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Adaptive synchronization of hyperchaotic Chen system with uncertain parameters

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With my cordial regards, this paper is dedicated to K.S. Gang on the occasion of her 85th birthday

Abstract

This article addresses control for the chaos synchronization of hyperchaotic Chen system with five uncertain parameters. Based on the Lyapunov stability theory, an adaptive control law is derived to make the states of two identical hyperchaotic Chen systems asymptotically synchronized. Finally, a numerical simulations is presented to show the effectiveness of the proposed chaos synchronization scheme.

1. Introduction

Since Pecora and Carroll [1] introduced a method to synchronization two identical chaotic systems with different initial conditions, chaos synchronization have attracted a great deal of attention from various fields during the last three decades [2–13]. The idea of synchronization is to use the output of the master system to control the slave system so that the output of the response system follows the output of the master system asymptotically. Many methods and techniques for handling chaos control and synchronization have been developed, such as PC method [1], OGY method [3], feedback approach [6,7], adaptive method [14], time-delay feedback approach [5], and backstepping design technique [10], etc. On the other hand, when the chaotic systems have some uncertain parameters, it is generally difficult to control the system. In this case, it is well known that the adaptive control scheme is an effective method for the synchronization. In this regard, the adaptive feedback synchronization for several chaotic systems has been investigated by Wang et al. [14], Han et al. [16], Elabbasy et al. [17], Lu et al. [15], and Park [18].

Recently, because the presence of more than positive Lyapunov exponent clearly improves security by generating more complex dynamics, synchronization schemes of hyperchaotic systems have been investigated [19–21].

In this article, we consider the problem of chaos synchronization of hyperchaotic Chen system with uncertain parameters. For chaotic synchronization of the uncertain hyperchaotic system, a class of novel adaptive control scheme is proposed. Then, chaos synchronization of the system is proved by the Lyapunov stability theory.

Throughout the article, diag$\{\cdots\}$ denotes the diagonal matrix. $\|\cdot\|$ denotes Euclidean norm of given vector. The organization of this article is as follows. In Section 2, the problem statement and master-slave synchronization scheme

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are presented for the hyperchaotic Chen system. In Section 3, we provide a numerical example to demonstrate the effectiveness of the proposed method. Finally concluding remark is given.

2. Chaos synchronization of hyperchaotic Chen system

The hyperchaotic Chen system [21] is given by

$$\begin{cases} \dot{x} = a(y-x) + w, \\ \dot{y} = dx - xz + cy, \\ \dot{z} = xy - bz, \\ \dot{w} = yz + rw, \end{cases} \tag{1}$$

where x, y, z, and w are state variables, and a, b, c, d and r are the real constants.

When $a = 35$, $b = 3$, $c = 12$, $d = 7$, and $0 \leqslant r \leqslant 0.085$, the system (1) is chaotic; when $a = 35$, $b = 3$, $c = 124$, $d = 7$, and $0.085 < r \leqslant 0.798$, the system (1) is hyperchaotic; when $a = 35$, $b = 3$, $c = 12$, $d = 7$, and $0.798 < r \leqslant 0.90$, the system (1) is periodic [20]. For the projections of hyperchaotic attractor of the system, see the papers [20,21]. Also, it is shown that when $a = 35$, $b = 3$, $c = 12$, $d = 7$, and $0.085 < r \leqslant 0.798$, the hyperchaotic Chen system (1) only has one equilibrium $\mathcal{O}(0.0.0.0)$, and is a forced dissipative system, which implies that the solutions of the system are bounded as $t \to \infty$.

Now, we assume that we have two hyperchaotic Chen systems where the master system with the subscript m drives the slave system having identical equations denoted by the subscript s. For the systems (1), the master (or drive) and slave (or response) systems are defined below, respectively,

$$\begin{cases} \dot{x}_{\mathrm{m}} = a(y_{\mathrm{m}} - x_{\mathrm{m}}) + w_{\mathrm{m}}, \\ \dot{y}_{\mathrm{m}} = dx_{\mathrm{m}} - x_{\mathrm{m}}z_{\mathrm{m}} + cy_{\mathrm{m}}, \\ \dot{z}_{\mathrm{m}} = x_{\mathrm{m}}y_{\mathrm{m}} - bz_{\mathrm{m}}, \\ \dot{w}_{\mathrm{m}} = y_{\mathrm{m}}z_{\mathrm{m}} + rw_{\mathrm{m}}, \end{cases} \tag{2}$$

and

$$\begin{cases} \dot{x}_{\mathrm{s}} = a_1(y_{\mathrm{s}} - x_{\mathrm{s}}) + w_{\mathrm{s}} + u_1, \\ \dot{y}_{\mathrm{s}} = d_1x_{\mathrm{s}} - x_{\mathrm{s}}z_{\mathrm{s}} + c_1y_{\mathrm{s}} + u_2, \\ \dot{z}_{\mathrm{s}} = x_{\mathrm{s}}y_{\mathrm{s}} - b_1z_{\mathrm{s}} + u_3, \\ \dot{w}_{\mathrm{s}} = y_{\mathrm{s}}z_{\mathrm{s}} + r_1w_{\mathrm{s}} + u_4, \end{cases} \tag{3}$$

where a_1, b_1, c_1, d_1 and r_1 are parameters of the slave system which needs to be estimated, and u_1, u_2, u_3 and u_4 are the nonlinear controller such that two hyperchaotic systems can be synchronized.

Subtracting Eq. (2) from Eq. (3) yields error dynamical system between Eqs. (2) and (3)

$$\begin{cases} \dot{e}_1(t) = a_1(y_{\mathrm{s}} - x_{\mathrm{s}}) - a(y_{\mathrm{m}} - x_{\mathrm{m}}) + e_4 + u_1, \\ \dot{e}_2(t) = d_1x_{\mathrm{s}} - dx_{\mathrm{m}} - x_{\mathrm{s}}z_{\mathrm{s}} + x_{\mathrm{m}}z_{\mathrm{m}} + c_1y_{\mathrm{s}} - cy_{\mathrm{m}} + u_2, \\ \dot{e}_3(t) = x_{\mathrm{s}}y_{\mathrm{s}} - x_{\mathrm{m}}y_{\mathrm{m}} - b_1z_{\mathrm{s}} + bz_{\mathrm{m}} + u_3, \\ \dot{e}_4(t) = y_{\mathrm{s}}z_{\mathrm{s}} - y_{\mathrm{m}}z_{\mathrm{m}} + r_1w_{\mathrm{s}} - rw_{\mathrm{m}} + u_4, \end{cases} \tag{4}$$

where

$$\begin{cases} e_1(t) = x_{\mathrm{s}}(t) - x_{\mathrm{m}}(t), \\ e_2(t) = y_{\mathrm{s}}(t) - y_{\mathrm{m}}(t), \\ e_3(t) = z_{\mathrm{s}}(t) - z_{\mathrm{m}}(t), \\ e_4(t) = w_{\mathrm{s}}(t) - w_{\mathrm{m}}(t). \end{cases} \tag{5}$$

Here, our goal is to make synchronization between two hyperchaotic Chen systems by using adaptive control scheme u_i, $i = 1,2,3,4$ when the parameter of the drive system is unknown and different with those of the response system, i.e.,

$$\lim_{t\to\infty} \|e(t)\| = 0,$$

where $e = [e_1\ \ e_2\ \ e_3\ \ e_4]^{\mathrm{T}}$.

For two hyperchaotic Chen systems without control ($u_i = 0$, $i = 1,2,3,4$), if the initial condition $(x_{\mathrm{m}}(0), y_{\mathrm{m}}(0), z_{\mathrm{m}}(0), w_{\mathrm{m}}(0)) \neq (x_{\mathrm{s}}(0), y_{\mathrm{s}}(0), z_{\mathrm{s}}(0), w_{\mathrm{s}}(0))$, the trajectories of the two identical systems will quickly separate each other and become irrelevant. However, for the two-controlled hyperchaotic Chen systems, the two systems will approach synchronization for any initial condition by appropriate control gain. For this end, we propose the following control law for the system (3):

$$\begin{aligned} u_1 &= -(k_1 - a_1)e_1 - (a_1 + d_1 - z_s)e_2 - e_4, \\ u_2 &= -(k_2 + c_1)e_2 - e_1 e_3, \\ u_3 &= -(k_3 - b_1)e_3 + y_m e_1, \\ u_4 &= -(k_4 - r_1)e_4 - y_s e_3 - z_m e_2, \end{aligned} \tag{6}$$

where k_i are the positive scalars, and the update rule for five unknown parameters a, b, c, d and r:

$$\begin{aligned} \dot{a}_1 &= -(y_m - x_m)e_1, \\ \dot{b}_1 &= z_m e_3, \\ \dot{c}_1 &= -y_m e_2, \\ \dot{d}_1 &= -x_m e_2, \\ \dot{r}_1 &= -w_m e_4. \end{aligned} \tag{7}$$

Then, we have the following main result.

Theorem. *For any initial conditions, the two systems (2) and (3) are globally asymptotically synchronized by the control law (6) and the update law (7).*

Proof. Choose the following Lyapunov candidate:

$$V = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2 + e_c^2 + e_d^2 + e_r^2\right), \tag{8}$$

where $e_a = a_1 - a$, $e_b = b_1 - b$, $e_c = c_1 - c$, $e_d = d_1 - d$, and $e_r = r_1 - r$.

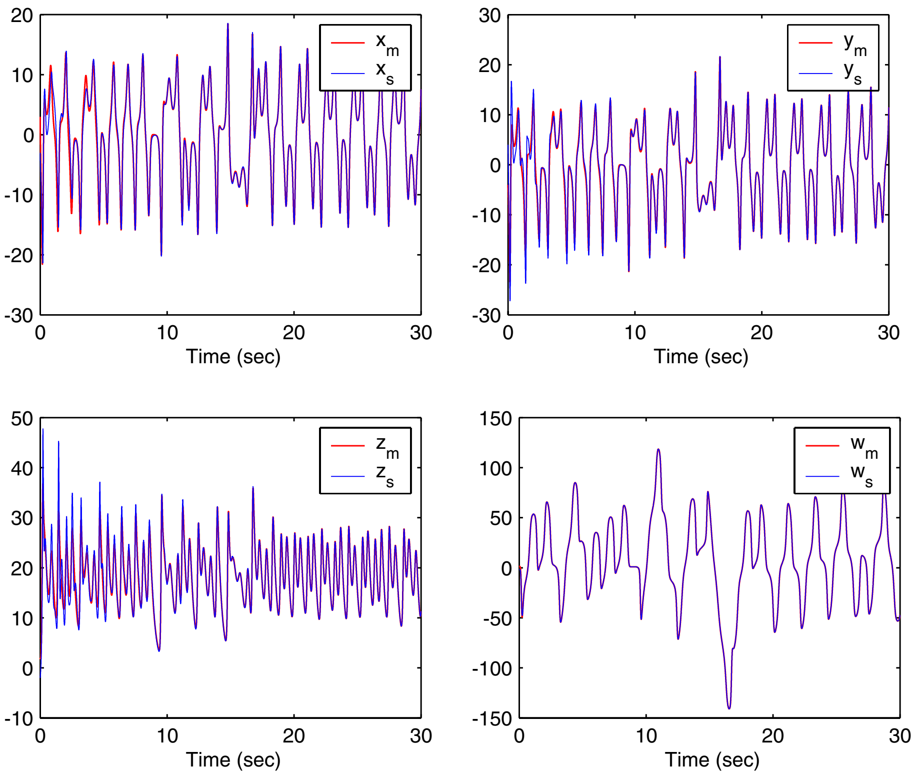

Fig. 1. State trajectories of master and slave systems.

The differential of the Lyapunov function along the trajectory of error system (4) is

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &= \dot{e}_1e_1 + \dot{e}_2e_2 + \dot{e}_3e_3 + \dot{e}_ae_a + \dot{e}_be_b + \dot{e}_ce_c + \dot{e}_de_d + \dot{e}_re_r \\ &= e_1[a_1(y_\mathrm{s} - x_\mathrm{s}) - a(y_\mathrm{m} - x_\mathrm{m}) + e_4 + u_1] + e_2[d_1x_\mathrm{s} - dx_\mathrm{m} - x_\mathrm{s}z_\mathrm{s} + x_\mathrm{m}z_\mathrm{m} + c_1y_\mathrm{s} - cy_\mathrm{m} + u_2] \\ &\quad + e_3[x_\mathrm{s}y_\mathrm{s} - x_\mathrm{m}y_\mathrm{m} - b_1z_\mathrm{s} + bz_\mathrm{m} + u_3] + e_4[y_\mathrm{s}z_\mathrm{s} - y_\mathrm{m}z_\mathrm{m} + r_1w_\mathrm{s} - rw_\mathrm{m} + u_4] + \dot{e}_a(a_1 - a) + \dot{e}_b(b_1 - b) \\ &\quad + \dot{e}_c(c_1 - c) + \dot{e}_d(d_1 - d) + \dot{e}_r(r_1 - r). \end{aligned} \tag{9}$$

Here, note that

$$\begin{aligned} &-x_\mathrm{s}z_\mathrm{s} + x_\mathrm{m}z_\mathrm{m} = -z_\mathrm{s}e_1 - x_\mathrm{m}e_3, \\ &x_\mathrm{s}y_\mathrm{s} - x_\mathrm{m}y_\mathrm{m} = x_\mathrm{s}e_2 - y_\mathrm{m}e_1, \\ &y_\mathrm{s}z_\mathrm{s} - y_\mathrm{m}z_\mathrm{m} = y_\mathrm{s}e_3 + z_\mathrm{m}e_2. \end{aligned} \tag{10}$$

Substituting Eqs. (7) and (10) into Eq. (9) give that

$$\begin{aligned}\frac{\mathrm{d}V}{\mathrm{d}t} &= -a_1e_1^2 + (a_1 + d_1 - z_\mathrm{s})e_1e_2 + e_1e_4 + e_1u_1 + c_1e_2^2 + e_2u_2 - y_\mathrm{m}e_1e_3 - b_1e_3^2 + e_3u_3 + y_\mathrm{s}e_3e_4 + z_\mathrm{m}e_2e_4 \\ &\quad + r_1e_4^2 + e_4u_4 + e_1e_2e_3. \end{aligned} \tag{11}$$

Again, substituting Eq. (6) into Eq. (11) gives that

$$\frac{\mathrm{d}V}{\mathrm{d}t} = -k_1e_1^2 - k_2e_2^2 - k_3e_3^2 - k_4e_4^2 = -e^\mathrm{T}Pe, \tag{12}$$

where $P = \mathrm{diag}\{k_1, k_2, k_3, k_4\}$.

Since $\dot{V}$ is negative semidefinite, we cannot immediately obtain that the origin of error system (5) is asymptotically stable. In fact, as $\dot{V} \leqslant 0$, then $e_1, e_2, e_3, e_4 \in \mathscr{L}_\infty$ and $e_a, e_b, e_c, e_d, e_r \in \mathscr{L}_\infty$. From the error system (4), we have $\dot{e}_1, \dot{e}_2, \dot{e}_3, \dot{e}_4 \in \mathscr{L}_\infty$. Since $\dot{V} = -e^\mathrm{T}Pe$, then we have

$$\int_0^t \lambda_{\min}(P)\|e\|^2\,\mathrm{d}t \leqslant \int_0^t e^\mathrm{T}Pe\,\mathrm{d}t \leqslant \int_0^t -\dot{V}\,\mathrm{d}t = V(0) - V(t) \leqslant V(0),$$

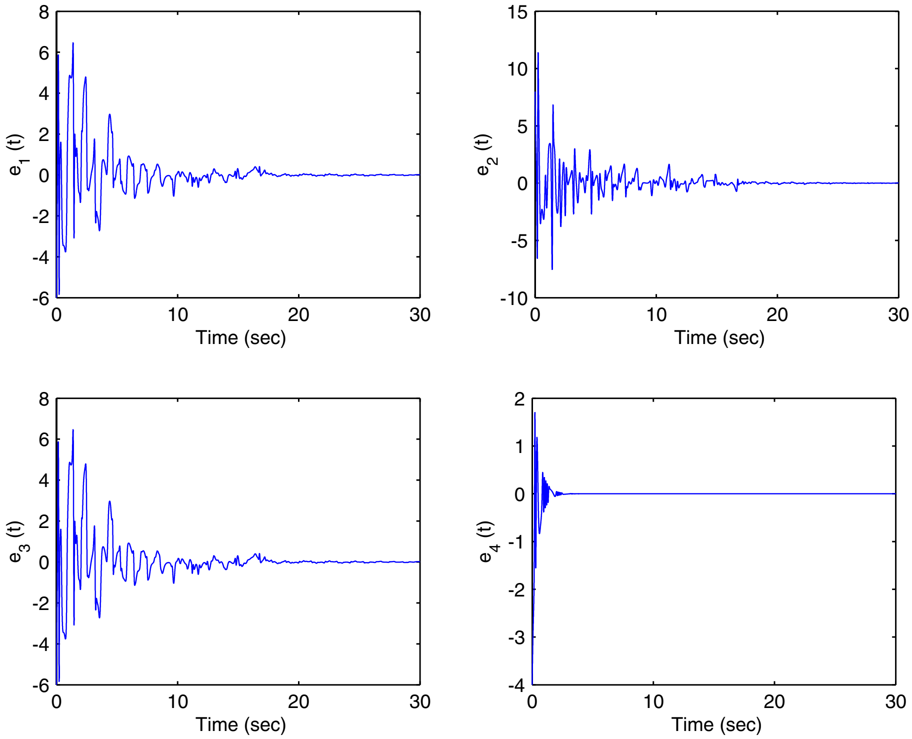

Fig. 2. Synchronization errors, e_1, e_2, e_3, e_4 with time t.

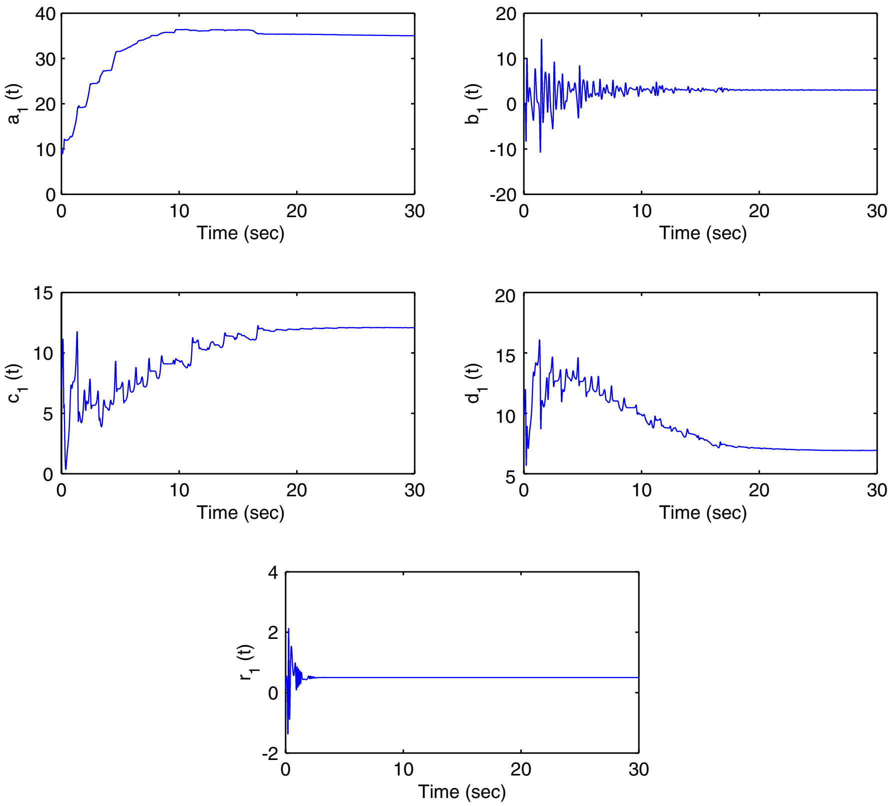

Fig. 3. Changing parameters a_1, b_1, c_1, d_1, r_1 of response system with time t.

where $\lambda_{\min}(P)$ is the minimum eigenvalue of positive-definite matrix P. Thus $e_1, e_2, e_3, e_4 \in \mathscr{L}_2$. According to the Barbalat's lemma, we have $e_1(t), e_2(t), e_3(t), e_4(t) \to 0$ as $t \to \infty$, i.e., $\lim_{t\to\infty}\|e(t)\| = 0$. Therefore, the slave system (3) synchronize the master system (2) by the controller (6). This completes the proof. □

3. Numerical example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for hyperchaotic Chen system. In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001.

For this numerical simulation, we assume that the initial condition, $(x_m(0), y_m(0), z_m(0), w_m(0)) = (3, -4, 2, 2)$, and $(x_s(0), y_s(0), z_s(0), w_s(0)) = (-3, 4, -2, -2)$ and control gains, $(k_1, k_2, k_3, k_4) = (7, 5, 5, 5)$ are employed. Hence the error system has the initial values $e_1(0) = -3$, $e_2(0) = 8$, $e_3(0) = -4$, and $e_4(0) = -4$. The five unknown parameters are chosen as $a = 35$, $b = 3$, $c = 12$, $d = 7$ and $r = 0.5$ in simulations so that the system (1) exhibits a hyperchaotic behavior. Synchronization of the systems (2) and (3) via adaptive control law (6) and (7) with the initial estimated parameters $a_1(0) = 10$, $b_1(0) = 0.5$, $c_1(0) = 7$, $d_1(0) = 10$ and $r_1(0) = 0.1$ are shown in Figs. 1–3. Figs. 1 and 2 display the state response and synchronization errors of systems (2), (3). Fig. 3 shows that the estimates $a_1(t)$, $b_1(t)$, $c_1(t)$, $d_1(t)$, $r_1(t)$ of the unknown parameters converges to $a = 35$, $b = 3$, $c = 12$, $d = 7$ and $r = 0.5$ as $t \to \infty$.

4. Concluding remark

In this article, we investigate the synchronization of controlled hyperchaotic Chen chaotic systems with five uncertain parameters. We have proposed a novel adaptive nonlinear control scheme for asymptotic chaos synchronization using the Lyapunov stability theory. Finally, a numerical simulation is provided to show the effectiveness of the method proposed in this work.

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