On synchronization of unified chaotic systems via nonlinear Control

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Abstract

In this paper, a simple but efficient nonlinear control method is applied to the synchronization of unified chaotic systems using the Lyapunov method. A numerical example is given to illuminate the design procedure and advantage of the result derived.

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1. Introduction

Chaos is very interesting nonlinear phenomenon and has been intensively studied in the last three decades [1–20]. It is found to be useful or has great potential in many disciplines [1]. Especially, the subject of chaotic synchronization has received considerable attentions in recent years. Synchronization has been widely explored in a variety of fields including physical, chemical and ecological systems, secure communications, etc [10–20]. For example, the chaos synchronization of linearly coupled chaotic systems is investigated by Li et al. [17], Lü et al. [18], and Park [19]; the synchronization problem via nonlinear control scheme is studied by Chen and Han [13] and Chen [20].

In this paper, we will consider the study of the master-slave type of chaos synchronization problems for unified chaotic systems. The unified chaotic system which contains the Lorenz and Chen systems as two extremes and the Lü system as a special case has been produced by Lü et al. [14]. More recently, there are some results reported about the unified system [15–17]. In this paper, for chaotic synchronization of the unified chaotic systems, a class of novel nonlinear control scheme is proposed. Then, chaos synchronization of the system is proved by the Lyapunov method.

The organization of this paper is as follows. In Section 2, the problem statement and master-slave synchronization scheme are presented for unified chaotic systems. In Section 3, we provide an numerical example to demonstrate the effectiveness of the proposed method. Finally concluding remark is given.
2. Synchronization of unified chaotic systems

Consider the unified chaotic system described by

\[
\begin{aligned}
\dot{x} &= (25a + 10)(y - x), \\
\dot{y} &= (28 - 35a)x - xz + (29a - 1)y, \\
\dot{z} &= xy - ((8 + a)/3)z,
\end{aligned}
\]

where \(x, y, z\) are status variables and \(a \in [0, 1]\).

The system is chaotic for any \(a \in [0, 1]\). System (1) is called the general Lorenz, Lü, and Chen system, when \(a \in [0, 0.8)\), when \(a = 0.8\), and when \(a \in (0.8, 1]\), respectively.

Recently, there are some results reported about this unified chaotic systems. Lu et al. further investigated the PC synchronization and its application in secure communication [15]. Wu and Lu proposed a backstepping control scheme [16]. Park [19] utilize linear matrix inequality approach to analyze the stability problem for chaotic synchronization of the system.

Our goal is to make synchronization for two identical unified chaotic systems based on the Lyapunov method. For the unified chaotic systems (1), the master and slave systems are defined below, respectively,

\[
\begin{aligned}
\dot{x}_m &= (25a + 10)(y_m - x_m), \\
\dot{y}_m &= (28 - 35a)x_m - x_mz_m + (29a - 1)y_m, \\
\dot{z}_m &= x_my_m - ((8 + a)/3)z_m,
\end{aligned}
\]

Fig. 1. State trajectories of drive system.
and
\[
\begin{align*}
\dot{x}_s &= (25x + 10)(y_s - x_s) + u_1, \\
\dot{y}_s &= (28 - 35x)x_s - x_s z_s + (29x - 1)y_s + u_2, \\
\dot{z}_s &= x_s y_s - ((8 + x)/3)z_s + u_3,
\end{align*}
\]
where the lower scripts m and s stand for the master (or drive) systems, the slave (or response) one, respectively, $u_1, u_2$ and $u_3$ are the nonlinear controller such that two chaotic systems can be synchronized.

Define the error signal as
\[
\begin{align*}
e_1(t) &= x_s(t) - x_m(t), \\
e_2(t) &= y_s(t) - y_m(t), \\
e_3(t) &= z_s(t) - z_m(t),
\end{align*}
\]
which gives that
\[
\begin{align*}
x_m z_m / C_0 x_s z_s &= -y_m e_1 - x_s e_3, \\
-x_m y_m + x_s y_s &= y_m e_1 + x_s e_2.
\end{align*}
\]
From Eqs. (4) and (5), we have the following error dynamics:
\[
\begin{align*}
\dot{e}_1(t) &= (25x + 10)(e_2 - e_1) + u_1, \\
\dot{e}_2(t) &= (28 - 35x)e_1 + (29x - 1)e_2 - z_m e_1 - x_s e_3 + u_2, \\
\dot{e}_3(t) &= y_m e_1 + x_s e_2 - ((8 + x)/3)e_3 + u_3.
\end{align*}
\]

Fig. 2. Synchronization errors.
For two identical unified chaotic system without control \((u_i = 0)\), if the initial condition \((x_m(0), y_m(0), z_m(0)) \neq (x_s(0), y_s(0), z_s(0))\), the trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two controlled unified chaotic systems, the two systems will approach synchronization for any initial condition by appropriate control gain. For this end, we propose the following control law for the system (3):

\[
\begin{align*}
  u_1 &= - (\beta_1 + \beta_2 - z_m) e_2, \\
  u_2 &= - (\beta_3 + k) e_2, \\
  u_3 &= - y_m e_1,
\end{align*}
\]

(7)

where \(\beta_1 = 25x + 10 > 0\), \(\beta_2 = 28 - 35x\), \(\beta_3 = 29x - 1\), \(\beta_4 = (8 + x)/3 > 0\) and \(k > 0\) is a free control gain.

Then, we have the following theorem.

**Theorem 1.** For given \(0 \leq \alpha \leq 1\), the controlled unified chaotic systems (2) and (3) will approach synchronization for any initial condition \((x_m(0), y_m(0), z_m(0)), (x_s(0), y_s(0), z_s(0))\) by the control law (7).

**Proof.** Construct a Lyapunov function

\[
V = (1/2)(e_1^2 + e_2^2 + e_3^2).
\]

(8)

The differential of the Lyapunov function along the trajectory of system (4) is

\[
\frac{dV}{dt} = \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3.
\]

(9)

Substituting Eq. (6) into Eq. (9) gives that

\[
\frac{dV}{dt} = -\beta_1 e_1^2 - k e_2^2 - \beta_4 e_3^2 < 0,
\]

(10)

Fig. 3. State trajectories of drive system.
which gives asymptotic stability of the system by Lyapunov stability theory. This means that the controlled unified chaotic systems (2) and (3) is synchronized for any initial condition. This completes the proof. □

Remark 1. Note that the nonlinear control law given in (7) utilizes only two error information, i.e., $e_1$ and $e_2$. Also, a control parameter $k$ can be chosen by considering the convergence speed of error signals.

3. Numerical example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for Chen and Lorenz system. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001.

Case I: Chen system. When $a = 1$, Eqs. (2) and (3) is Chen’s system. First, let us take a control parameter as $k = 5$ for the unified system. For this numerical simulation, we assume that the initial condition, $(x_m(0),y_m(0),z_m(0)) = (3,5,7)$, and $(x_s(0),y_s(0),z_s(0)) = (−2,2,3)$ is employed. Figs. 1 and 2 display the state response and synchronization errors of systems (2) and (3). It can be seen that the synchronization errors converge to zero rapidly.

Case II: Lorenz system. When $a = 0.1$, Eqs. (2) and (3) is Lorenz’s system. Under the same simulation condition of Case I, the simulation results are given in Figs. 3 and 4.
4. Concluding remark

In this paper, we investigate the synchronization of controlled unified chaotic systems. We have proposed a novel nonlinear control scheme for asymptotic chaos synchronization using the Lyapunov method. Finally, a numerical simulation is provided to show the effectiveness of our method.

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References