Novel delay-dependent robust stability criterion of delayed cellular neural networks

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Abstract

In this paper, we consider the problem of global robust stability for cellular neural networks which have time-varying delay and parametric uncertainties. Using the Lyapunov method and linear matrix inequality (LMI) framework, the delay-dependent criterion is presented in terms of LMIs. Two numerical examples are presented to illustrate the effectiveness of our result.

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1. Introduction

Cellular neural networks (CNNs) have been a subject of intense research activities in the literature over the decades and have found extensive applications in image processing, pattern recognition and classification, solving certain optimization problems, and so on. Thus, the study of the stability problem of CNNs has received great attention in recent years and a number of remarkable results have been reported [1–5]. In hardware implementation of CNNs, time delays occur frequently during the processing and transmission of the signals among the cells. This leads to the model of delayed cellular neural networks (DCNNs). It is well known that time delay can easily cause instability and oscillations in system. Therefore, the stability analysis of DCNNs has become an important topic of theoretical studies in neural networks [6–21].

On the other hand, in many practical situations, the weight coefficients of the neurons depend on certain resistance and capacitance value which contain uncertainties. It is important and interesting to ensure the global robust stability of neural networks; see [22] and the references therein.

This paper is concerned with the problem of global robust asymptotic stability for DCNNs with time-varying delays and uncertainties. The stability of the DCNNs is analyzed based on the Lyapunov theory and LMI framework. Then, the delay-dependent criterion, which is usually less conservative than delay-independent one, is derived in terms of LMIs. The advantage of the proposed approach is that the resulting stability criterion can be solved efficiently via existing numerical convex optimization algorithms. The numerical examples have shown that our criterion is more effective than the existing results.
Throughout the paper, \( \mathbb{R}^n \) denotes the \( n \) dimensional Euclidean space, and \( \mathbb{R}^{n \times m} \) is the set of all \( n \times m \) real matrices. \( I \) denotes the identity matrix with appropriate dimensions. \( \|x\| \) denotes the Euclidean norm of vector \( x \). \( \star \) denotes the elements below the main diagonal of a symmetric block matrix. \( \text{diag} [\cdot] \) denotes the block diagonal matrix. For symmetric matrices \( X \) and \( Y \), the notation \( X > Y \) (respectively, \( X \geq Y \)) means that the matrix \( X - Y \) is positive definite (respectively, nonnegative).

### 2. Main results

Consider a continuous-time cellular neural network with a time-varying delay described by the following nonlinear retarded functional differential state equation:

\[
\dot{y}(t) = -(a_i + \Delta a_i(t))y_i(t) + \sum_{j=1}^{n} (w_{ij} + \Delta w_{ij}(t))f_j(y_j(t)) + \sum_{j=1}^{n} (w_{ij}^1 + \Delta w_{ij}^1(t))f_j(y_j(t - h(t))) + b_i, \\
i = 1, 2, \ldots, n
\]

or equivalently

\[
\dot{y}(t) = -(A + \Delta A(t))y(t) + (W + \Delta W(t))f(y(t)) + (W_1 + \Delta W_1(t))y(t - h(t)) + b,
\]

where \( y(t) = [y_1(t), \ldots, y_n(t)]^T \in \mathbb{R}^n \) is the neuron state vector, \( f(y(t)) = [f_1(y_1(t)), \ldots, f_n(y_n(t))]^T \in \mathbb{R}^n \) is the activation functions, \( f(y(t - h(t))) = [f_1(y_1(t - h(t))), \ldots, f_n(y_n(t - h(t)))]^T \in \mathbb{R}^n \), \( b = [b_1, \ldots, b_n]^T \) is a constant input vector, \( A = \text{diag}(a) \) is a positive diagonal matrix, \( W = (w_{ij})_{n \times n} \) and \( W_1 = (w_{ij}^1)_{n \times n} \) are the interconnection matrices representing the weight coefficients of the neurons, \( \Delta A, \Delta W, \Delta W_1 \) are parametric uncertainties, the time delays \( h(t) \) is bounded nonnegative functions satisfying \( 0 \leq h(t) \leq \bar{h} \), and \( \bar{h}(t) < h_\delta < 1 \).

In this paper, it is assumed that the continuous matrix-valued functions \( \Delta A(t), \Delta W(t), \Delta W_1(t) \) are of the form

\[
\Delta A(t) = H_1 F_1(t) E_1, \quad \Delta W(t) = H_2 F_2(t) E_2, \quad \Delta W_1(t) = H_3 F_3(t) E_3,
\]

where \( H_1, H_2, H_3, E_1, E_2, E_3 \) are known constant matrices of appropriate dimensions, and \( F_1(t), F_2(t), F_3(t) \) are unknown time-varying matrix with Lebesgue measurable elements bounded by

\[
F_i^T(t) F_i(t) \leq I, \quad i = 1, 2, 3.
\]

The activation function \( f(y) \) is nondecreasing, bounded and globally Lipschitz; that is

\[
0 \leq \frac{f_i(\xi_1) - f_i(\xi_2)}{\xi_1 - \xi_2} \leq k_i, \quad i = 1, 2, \ldots, n.
\]

Then, by using the well-known Brouwer’s fixed point theorem [10], one can easily prove that there exists at least one equilibrium point for Eq. (2).

For the sake of simplicity in the stability analysis of the system (2), we make the following transformation to the system (2):

\[
x(t) = y(t) - y^*,
\]

where \( y^* = (y_1^*, y_2^*, \ldots, y_n^*)^T \) is an equilibrium point of Eq. (2). Under the transformation, it is easy to see that the system (2) becomes

\[
\dot{x}(t) = -(A + \Delta A(t))x(t) + (W + \Delta W(t))g(x(t)) + (W_1 + \Delta W_1(t))g(x(t - h(t))),(4)
\]

where \( x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n \) is the state vector of the transformed system, \( g(x) = [g_1(x), \ldots, g_n(x)]^T \) and \( g_j(x_j(t)) = f_j(x_j(t) + y_j^*) - f_j(y_j^*)^T \) with \( g(0) = 0, \forall j \). It is noted that each activation function \( g_i(\cdot) \) satisfies the following sector condition:

\[
0 \leq \frac{g_i(\xi_1) - g_i(\xi_2)}{\xi_1 - \xi_2} \leq k_i, \quad i = 1, 2, \ldots, n.
\]

The following fact and lemma will be used for deriving main result.

**Fact 1 (Schur complement).** Given constant symmetric matrices \( \Sigma_1, \Sigma_2, \Sigma_3 \) where \( \Sigma_1 = \Sigma_1^T \) and \( 0 < \Sigma_2 = \Sigma_2^T \), then \( \Sigma_1 + \Sigma_1^T \Sigma_2^{-1} \Sigma_3 < 0 \) if and only if
D=diag{asymptotically stable if there exist positive definite matrices P, Q, R, Z, X, a positive diagonal matrix Y

Theorem 1. For given

Lemma 1 (see [24]). Assume that a ∈ ℝ⁺, b ∈ ℝ⁺, and N ∈ ℝ⁺⁺ are defined, then for any matrices X ∈ ℝ⁺⁺, Y ∈ ℝ⁺⁺ and Z ∈ ℝ⁺⁺, the following holds

Fact 2. For given matrices H, E and F with FᵀF ≤ I and a scalar ε > 0, the following inequality is always satisfied.

HFE + EᵀFᵀDᵀ ≤ εDDᵀ + ε⁻¹EᵀE.

Now we will present a new result for asymptotic stability of Eq. (4).

Theorem 1. For given 0 ≤ h(t) ≤ ¯h, h(t) ≤ h₁ < 1 and K = diag{k₁, k₂, ..., kₙ}, the equilibrium point of (4) is globally asymptotically stable if there exist positive definite matrices P, Q, R, Z, X, a positive diagonal matrix D = diag{d₁, d₂, ..., dₙ} and any matrix Y, satisfying the following LMIs:

<table>
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<tr>
<th>Π₁</th>
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where

Π₁ = −AᵀP − PA + R + ε₁E₁ᵀE₁ + ε₂E₂ᵀE₂ + ε₃hE₃ᵀE₃,

Π₂ = −2DAK⁻¹ + DW + WᵀD + Q + ε₁E₁ᵀE₂ + ε₂E₂ᵀE₂ + ε₃hE₃ᵀE₂,

Π₃ = −(1 − h₃)Q − YK⁻¹ − K⁻¹Y + h₃X − (1 − h₃)K⁻¹RK⁻¹ + ε₁E₁ᵀE₃ + ε₂E₂ᵀE₃ + ε₃hE₃ᵀE₃,

Π₄ = [PH₁ PH₂ PH₃],

Π₅ = [DH₁ DH₂ DH₃],

Π₆ = [hZH₁ hZH₂ hZH₃],

Π₇ = diag{−ε₁I, −ε₂I, −ε₃I},

Π₈ = diag{−ε₄I, −ε₅I, −ε₆I},

Π₉ = diag{−ε₇I, −ε₈I, −ε₉I}.

Proof. Define a Lyapunov functional candidate for system (4) as

\[ V = xᵀ(t)x(t) + 2 \sum_{i=1}^{n} d_i \int_0^{x_i(t)} g_i(s)ds + \int_{-h}^{t} gᵀ(x(s))Qg(x(s))ds + \int_{-h}^{t} xᵀ(s)Rx(s)ds \]

+ \int_{-h}^{t} \int_{-h}^{t} xᵀ(s)Zx(s)dsdθ.

Then, the time derivative of V along the trajectory of (4) gives
\[ \dot{V} = 2x^T(t)P \dot{x}(t) + 2g^T(x(t))D \dot{x}(t) + g^T(x(t))Qg(x(t)) - (1 - \dot{h}(t))g^T(x(t-h(t)))Qg(x(t-h(t))) + x^T(t)Rx(t) \\
- (1 - \dot{h}(t))x^T(t-h(t))Rx(t-h(t)) + \dot{h}x^T(t)Z \dot{x}(t) - \int_{t-h}^{t} \dot{x}^T(s)Z \dot{x}(s) ds \leq 2x^T(t)(P(-A + \Delta A)x(t) + (W + \Delta W)g(x(t)) + (W_1 + \Delta W_1)g(x(t-h(t)))) + 2g^T(x(t))D(-A + \Delta A)x(t) \\
+ (W + \Delta W)g(x(t)) + (W_1 + \Delta W_1)g(x(t-h(t))) + g^T(x(t))Qg(x(t)) - (1 - \dot{h}_d)g^T(x(t-h(t)))Qg(x(t-h(t))) \\
+ x^T(t)Rx(t) - (1 - \dot{h}_d)x^T(t-h(t))Rx(t-h(t)) + \dot{h}x^T(t)Z \dot{x}(t) - \int_{t-h(t)}^{t} \dot{x}^T(s)Z \dot{x}(s) ds, \] (9)

where the following inequality is used:

\[ - \int_{t-h}^{t} \dot{x}^T(s)Z \dot{x}(s) ds \leq - \int_{t-h}^{t} \dot{x}^T(s)Z \dot{x}(s) ds. \]

By the Leibniz–Newton formula, the following equation satisfies:

\[ 2g^T(x(t-h(t))) \left( x(t) - x(t-h(t)) - \int_{t-h(t)}^{t} \dot{x}(s) ds \right) = 0. \] (10)

By adding the relation (10) to Eq. (9), we have

\[ \dot{V} \leq 2x^T(t)(P(-A + H_1 F_1(t) E_1)x(t) + (W + H_2 F_2(t) E_2)g(x(t)) + (W_1 + H_2 F_3(t) E_3)g(x(t-h(t)))) \\
+ 2g^T(x(t))D(-A + H_1 F_1(t) E_1)x(t) + (W + H_2 F_2(t) E_2)g(x(t)) + (W_1 + H_2 F_3(t) E_3)g(x(t-h(t)))) \\
+ g^T(x(t))Qg(x(t)) - (1 - \dot{h}_d)g^T(x(t-h(t)))Qg(x(t-h(t))) + x^T(t)Rx(t) - (1 - \dot{h}_d)x^T(t-h(t))Rx(t-h(t)) \\
- \int_{t-h(t)}^{t} \dot{x}^T(s)Z \dot{x}(s) ds + \dot{h}(-A + H_1 F_1(t) E_1)x(t) + (W + H_2 F_2(t) E_2)g(x(t)) + (W_1 \\
+ H_2 F_3(t) E_3)g(x(t-h(t)))) + 2g^T(x(t-h(t)))x(t) - 2g^T(x(t-h(t)))x(t-h(t)) - 2g^T(x(t-h(t))) \\
\times \int_{t-h(t)}^{t} \dot{x}(s) ds. \] (11)

Applying Lemma 1 to a term in (11) gives that

\[ -2 \int_{t-h(t)}^{t} g^T(x(t-h(t)))\dot{x}(s) ds \leq \dot{h}g^T(x(t-h(t)))Xg(x(t-h(t))) + 2g^T(x(t-h(t)))(Y-1)(x(t) \\
- x(t-h(t))) + \int_{t-h(t)}^{t} \dot{x}^T(s)Z \dot{x}(s) ds, \] (12)

where

\[ \begin{bmatrix} X & Y \\ \star & Z \end{bmatrix} \geq 0. \] (13)

Here note that

\[ -2g^T(x(t))DAx(t) \leq -2g^T(x(t))DAK^{-1}g(x(t)), \]
\[ -(1 - \dot{h}_d)x^T(t-h(t))Rx(t-h(t)) \leq -(1 - \dot{h}_d)g^T(x(t-h(t)))K^{-1}RK^{-1}g(x(t-h(t))), \]
\[ -2g^T(x(t-h(t)))Yx(t-h(t)) \leq -2g^T(x(t-h(t)))YK^{-1}g(x(t-h(t))), \] (14)
Then, using (12) and (14), it can be shown that

\[
\dot{V} \leq 2x^T(t)\left( - (A + H_1F_1(t)E_1)x(t) + (W + H_2F_2(t)E_2)g(x(t)) + (W_1 + H_3F_3(t)E_3)g(x(t-h(t))) \\
- 2g^T(x(t))DAK^{-1}g(x(t)) + 2g^T(x(t))D(-H_1F_1(t)E_1)x(t) + (W + H_2F_2(t)E_2)g(x(t)) \\
+ (W_1 + H_3F_3(t)E_3)g(x(t-h(t))) + g^T(x(t))Qg(x(t)) - (1 - h_o)g^T(x(t-h(t)))Qg(x(t-h(t))) \\
+ x^T(t)RX(t) - (1 - h_o)g^T(x(t-h(t)))K^{-1}RK^{-1}g(x(t-h(t))) + \tilde{h}(- (A + H_1F_1(t)E_1)x(t) \\
+ (W + H_2F_2(t)E_2)g(x(t)) + (W_1 + H_3F_3(t)E_3)g(x(t-h(t))))^T Z \times (- (A + H_1F_1(t)E_1)x(t) \\
+ (W + H_2F_2(t)E_2)g(x(t)) + (W_1 + H_3F_3(t)E_3)g(x(t-h(t)))) + \tilde{h}g^T(x(t-h(t)))Xg(x(t-h(t))) \\
+ 2g^T(x(t-h(t)))Yx(t) - 2g^T(x(t-h(t)))YK^{-1}g(x(t-h(t))) = \zeta^T(t)(\Sigma_1 + \mathcal{M}^T(\tilde{h}Z)^{-1}\mathcal{M})\zeta,
\]

where

\[
\Sigma_1 = \begin{bmatrix}
\bar{\Pi}_1 & -2PH_1F_1(t)E_1 & PW - E_1^T F_1^T(t)H_1^T D + PH_2F_2(t)E_2 & PW_1 + PH_3F_3(t)E_3 + Y^T \\
\bar{\Pi}_2 + 2DH_2F_2(t)E_2 & DW_1 + DH_3F_3(t)E_3 & \\
\bar{\Pi}_3 & & & \\
\end{bmatrix},
\]

\[
\mathcal{M}^T = \begin{bmatrix}
-\tilde{h}(A + H_1F_1(t)E_1)^T Z \\
\tilde{h}(W + H_2F_2(t)E_2)^T Z \\
\tilde{h}(W_1 + H_3F_3(t)E_3)^T Z \\
\end{bmatrix},
\]

\[
\zeta(t) = \begin{bmatrix}
x^T(t) & g^T(x(t)) & g^T(x(t-h(t)))
\end{bmatrix}^T,
\]

\[
\bar{\Pi}_1 = -A^TP - PA + R,
\]

\[
\bar{\Pi}_2 = -2DAK^{-1} + DW + W^TD + Q,
\]

\[
\bar{\Pi}_3 = -(1 - h_o)Q - YK^{-1} - K^{-1}Y + \tilde{h}X - (1 - h_o)K^{-1}RK^{-1}.
\]

If the matrix \( \Sigma_1 + \mathcal{M}^T(\tilde{h}Z)^{-1}\mathcal{M} \) is a negative definite matrix, then there exist a positive scalar \( \delta \) such that \( \dot{V} < -\delta\|x(t)\| \), which guarantees the stability of the system [26]. By Fact 1 (Schur complement), the inequality \( \Sigma_1 + \mathcal{M}^T(\tilde{h}Z)^{-1}\mathcal{M} < 0 \) is equivalent to

\[
\Sigma_2 \equiv \phi_1 + \phi_2 < 0,
\]

where

\[
\phi_1 = \begin{bmatrix}
\bar{\Pi}_1 & PW & PW_1 + Y^T & -\tilde{h}A^T Z \\
\star & \bar{\Pi}_2 & DW_1 & \tilde{h}W^T Z \\
\star & \star & \bar{\Pi}_3 & \tilde{h}W^T Z \\
\star & \star & \star & -Z \\
\end{bmatrix},
\]

\[
\phi_2 = \begin{bmatrix}
-2PH_1F_1(t)E_1 & -E_1^T F_1^T(t)H_1^T D + PH_2F_2(t)E_2 & PH_3F_3(t)E_3 & -\tilde{h}(H_1F_1(t)E_1)^T Z \\
\star & 2DH_2F_2(t)E_2 & DH_3F_3(t)E_3 & 0 \\
\star & \star & \star & \star & \tilde{h}(H_3F_3(t)E_3)^T Z \\
\end{bmatrix}.
\]

Here, let us apply Fact 2 to the all the terms in \( \phi_2 \) to remove the uncertain factors \( F_i(t) \), i.e., the following inequalities hold:

\[
-2\xi_{i1}^T PH_1F_1(t)E_1\zeta_1 \leq e_1^T \xi_{i1}^T PH_1H_1^T P\xi_1 + e_1^T \xi_{i1}^T E_1\zeta_1,
\]

\[
2\xi_{i1}^T PH_2F_2(t)E_2\zeta_2 \leq e_2^T \xi_{i2}^T PH_2H_2^T P\xi_2 + e_2^T \xi_{i2}^T E_2\zeta_2,
\]

\[
2\xi_{i1}^T PH_3F_3(t)E_3\zeta_3 \leq e_3^T \xi_{i3}^T PH_3H_3^T P\xi_3 + e_3^T \xi_{i3}^T E_3\zeta_3,
\]

\[
-2\xi_{i1}^T DH_1F_1(t)E_1\zeta_1 \leq e_4^T \xi_{i4}^T DH_1H_1^T D\zeta_2 + e_4^T \xi_{i4}^T E_1\zeta_1,
\]

\[
2\xi_{i1}^T DH_2F_2(t)E_2\zeta_2 \leq e_5^T \xi_{i5}^T DH_2H_2^T D\zeta_2 + e_5^T \xi_{i5}^T E_2\zeta_2,
\]

\[
2\xi_{i1}^T DH_3F_3(t)E_3\zeta_3 \leq e_6^T \xi_{i6}^T DH_3H_3^T D\zeta_2 + e_6^T \xi_{i6}^T E_3\zeta_3,
\]
The LMI solutions of Theorem 1 can be obtained by solving the eigenvalue problem with respect to solution variables, which is a convex optimization problem [23]. In this letter, we utilize Matlab’s LMI Control Toolbox [25] algorithms [23] than those in literature.

The criterion given in Theorem 1 is dependent on the time delay. It is well known that the delay-dependent criteria are less conservative than delay-independent criteria when the delay is small. By iteratively solving the LMIs given in Theorem 1 with respect to time delay \( h \), one can find the maximum allowable upper bound \( \tilde{h} \) of time delay \( h(t) \) for guaranteeing asymptotic stability of system (4).

Remark 1. The LMI solutions of Theorem 1 can be obtained by solving the eigenvalue problem with respect to solution variables, which is a convex optimization problem [23]. In this letter, we utilize Matlab’s LMI Control Toolbox [25] which implements interior-point algorithm. This algorithm is significantly faster than classical convex optimization algorithms [23].

The following examples are given to show the less conservatism of our delay-dependent asymptotic stability results than those in literature.

**Example 1.** Consider the following DCNNs (4) with any time-varying delay \( h(t) \) and

\[
A = \begin{bmatrix} 2.6 & 0 \\ 0 & 1.1 \end{bmatrix}, \quad W = \begin{bmatrix} 1.1 & 1 \\ -0.2 & 0.1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.9 & 0.1 \\ -0.1 & 0.1 \end{bmatrix}, \quad H_1 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix},
\]

\[
H_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, \quad H_3 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad E_1 = H_1, \quad E_2 = H_2, \quad E_3 = H_3, \quad h_d = 0.2
\]

and the activation function \( g(x) = 0.5(|x+1|-|x-1|) \).

For this example, it is easy to check that the condition in [22] is not satisfied. It means that they fail to conclude whether this system is asymptotically stable or not. On the other hand, our delay-dependent criterion in Theorem 1 presents \( h = \infty \). It means that for this example our criterion show that the system is actually delay-independent stable, which shows that our criterion is less conservative than that in [22].

**Example 2.** [19] Consider a nominal DCNNs (4) with constant delay \( h \) and the parameters

\[
A = \begin{bmatrix} 1.2769 & 0 & 0 & 0 \\ 0 & 0.6231 & 0 & 0 \\ 0 & 0 & 0.9230 & 0 \\ 0 & 0 & 0 & 0.4480 \end{bmatrix},
\]

\[
W = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix},
\]

Thus if \( \Phi_1 + \Phi_3 < 0 \), the system (4) is asymptotically stable. By Fact 1, the inequality \( \Phi_1 + \Phi_3 < 0 \) is equivalent to the LMI (6). This completes our proof. \( \square \)

**Remark 2.** The LMI solutions of Theorem 1 can be obtained by solving the eigenvalue problem with respect to solution variables, which is a convex optimization problem [23]. In this letter, we utilize Matlab’s LMI Control Toolbox [25] which implements interior-point algorithm. This algorithm is significantly faster than classical convex optimization algorithms [23].
\[ W_1 = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix} \]

and \( k_1 = 0.1137, k_2 = 0.1279, k_3 = 0.7994, k_4 = 0.2386 \).

For this example, it can be checked that the conditions in [15,18], Theorem 1 in [16], Theorem 2 in [17], and Theorem 1 in [19] are not satisfied. It means that they fail to conclude whether this system is asymptotically stable or not. However, by applying Theorem 2 in [19] to this example, the maximum allowable bound \( h \) of \( h \) is \( h = 1.4224 \), while by Theorem 1 in this letter, we have \( h = 1.9321 \), which shows that our criterion is less conservative than those in [15–19].

3. Concluding remarks

In this paper, the global robust stability criterion for delayed neural networks with time-varying delay and parametric uncertainties has been investigated using the Lyapunov stability theory. An LMI-based method for delay-dependent criterion for the stability of the system have been developed. It can be shown that our result is less conservative than previously existing results through the numerical examples.

References