations, which is bounded in magnitude as

$$||f_0(t,x(t))|| \leq \beta_0||x(t)||, \quad ||f_1(t,x(t-h))|| \leq \beta_1||x(t-h)||,$$

where $\beta_0$ and $\beta_1$ are positive scalars.

In this article, it is assumed that the pair $(A_0 + A_1, B)$ is completely controllable. This is a basic requirement for controller design.

Now, we are interested in designing a memory retarded integral state feedback controller for the system (1) as

$$u(t) = -K \left( x(t) + \int_{t-h}^{t} A_1 x(s) \, ds - A_2 x(t-\tau) \right),$$

where $K$ is a control gain to be designed.

Associated with the system (1) is the following quadratic cost function:

$$J = \int_0^\infty (x^T(t)Qx(t) + u^T(t)Su(t)) \, dt,$$

where $Q \in \mathbb{R}^{n \times n}$ and $S \in \mathbb{R}^{m \times m}$ are given positive-definite matrices.

Here, the objective of this article is to develop a procedure to design a memory state feedback controller $u(t)$ for the system (1) and cost function (3) such that the resulting closed-loop system is asymptotically stable and the closed-loop value of the cost function (3) satisfies $J \leq J^*$, where $J^*$ is some specified constant.

**Definition 1.** For the neutral system (1) and cost function (3), if there exist a control law $u^*(t)$ and a positive $J^*$ such that for all admissible delays, the system (1) is asymptotically stable and the closed-loop value of the cost function (3) satisfies $J \leq J^*$, then $J^*$ is said to be a guaranteed cost and $u^*(t)$ is said to be a guaranteed cost control law of the system (1) and cost function (3).

Before proceeding further, we will state a well known fact and two lemmas.

**Fact 1** (Schur complement). Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_1^T \Sigma_2^{-1} \Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_1^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0,$$

or

$$\begin{bmatrix} -\Sigma_2 & \Sigma_1 \\ \Sigma_3 & \Sigma_1^T \end{bmatrix} < 0.$$
Fact 2. For any positive scalar $\epsilon$ and vectors $x$ and $y$, the following inequality holds:
\[ x^T y + y^T x \leq \epsilon x^T x + \epsilon^{-1} y^T y. \]

Lemma 1 [12]. For any constant matrix $M \in \mathbb{R}^{n \times n}$, $M = M^T > 0$, a scalar $\gamma > 0$, and a vector function $\omega : [0, \gamma] \rightarrow \mathbb{R}^n$ such that the integrations concerned are well defined, then
\[
\left( \int_0^\gamma \omega(s) \, ds \right)^T M \left( \int_0^\gamma \omega(s) \, ds \right) \leq \gamma \int_0^\gamma \omega(s)^T M \omega(s) \, ds.
\]

Lemma 2 [7]. For given positive scalars $h$ and $\tau$ and any $A_1, A_2 \in \mathbb{R}^{n \times n}$, the operator $\mathcal{D}(x_t) : \mathcal{C}_0 \rightarrow \mathbb{R}^n$ defined by
\[
\mathcal{D}(x_t) = x(t) + \int_{t-h}^t A_2 x(s) \, ds - A_1 x(t-x(t)) - A_2 \hat{x}(t-x(t))
\]
is stable if there exist a positive definite matrix $\Gamma_0$ and positive scalars $\alpha_1$ and $\alpha_2$ such that
\[
\alpha_1 + \alpha_2 < 1,
\]
\[
\begin{bmatrix}
  A_1^2 \Gamma_0 A_2 - \alpha_1 \Gamma_0 & h A_1^2 \Gamma_0 A_1 \\
  h^2 A_1^2 \Gamma_0 A_1 - \alpha_2 \Gamma_0
\end{bmatrix} < 0.
\]

Now, differentiating $\mathcal{D}(x_t)$ and combining Eqs. (1) and (2) leads to
\[
\begin{aligned}
\dot{\mathcal{D}}(x_t) &= \dot{x}(t) + A_1 x(t) - A_1 x(t-h) - A_2 \dot{x}(t-x(t)) \\
&= (A_0 + A_1 - BK)x(t) - BK \int_{t-h}^t A_2 x(s) \, ds + BKA_2 x(t-x(t)) + f_0(t, x(t)) + f_1(t, x(t-h)) \\
&= (A_0 + A_1 - BK)x(t) - A \int_{t-h}^t A_2 x(s) \, ds + AA_2 x(t-x(t)) + f_0(t, x(t)) + f_1(t, x(t-h)),
\end{aligned}
\]
where $A = A_0 + A_1$.

Now, we establish a criterion in terms of LMIs, for asymptotic stabilization of (1) using the Lyapunov method.

Theorem 1. Suppose that there exist $\Gamma_0 > 0$, $\alpha_1 > 0$, and $\alpha_2 > 0$ satisfying (5). Then, for given $Q > 0$ and $S > 0$, the controller $u(t)$ given in (2) is a guaranteed cost controller for the system (1) if there exist the positive-definite matrices $X$, $Z_1$, $Z_2$, $Z_3$, $\alpha_0$, $\alpha_1$, and a matrix $Y$ satisfying the following LMI:
\[
\begin{bmatrix}
  AX + XA - BY \\
  -Y^T B^T + \epsilon_0 I + \epsilon_1 I
\end{bmatrix} \begin{bmatrix}
  Y^T S & -AA_1 Z_1 & AA_2 Z_2 & 0 & 0 & hX & X & X & X & X & X & X & X & X & X & Q
\end{bmatrix}
\leq 0.
\]

Also, the gain matrix of the controller (2) is $K = YX^{-1}$, and the upper bound of the quadratic cost function $J$ is
\[
J^* = \mathcal{D}(0) X^{-1} \mathcal{D}(0) + h \int_{t-h}^t (x(s) + h) x^T(s) Z_1^{-1} x(s) \, ds + \int_{t-h}^t x^T(s) Z_2^{-1} x(s) \, ds + \int_{t-h}^t x^T(s) Z_3^{-1} x(s) \, ds,
\]
where $\mathcal{D}(0)$ denotes $\mathcal{D}(x_t)|_{t=0}$. 

Proof. For $P > 0$, $R_1 > 0$, $R_2 > 0$, and $R_3 > 0$, the functional given by

$$V = V_1 + V_2 + V_3 + V_4,$$

(9)

where

$$V_1 = \mathcal{D}(x_t)^T P \mathcal{D}(x_t),$$

(10)

$$V_2 = \int_{t-h}^t (s - t + h)x^T(s)R_1x(s)\,ds,$$

(11)

$$V_3 = \int_{t-\tau}^t x^T(s)R_2x(s)\,ds,$$

(12)

$$V_4 = \int_{t-h}^t x^T(s)R_3x(s)\,ds$$

(13)

is a legitimate Lyapunov functional candidate [1].

Taking the time derivative of $V$ along the solution of (6) gives that

$$\frac{dV_1}{dt} = 2\mathcal{D}(x_t)^T P \left[ (A - BK)\mathcal{D}(x_t) - A \int_{t-h}^t A_1x(s)\,ds + AA_2x(t - \tau) + f_0(t, x(t)) + f_1(t, x(t-h)) \right]$$

$$\leq 2\mathcal{D}(x_t)^T P \left[ (A - BK)\mathcal{D}(x_t) - A \int_{t-h}^t A_1x(s)\,ds + AA_2x(t - \tau) \right] + c_0\mathcal{D}(x_t)^T PP\mathcal{D}(x_t) + c_1^2\beta_0 x^T(t)x(t)$$

$$+ \epsilon_1\mathcal{D}(x_t)^T PP\mathcal{D}(x_t) + c_1^2\beta_0 x^T(t-h)x(t-h)$$

(14)

$$\frac{dV_2}{dt} = hx^T(t)R_1x(t) - \int_{t-h}^t x^T(s)R_1x(s)\,ds \leq hx^T(t)R_1x(t) - \left( \int_{t-h}^t x(s)\,ds \right)^T \left( hh^{-1}R_1 \right) \left( \int_{t-h}^t x(s)\,ds \right).$$

(15)

$$\frac{dV_3}{dt} = x^T(t)R_2x(t) - x^T(t - \tau)R_2x(t - \tau),$$

(16)

$$\frac{dV_4}{dt} = x^T(t)R_3x(t) - x^T(t - h)R_3x(t - h),$$

(17)

where Fact 2 and Lemma 1 are utilized in (14) and (15), respectively.

Here, let $M = hR_1 + R_2 + R_3 + c_0^{-1}\beta_0 I$ and note that

$$x^T(t)Mx(t) = \left( \mathcal{D}(x_t) - \int_{t-h}^t A_1x(s)\,ds + AA_2x(t - \tau) \right)^T M \left( \mathcal{D}(x_t) - \int_{t-h}^t A_1x(s)\,ds + AA_2x(t - \tau) \right)$$

$$= \mathcal{D}(x_t)^T M \mathcal{D}(x_t) - 2\mathcal{D}(x_t)^T M \int_{t-h}^t A_1x(s)\,ds + 2\mathcal{D}(x_t)^T MA_2x(t - \tau) + \left( \int_{t-h}^t A_1x(s)\,ds \right)^T M \left( \int_{t-h}^t A_1x(s)\,ds \right)$$

$$- 2 \left( \int_{t-h}^t A_1x(s)\,ds \right)^T MA_2x(t - \tau) + x(t - \tau)^T A_2^T MA_2x(t - \tau).$$

(18)

Then, a new bound of the time-derivative of $V$ is as follows:

$$\frac{dV}{dt} = \sum_{i=1}^4 \frac{dV_i}{dt} \leq \chi^T(t)\Omega\chi(t),$$

(19)

where

$$\chi(t) = \begin{bmatrix} \mathcal{D}(x_t) \\ \int_{t-h}^t x(s)\,ds \\ x(t - \tau) \\ x(t - h) \end{bmatrix}$$

(20)
\[ \Sigma = P(A - BK) + (A - BK)^T P + \epsilon_0 PP + \epsilon_1 PP. \]

Again, applying the relation (18) to the terms \( x^T(t)Qx(t) \) and using \( u^T(t)Su(t) = \mathcal{D}(x_t)K^TSK \mathcal{D}(x_t) \), gives that
\[
\frac{dV}{dt} \leq x^T(t) \Omega_1 x(t) - (x^T(t)Qx(t) + u^T(t)Su(t)),
\]
where
\[
\Omega_1 = \begin{bmatrix}
\Sigma + K^T SK & -PAA_1 & PAA_2 & 0 \\
-\epsilon^{-1}R_1 & 0 & 0 & 0 \\
-\epsilon^{-1}R_2 & 0 & 0 & 0 \\
-\epsilon^{-1}R_3 + \epsilon^{-1}_1 \beta^2 I & 0 & 0 & 0 \\
\end{bmatrix} + \begin{bmatrix}
I \\
-A_1^T \\
-A_1^T \\
-A_1^T \\
\end{bmatrix} (M + Q) \begin{bmatrix}
I \\
-A_1 \\
A_2 \\
0 \\
\end{bmatrix}.
\]

Therefore, if \( \Omega_1 < 0 \), there exists the positive scalar \( \gamma \) such that
\[
\frac{dV}{dt} \leq -\gamma \| \mathcal{D}(x_t) \|^2.
\]

By Fact 1, the inequality \( \Omega_1 < 0 \) is equivalent to
\[
\Omega_2 = \begin{bmatrix}
\Sigma + K^T SK & -PAA_1 & PAA_2 & 0 & hI & I & I & I & \epsilon^{-1}_0 I & I \\
-\epsilon^{-1}R_1 & 0 & 0 & 0 & hA_1^T & I & I & I & -\epsilon^{-1}_0 A_1^T & -A_1^T \\
-\epsilon^{-1}R_2 & 0 & 0 & 0 & I & hA_2^T & A_2^T & I & \epsilon^{-1}_0 A_2^T & A_2^T \\
-\epsilon^{-1}R_3 + \epsilon^{-1}_1 \beta^2 I & 0 & 0 & 0 & I & I & I & -\epsilon^{-1}_0 R_3^{-1} I & 0 \\
\end{bmatrix} < 0.
\]

Letting \( X = P^{-1}, Y = KY, Z_1 = hR_1^{-1}, Z_2 = R_2^{-1}, Z_3 = R_3^{-1} \), and pre- and post-multiplying the matrix \( \Omega_2 \) by \( \text{diag}(X,Z_1,Z_2,Z_3,I,I,I,I,I,I,Q) \), give that \( \Omega_2 < 0 \) is equivalent to the following inequality:
\[
\Omega_3 = \begin{bmatrix}
X \Sigma X + Y^T SY & -AA_1 Z_1 & AA_2 Z_2 & 0 & hX & X & X & X & XQ \\
-\epsilon^{-1}R_1 & 0 & 0 & 0 & hZ_1 A_1^T & -Z_1 A_1^T & -Z_1 A_1^T & -Z_1 A_1^T & -Z_1 A_1^T Q \\
-\epsilon^{-1}R_2 & 0 & 0 & 0 & hZ_2 A_2^T & Z_2 A_2^T & Z_2 A_2^T & Z_2 A_2^T & Z_2 A_2^T Q \\
-\epsilon^{-1}R_3 + \epsilon^{-1}_1 \beta^2 Z_3 & 0 & 0 & 0 & \epsilon^{-1}_0 R_3^{-1} I & 0 \\
\end{bmatrix} < 0.
\]

Again, by Fact 1, the fact that \( \Omega_3 < 0 \) is equivalent to the LMI (7). This implies that both the system (1) and (6) with stable operator \( \mathcal{D}(x_t) \) are asymptotically stable by Theorem 9.8.1 in [1]. Furthermore, we have

\[ \vdots \]
Integrating both sides of the above inequality from 0 to $T_f$ leads to

$$\int_0^{T_f} (x^T(t)Qx(t) + u^T(t)Su(t)) \, dt < V(0) - V(T_f)$$

$$= (\mathcal{D}(0)PD(0) - \mathcal{D}^T(T_f)PD(T_f)) + \left( \int_{-h}^{0} (s+h)x^T(s)R_1x(s) \, ds - \int_{T_f-h}^{T_f} (s+h)x^T(s)R_1x(s) \, ds \right)$$

$$+ \left( \int_{-\tau}^{0} x^T(s)R_2x(s) \, ds - \int_{T_f-\tau}^{T_f} x^T(s)R_2x(s) \, ds \right) + \left( \int_{-h}^{0} x^T(s)R_3x(s) \, ds - \int_{T_f-h}^{T_f} x^T(s)R_3x(s) \, ds \right).$$

As both the operator $\mathcal{D}(x)$ and the system (1) are stable, when $T_f \to \infty$,

$$\mathcal{D}^T(T_f)PD(T_f) \to 0, \quad \int_{T_f-h}^{T_f} x^T(s)R_1x(s) \, ds \to 0$$

$$\int_{T_f-\tau}^{T_f} x^T(s)R_2x(s) \, ds \to 0, \quad \int_{T_f-h}^{T_f} x^T(s)R_3x(s) \, ds \to 0.$$

Therefore we have

$$\int_0^{\infty} (x^T(t)Qx(t) + u^T(t)Su(t)) \, dt < V(0)$$

$$= \mathcal{D}(0)PD(0) + \int_{-h}^{0} (s+h)x^T(s)R_1x(s) \, ds + \int_{-\tau}^{0} x^T(s)R_2x(s) \, ds + \int_{-h}^{0} x^T(s)R_3x(s) \, ds \triangleq J'.$$  \hspace{1cm} (27)

This completes our proof. \hfill \Box

Theorem 1 presents a method of designing a state feedback guaranteed cost controller. The following theorem presents a method of selecting a controller minimizing the upper bound of the guaranteed cost (8).

**Theorem 2.** Consider the system (1) with cost function (3). If the following optimization problem

$$\min_{\Gamma_1, \Gamma_2, \Gamma_3, Z_1, Z_2, Z_3, \alpha, \lambda} \{ \alpha + \text{tr}(\Gamma_1) + \text{tr}(\Gamma_2) + \text{tr}(\Gamma_3) \}$$

subject to

(i) LMI(7) \hspace{1cm} (28)

(ii) \begin{bmatrix} -\alpha & \mathcal{D}(0) \\ \mathcal{D}^T(0) & -X \end{bmatrix} < 0,

(iii) \begin{bmatrix} -\Gamma_1 & hN_1^T \\ hN_1 & -hZ_1 \end{bmatrix} < 0,

(iv) \begin{bmatrix} -\Gamma_2 & N_2^T \\ N_2 & -Z_2 \end{bmatrix} < 0,

(v) \begin{bmatrix} -\Gamma_3 & N_3^T \\ N_3 & -Z_3 \end{bmatrix} < 0,

has a positive solution set $(X, \Gamma_1, \Gamma_2, \Gamma_3, Z_1, Z_2, Z_3, \alpha, \lambda, Y, \lambda)$, then the control law (2) is an optimal robust guaranteed cost control law which ensures the minimization of the guaranteed cost (8) for neutral system (1), where $\int_{-h}^{0} (s+h)x(s)x^T(s) \, ds = N_1N_1^T$, $\int_{-\tau}^{0} x(s)x^T(s) \, ds = N_2N_2^T$, and $\int_{-h}^{0} x(s)x^T(s) \, ds = N_3N_3^T$.

**Proof.** By Theorem 1, (i) in (28) is clear. Also, it follows from the Lemma 1 that (ii), (iii), and (iv) in (28) are equivalent to $\mathcal{D}(0)X^{-1}\mathcal{D}(0) < \alpha$, $hN_1^TZ_11N_1 < \Gamma_1$, $N_1^TZ_11N_2 < \Gamma_2$, and $N_1^TZ_11N_3 < \Gamma_3$, respectively. On the other hand,
where algorithms can be used to check whether the matrix inequality (7) is feasible. In this article, in order to solve the matrix which is a convex optimization problem. For details, see Boyd et al. [11]. Various efficient convex optimization algo-

Actually, when the control input is not forced to the system (29), the system go to infinity as

\[ x(t) = 0.1 \cos t [x_1(t)], \dot{x}_2 = 0.2 \sin t [x_1(t)] \]

Thus, we have

\[ f_i(t, x(t-h)) = (\gamma_1 \cos t [x_1(t-h)], \gamma_2 \sin t [x_1(t-h)])^T \]

with \( |\delta_i| \leq \beta_0 = 0.1 \), \( |\gamma_i| \leq \beta_1 = 0.1 \) (i = 1, 2).

Thus, we have

\[ \| f(t, x(t)) \| \leq \beta_0 \| x(t) \|, \quad \| f_i(t, x(t-h)) \| \leq \beta_1 \| x(t-h) \|. \]

The initial condition of the system is as follows:

\[ x(t) = [0.5 e^{-t} - 0.5 e^{-t}]^T, \quad \text{for} \quad -0.3 \leq t \leq 0. \]

Actually, when the control input is not forced to the system (29), i.e., \( u(t) = 0 \), the system is unstable since the states of the system go to infinity as \( t \to \infty \).

Here, associated with this system is the cost function of (3) with \( Q = I \) and \( S = I \).

From the relations \( D(0) = x(0) + A_1 \int_0^t x(s) \, ds - A_2 x(t) \), \( N_1 N_1^T = \int_{-0.3}^0 (s + 0.3) x(s)x^T(s) \, ds \), \( N_2 N_2^T = \int_{-0.3}^0 x(s)x^T(s) \, ds \) and \( N_3 N_3^T = \int_{-0.3}^0 x(s)x^T(s) \, ds \), we have

\[ D(0) = \begin{bmatrix} 0.3450 \\ -0.3559 \end{bmatrix}, \quad N_1 = \begin{bmatrix} 0.0876 & -0.0402 \\ -0.0402 & 0.1919 \end{bmatrix}, \]

\[ N_2 = N_3 = \begin{bmatrix} 0.1614 & -0.1742 \\ -0.1742 & 0.2691 \end{bmatrix}. \]

First, checking the stability condition (5) for operator \( D(x_i) \) gives the solutions:

\[ \Gamma_0 = \begin{bmatrix} 0.6284 & -0.0002 \\ -0.0002 & 0.6280 \end{bmatrix}, \quad \alpha_1 = 0.3333, \quad \alpha_2 = 0.3333. \]
Next, by solving the optimization problem of Theorem 2, we find the solutions of the LMIs (28) for the system as

\[
X = \begin{bmatrix}
0.1611 & -0.0682 \\
-0.0682 & 0.1232
\end{bmatrix}, \quad Z_1 = \begin{bmatrix}
0.9096 & -0.3525 \\
-0.3525 & 0.2226
\end{bmatrix},
\]

\[
Z_2 = \begin{bmatrix}
1.8923 & -0.0705 \\
-0.0705 & 0.9219
\end{bmatrix}, \quad Z_3 = 10^8 \times \begin{bmatrix}
2.4290 & 0.0313 \\
0.0313 & 2.5716
\end{bmatrix},
\]

\[
Y = \begin{bmatrix}
-0.00000007657712 & 0.9999997138596 \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix}
0.0027 & -0.0050 \\
-0.0050 & 0.1090
\end{bmatrix},
\]

\[
\Gamma_2 = \begin{bmatrix}
0.0445 & -0.0629 \\
-0.0629 & 0.0911
\end{bmatrix}, \quad \Gamma_3 = 10^{-8} \times \begin{bmatrix}
0.1941 & -0.0144 \\
-0.0144 & 0.2028
\end{bmatrix},
\]

\[
z = 1.2059, \quad \epsilon_0 = 0.0289, \quad \epsilon_1 = 10^{-9} \times 1.0703.
\]

Therefore, the gain matrix of stabilizing optimal guaranteed cost controller \(u(t)\) for the system (29) is

\[
K = YX^{-1} = \begin{bmatrix}
4.4835 & 10.5937
\end{bmatrix},
\]

and the optimal guaranteed cost of the closed-loop system is as follows:

\[
J^* = z + \text{tr}(\Gamma_1) + \text{tr}(\Gamma_2) + \text{tr}(\Gamma_3) = 1.4532.
\]

3. Conclusions

In this article, the optimal guaranteed cost control problem via a retarded integral state feedback controller for uncertain neutral delay-differential systems has been investigated using the Lyapunov method and the LMI framework. The controller can be obtained through a convex optimization problem which can be solved by various efficient convex optimization algorithms.

References