Available online at www.sciencedirect.com
ScienceDirect
Chaos, Solitons and Fractals 33 (2007) 436–442
CHAOS SOLITONS & FRACTALS
www.elsevier.com/locate/chaos

A delay-dependent asymptotic stability criterion of cellular neural networks with time-varying discrete and distributed delays

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Accepted 15 December 2005

Abstract

Based on the Lyapunov functional stability analysis for differential equations and the linear matrix inequality (LMI) optimization approach, A novel criterion for the global asymptotic stability of cellular neural networks with time-varying discrete and distributed delays is derived to guarantee global asymptotic stability. The criterion is expressed in terms of LMIs, which can be solved easily by various convex optimization algorithms. Some numerical examples are given to show the effectiveness of proposed method.

1. Introduction

It is well known that a cellular neural network (CNN) is formed by many cells, and that a cell contains linear and nonlinear circuit elements, which typically are capacitors, resistors, linear and nonlinear controlled sources, and independent sources. The structure of a CNN is similar to that found in cellular automata. Namely any cell in a CNN is connected only to its neighbor cells [1–14]. Nowadays, CNNs are widely used in signal processing, image processing, pattern classification, associative memories, fixed-point computation, and so on. On the other hand, in order to deal with moving images, one must introduce the time delays in the signal transmission among the cells. This leads to the model of delayed neural networks (DCNNs). Thus the stability analysis of DCNNs has become an important topic of theoretical studies in neural networks [15–23]. It is noticed that most works on delayed neural networks have dealt with the stability analysis problem for neural networks with discrete time delays. Very recently, there have been some initial studies on the stability analysis issue for various neural networks with distributed delays [24,8,25].

In this paper, we deal with the problem of global asymptotic stability for a class of neural networks with time-varying discrete and distributed delays using the Lyapunov theory and LMI framework. Then, a novel less conservative criterion is given in terms of LMIs. The advantage of the proposed approach is that the resulting stability criterion

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doi:10.1016/j.chaos.2006.01.015

can be used efficiently via existing numerical convex optimization algorithms such as the interior-point algorithms for solving LMIs [26].

Throughout the paper, $\mathscr{R}^n$ denotes the n dimensional Euclidean space, and $\mathscr{R}^{n\times m}$ is the set of all $n\times m$ real matrices. I denotes the identity matrix with appropriate dimensions. $\|x\|$ denotes the Euclidean norm of vector x. $\star$ denotes the elements below the main diagonal of a symmetric block matrix. diag$\{\cdot\}$ denotes the block diagonal matrix. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \geqslant Y$) means that the matrix $X - Y$ is positive definite (respectively, nonnegative).

2. Main results

Consider the following delayed neural networks with n neurons:

$$\dot{y}(t) = -Ay(t) + W_0 f(y(t)) + W_1 f(y(t-h(t))) + W_2 \int_{t-\tau(t)}^{t} f(y(s))\mathrm{d}s + b, \tag{1}$$

where $y(t) = [y_1(t), \ldots, y_n(t)]^{\mathrm{T}} \in \mathbb{R}^n$ is the neuron state vector, $f(y(t)) = [f_1(y_1), f_2(y_2), \ldots, f_n(y_n)]^{\mathrm{T}}$ denotes the bounded neuron activation function with $f_i(0) = 0$, $b = [b_1, \ldots, b_n]^{\mathrm{T}}$ is a constant input vector, $A = \mathrm{diag}(a_i)$ is a positive diagonal matrix, and $W_0 = (w_{ij}^0)_{n\times n}$, $W_1 = (w_{ij}^1)_{n\times n}$, $W_2 = (w_{ij}^2)_{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons, the time delays $h(t)$ is bounded nonnegative functions satisfying $0 \leqslant h(t) \leqslant \bar{h}$, $\tau(t) > 0$ is the distributed time delay satisfying $0 \leqslant \tau(t) \leqslant \bar{\tau}$, and it is assumed that $\dot{h}(t) < h_d < 1$.

In this paper, it is assumed that the activation function $f(y)$ is nondecreasing, bounded and globally Lipschitz; that is

$$0 \leqslant \frac{f_i(\xi_1) - f_i(\xi_2)}{\xi_1 - \xi_2} \leqslant k_i, \quad i = 1, 2, \ldots, n. \tag{2}$$

Then, by using the well-known Brouwer's fixed point theorem [10], one can easily prove that there exists at least one equilibrium point for Eq. (1).

For the sake of simplicity in the stability analysis of the system (1), we make the following transformation to the system (1):

$$x(\cdot) = y(\cdot) - y^*,$$

where $y^* = (y_1^*, y_2^*, \ldots, y_n^*)^{\mathrm{T}}$ is an equilibrium point of Eq. (1). Under the transformation, it is easy to see that the system (1) becomes

$$\dot{x}(t) = -Ax(t) + W_0 g(x(t)) + W_1 g(x(t-h(t))) + W_2 \int_{t-\tau(t)}^{t} g(x(s))\mathrm{d}s, \tag{3}$$

where $x(t) = [\,x_1(t) \quad x_2(t) \quad \cdots \quad x_n(t)\,]^{\mathrm{T}} \in \mathscr{R}^n$ is the state vector of the transformed system, $g(x) = [g_1(x), \ldots, g_n(x)]^{\mathrm{T}}$ and $g_j(x_j(t)) = f_j(x_j(t) + y_j^*) - f_j(y_j^*)$ with $g_j(0) = 0$, $\forall j$. It is noted that each activation function $g_i(\cdot)$ satisfies the following sector condition:

$$0 \leqslant \frac{g_i(\xi_1) - g_i(\xi_2)}{\xi_1 - \xi_2} \leqslant k_i, \quad i = 1, 2, \ldots, n. \tag{4}$$

The following fact and lemmas will be used for deriving main result.

Fact 1. (Schur complement) Given constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$ where $\Sigma_1 = \Sigma_1^{\mathrm{T}}$ and $0 < \Sigma_2 = \Sigma_2^{\mathrm{T}}$, then $\Sigma_1 + \Sigma_3^{\mathrm{T}}\Sigma_2^{-1}\Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^{\mathrm{T}} \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^{\mathrm{T}} & \Sigma_1 \end{bmatrix} < 0.$$

Lemma 1. [27] *Assume that $a \in \mathscr{R}^{n_a}$, $b \in \mathscr{R}^{n_b}$, and $N \in \mathscr{R}^{n_a\times n_b}$ are defined, then for any matrices $X \in \mathscr{R}^{n_a\times n_a}$, $Y \in \mathscr{R}^{n_a\times n_b}$ and $Z \in \mathscr{R}^{n_b\times n_b}$, the following holds*:

$$-2a^{\mathrm{T}}Nb \leqslant \begin{bmatrix} a \\ b \end{bmatrix}^{\mathrm{T}} \begin{bmatrix} X & Y-N \\ \star & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix}, \quad \begin{bmatrix} X & Y \\ \star & Z \end{bmatrix} \geqslant 0.$$

Lemma 2. [28] *For any constant matrix $\Sigma \in \mathbb{R}^{n\times n}$, $\Sigma = \Sigma^T > 0$, scalar $\gamma > 0$, vector function $\omega : [0,\gamma] \to \mathbb{R}^n$ such that the integrations concerned are well defined, then*

$$\left(\int_0^{\gamma} \omega(s)\mathrm{d}s\right)^{\mathrm{T}} \Sigma \left(\int_0^{\gamma} \omega(s)\mathrm{d}s\right) \leqslant \gamma \int_0^{\gamma} \omega^{\mathrm{T}}(s)\Sigma\omega(s)\mathrm{d}s. \tag{5}$$

Now we will present a new result for asymptotic stability of Eq. (3).

Theorem 1. *For given $0 \leqslant h(t) \leqslant \bar{h}$, $\dot{h}(t) \leqslant h_d < 1$, $0 \leqslant \tau(t) \leqslant \bar{\tau}$, and $K = \mathrm{diag}\{k_1, k_2, \ldots, k_n\}$, the equilibrium point of (3) is globally asymptotically stable if there exist positive definite matrices P, Q, R, Z, S, X, a positive diagonal matrix $D = \mathrm{diag}\{d_1, d_2, \ldots, d_n\}$ and any matrix Y, satisfying the following two LMIs:*

$$\begin{bmatrix} \Pi_1 & PW_0 & PW_1 + Y^{\mathrm{T}} & PW_2 & -\bar{h}A^{\mathrm{T}}Z \\ \star & \Pi_2 & DW_1 & DW_2 & \bar{h}W_0^{\mathrm{T}}Z \\ \star & \star & \Pi_3 & 0 & \bar{h}W_1^{\mathrm{T}}Z \\ \star & \star & \star & -S & \bar{h}W_2^{\mathrm{T}}Z \\ \star & \star & \star & \star & -\bar{h}Z \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} X & Y \\ \star & Z \end{bmatrix} \geqslant 0, \tag{7}$$

where

$$\Pi_1 = -A^{\mathrm{T}}P - PA + R,$$
$$\Pi_2 = -2DAK^{-1} + DW_0 + W_0^{\mathrm{T}}D + Q + \bar{\tau}^2 S,$$
$$\Pi_3 = -(1-h_d)Q - YK^{-1} - K^{-1}Y + \bar{h}X - (1-h_d)K^{-1}RK^{-1}.$$

Proof. Consider the following Lyapunov functional:

$$\begin{aligned} V &= x^{\mathrm{T}}(t)Px(t) + 2\sum_{i=1}^{n} d_i \int_0^{x_i(t)} g_i(s)\mathrm{d}s + \int_{t-h(t)}^{t} g^{\mathrm{T}}(x(s))Qg(x(s))\mathrm{d}s + \int_{t-h(t)}^{t} x^{\mathrm{T}}(s)Rx(s)\mathrm{d}s \\ &\quad + \int_{-\bar{h}}^{0}\int_{t+\theta}^{t} \dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s\,\mathrm{d}\theta + \bar{\tau}\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} g^{\mathrm{T}}(x(s))Sg(x(s))\mathrm{d}s\,\mathrm{d}\theta, \end{aligned} \tag{8}$$

where P, Q, R, Z, S are positive definite matrices and d_i is a positive scalar.

Calculating the time derivative of V along the trajectory of (3), we have

$$\begin{aligned} \dot{V} &= 2x^{\mathrm{T}}(t)P\dot{x}(t) + 2g^{\mathrm{T}}(x(t))D\dot{x}(t) + g^{\mathrm{T}}(x(t))Qg(x(t)) - (1-\dot{h}(t))g^{\mathrm{T}}(x(t-h(t)))Qg(x(t-h(t))) \\ &\quad + x^{\mathrm{T}}(t)Rx(t) - (1-\dot{h}(t))x^{\mathrm{T}}(t-h(t))Rx(t-h(t)) + \bar{h}\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t) - \int_{t-\bar{h}}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s \\ &\quad + \bar{\tau}^2 g^{\mathrm{T}}(x(t))Sg(x(t)) - \bar{\tau}\int_{t-\bar{\tau}}^{t} g^{\mathrm{T}}(x(s))Sg(x(s))\mathrm{d}s \\ &\leqslant 2x^{\mathrm{T}}(t)P\dot{x}(t) + 2g^{\mathrm{T}}(x(t))D\dot{x}(t) + g^{\mathrm{T}}(x(t))Qg(x(t)) - (1-h_d)g^{\mathrm{T}}(x(t-h(t)))Qg(x(t-h(t))) \\ &\quad + x^{\mathrm{T}}(t)Rx(t) - (1-h_d)x^{\mathrm{T}}(t-h(t))Rx(t-h(t)) + \bar{h}\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t) \\ &\quad - \int_{t-h(t)}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s + \bar{\tau}^2 g^{\mathrm{T}}(x(t))Sg(x(t)) - \tau(t)\int_{t-\tau(t)}^{t} g^{\mathrm{T}}(x(s))Sg(x(s))\mathrm{d}s, \end{aligned} \tag{9}$$

where the following inequalities are used:

$$-\int_{t-\bar{h}}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s \leqslant -\int_{t-h(t)}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s,$$
$$-\bar{\tau}\int_{t-\bar{\tau}}^{t} g^{\mathrm{T}}(x(s))Sg(x(s))\mathrm{d}s \leqslant -\tau(t)\int_{t-\tau(t)}^{t} g^{\mathrm{T}}(x(s))Sg(x(s))\mathrm{d}s.$$

By well-known Leibniz–Newton formula, the following equation satisfies:

$$2g^{\mathrm{T}}(x(t-h(t)))\left(x(t)-x(t-h(t))-\int_{t-h(t)}^{t}\dot{x}(s)\mathrm{d}s\right)=0. \tag{10}$$

Using the relationship (10), we have a new bound of $\dot{V}$ as

$$\begin{aligned}
\dot{V} \leqslant\, & 2x^{\mathrm{T}}(t)P\left(-Ax(t)+W_0g(x(t))+W_1g(x(t-h(t)))+W_2\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\right)\\
&+2g^{\mathrm{T}}(x(t))D\left(-Ax(t)+W_0g(x(t))+W_1g(x(t-h(t)))+W_2\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\right)\\
&+g^{\mathrm{T}}(x(t))Qg(x(t))-(1-h_d)g^{\mathrm{T}}(x(t-h(t)))Qg(x(t-h(t)))+x^{\mathrm{T}}(t)Rx(t)-(1-h_d)x^{\mathrm{T}}(t-h(t))Rx(t-h(t))+\bar{h}\dot{x}^{\mathrm{T}}(t)Z\dot{x}(t)\\
&-\int_{t-h(t)}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s+2g^{\mathrm{T}}(x(t-h(t)))x(t)-2g^{\mathrm{T}}(x(t-h(t)))x(t-h(t))-2g^{\mathrm{T}}(x(t-h(t)))\\
&\times\int_{t-h(t)}^{t}\dot{x}(s)\mathrm{d}s+\bar{\tau}^2g^{\mathrm{T}}(x(t))Sg(x(t))-\left(\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\right)^{\mathrm{T}}S\left(\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\right),
\end{aligned} \tag{11}$$

where Lemma 2 is utilized in the last term in Eq. (9).

By applying Lemma 1 to a term in (11), we have the following relationship:

$$\begin{aligned}
-2\int_{t-h(t)}^{t}g^{\mathrm{T}}(x(t-h(t)))\dot{x}(s)\mathrm{d}s \leqslant\, & \bar{h}g^{\mathrm{T}}(x(t-h(t)))Xg(x(t-h(t)))+2g^{\mathrm{T}}(x(t-h(t)))(Y-I)(x(t)\\
&-x(t-h(t)))+\int_{t-h(t)}^{t}\dot{x}^{\mathrm{T}}(s)Z\dot{x}(s)\mathrm{d}s,
\end{aligned} \tag{12}$$

where

$$\begin{bmatrix} X & Y \\ \star & Z \end{bmatrix} \geqslant 0. \tag{13}$$

Also, the following inequalities hold from (4):

$$\begin{aligned}
&-2g^{\mathrm{T}}(x(t))DAx(t) \leqslant -2g^{\mathrm{T}}(x(t))DAK^{-1}g(x(t)),\\
&-(1-h_d)x^{\mathrm{T}}(t-h(t))Rx(t-h(t)) \leqslant -(1-h_d)g^{\mathrm{T}}(x(t-h(t)))K^{-1}RK^{-1}g(x(t-h(t))),\\
&-2g^{\mathrm{T}}(x(t-h(t)))Yx(t-h(t)) \leqslant -2g^{\mathrm{T}}(x(t-h(t)))YK^{-1}g(x(t-h(t))).
\end{aligned} \tag{14}$$

Then, using (12) and (14), it can be shown that

$$\begin{aligned}
\dot{V} \leqslant\, & 2x^{\mathrm{T}}(t)P\left(-Ax(t)+W_0g(x(t))+W_1g(x(t-h(t)))+W_2\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\right)-2g^{\mathrm{T}}(x(t))DAK^{-1}g(x(t))\\
&+2g^{\mathrm{T}}(x(t))D\left(W_0g(x(t))+W_1g(x(t-h(t)))+W_2\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\right)\\
&+g^{\mathrm{T}}(x(t))Qg(x(t))-(1-h_d)g^{\mathrm{T}}(x(t-h(t)))Qg(x(t-h(t)))\\
&+x^{\mathrm{T}}(t)Rx(t)-(1-h_d)g^{\mathrm{T}}(x(t-h(t)))K^{-1}RK^{-1}g(x(t-h(t)))\\
&+\bar{h}\left(-Ax(t)+W_0g(x(t))+W_1g(x(t-h(t)))+W_2\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\right)^{\mathrm{T}}Z\Bigg(-Ax(t)\\
&+W_0g(x(t))+W_1g(x(t-h(t)))+W_2\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\Bigg)+\bar{h}g^{\mathrm{T}}(x(t-h(t)))Xg(x(t-h(t)))\\
&+2g^{\mathrm{T}}(x(t-h(t)))Yx(t)-2g^{\mathrm{T}}(x(t-h(t)))YK^{-1}g(x(t-h(t)))+\bar{\tau}^2g^{\mathrm{T}}(x(t))Sg(x(t))\\
&-\left(\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\right)^{\mathrm{T}}S\left(\int_{t-\tau(t)}^{t}g(x(s))\mathrm{d}s\right)\\
=\,&\xi^{\mathrm{T}}(t)(\Sigma+\Gamma^{\mathrm{T}}(\bar{h}Z)^{-1}\Gamma)\xi
\end{aligned} \tag{15}$$

where

$$\Sigma = \begin{bmatrix} \Pi_1 & PW_0 & PW_1 + Y^{\mathrm{T}} & PW_2 \\ \star & \Pi_2 & DW_1 & DW_2 \\ \star & \star & \Pi_3 & 0 \\ \star & \star & \star & -S \end{bmatrix}, \quad \Gamma^{\mathrm{T}} = \begin{bmatrix} -\bar{h}A^{\mathrm{T}}Z \\ \bar{h}W_0^{\mathrm{T}}Z \\ \bar{h}W_1^{\mathrm{T}}Z \\ \bar{h}W_2^{\mathrm{T}}Z \end{bmatrix},$$

$$\xi(t) = \left[x^{\mathrm{T}}(t) \quad g^{\mathrm{T}}(x(t)) \quad g^{\mathrm{T}}(x(t-h(t))) \quad \left(\int_{t-\tau(t)}^{t} g(x(s))\mathrm{d}s \right)^{\mathrm{T}} \right]^{\mathrm{T}}.$$

If the matrix $\Sigma + \Gamma^{\mathrm{T}}(\bar{h}Z)^{-1}\Gamma$ is a negative definite matrix, then there exist a positive scalar δ such that $\dot{V} < -\delta\|x(t)\|$, which guarantees the stability of the system [30]. By Fact 1 (Schur complement), the inequality $\Sigma + \Gamma^{\mathrm{T}}(\bar{h}Z)^{-1}\Gamma < 0$ is equivalent to the LMI (6). This completes our proof. □

Remark 1. In the work of Wang et al. [25], the stability analysis of DCNNs with constant distributed time delays is considered. This is a special case of our work.

Remark 2. The criterion given in Theorem 1 is dependent on the time delay. It is well known that the delay-dependent criteria are less conservative than delay-independent criteria when the delay is small.

Remark 3. By iteratively solving the LMIs given in Theorem 1 with respect to $\bar{h}$ or $\bar{\tau}$, one can find the maximum allowable upper bounds of $h(t)$ or $\tau(t)$ for guaranteeing asymptotic stability of system (3).

Remark 4. The LMI solutions of Theorem 1 can be obtained by solving the eigenvalue problem with respect to solution variables, which is a convex optimization problem [26]. In this paper, we utilize Matlab's LMI Control Toolbox [29] which implements interior-point algorithm. This algorithm is significantly faster than classical convex optimization algorithms [26].

Three simple examples are presented here in order to illustrate the usefulness of our result.

Example 1. [19] Consider a delayed DCNNs (3) with constant delay $h(t) = h$ and the parameters

$$A = \begin{bmatrix} 1.2769 & 0 & 0 & 0 \\ 0 & 0.6231 & 0 & 0 \\ 0 & 0 & 0.9230 & 0 \\ 0 & 0 & 0 & 0.4480 \end{bmatrix}, \quad W = \begin{bmatrix} -0.0373 & 0.4852 & -0.3351 & 0.2336 \\ -1.6033 & 0.5988 & -0.3224 & 1.2352 \\ 0.3394 & -0.0860 & -0.3824 & -0.5785 \\ -0.1311 & 0.3253 & -0.9534 & -0.5015 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} 0.8674 & -1.2405 & -0.5325 & 0.0220 \\ 0.0474 & -0.9164 & 0.0360 & 0.9816 \\ 1.8495 & 2.6117 & -0.3788 & 0.8428 \\ -2.0413 & 0.5179 & 1.1734 & -0.2775 \end{bmatrix}, \quad W_2 = 0,$$

and $k_1 = 0.1137$, $k_2 = 0.1279$, $k_3 = 0.7994$, $k_4 = 0.2386$.

For this example, it can be checked that the conditions in [15,18], Theorem 1 in [16], Theorem 2 in [17], and Theorem 1 in [19] are not satisfied. It means that they fail to conclude whether this system is asymptotically stable or not. However, by applying Theorem 2 in [19] to this example, the maximum allowable bound $\bar{h}$ of h is $\bar{h} = 1.4224$, while by Theorem 1 in this paper, we have $\bar{h} = 1.9321$, which shows that our criterion is less conservative than those in [15–19].

Example 2. Consider a two-neuron neural network (3), where

$$A = \begin{bmatrix} 0.9 & 0 \\ 0 & 0.8 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 1 & -1.7 \\ -1.6 & 1 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 1 & 0.6 \\ 0.5 & 0.8 \end{bmatrix},$$

$$W_2 = \begin{bmatrix} 0.4 & 0.3 \\ 0.1 & 0.2 \end{bmatrix}, \quad k_1 = k_2 = 0.2, \quad h(t) = \tau(t) = H.$$

Now, we are about to get the maximum allowable bound $\bar{H}$ of the delay H for guaranteeing the stability of the system. Then by iteratively applying Theorem 1 to the system with respect to H, it is found that the LMI in Theorem 1 is feasible for $0 \leqslant H \leqslant 6.6789$. When $H = 6.6789$, the LMI solutions can be obtained as

Table 1
Maximum allowable bound $\bar{H}$

h_d	0.3	0.5	0.7	0.9
$\bar{H}$	23.2599	22.6761	21.5781	18.0082

$$
\begin{aligned}
P &= \begin{bmatrix} 8.6684 & 17.9975 \\ 17.9975 & 37.8523 \end{bmatrix}, \quad Y = \begin{bmatrix} -0.0115 & -0.0572 \\ -0.0967 & -0.4800 \end{bmatrix}, \\
Q &= \begin{bmatrix} 37.0423 & -92.3849 \\ -92.3849 & 230.4393 \end{bmatrix}, \quad D = \begin{bmatrix} 127.9263 & 0 \\ 0 & 214.3213 \end{bmatrix}, \\
R &= \begin{bmatrix} 7.5361 & 13.8536 \\ 13.8536 & 25.4675 \end{bmatrix}, \quad Z = \begin{bmatrix} 0.0146 & 0.0578 \\ 0.0578 & 0.2661 \end{bmatrix}, \\
X &= \begin{bmatrix} 0.0139 & 0.1142 \\ 0.1142 & 0.9524 \end{bmatrix}.
\end{aligned}
$$

Example 3. Consider a third-order delayed neural network (3) [25], where

$$
\begin{aligned}
A &= \begin{bmatrix} 2.3 & 0 & 0 \\ 0 & 3.4 & 0 \\ 0 & 0 & 2.5 \end{bmatrix}, \quad W_0 = \begin{bmatrix} 0.9 & -1.5 & 0.1 \\ -1.2 & 1 & 0.2 \\ 0.2 & 0.3 & 0.8 \end{bmatrix}, \quad W_1 = \begin{bmatrix} 0.8 & 0.6 & 0.2 \\ 0.5 & 0.7 & 0.1 \\ 0.2 & 0.1 & 0.5 \end{bmatrix}, \\
W_2 &= \begin{bmatrix} 0.3 & 0.2 & 0.1 \\ 0.1 & 0.2 & 0.1 \\ 0.1 & 0.1 & 0.2 \end{bmatrix}, \quad g_i(x_i) = 0.2 \times (|x_i + 1| - |x_i - 1|)/2, \quad i = 1, 2, 3, \;\; h = \tau = H.
\end{aligned}
$$

By applying Theorem 1 to this example, we could obtain the maximum delay bound $\bar{H} = 23.78$.

In case that the time delays are time-varying, i.e. $h(t) = \tau(t) = H(t)$, the maximum allowable bound $\bar{H}$ for several value of h_d is given in Table 1.

3. Concluding remarks

We have proposed a class of neural networks with time-varying distributed delays. An LMI approach has been developed to solve the problem addressed. The condition for the global asymptotic stability has been derived in terms of the solutions to the LMIs, and three numerical examples have been used to demonstrate the usefulness of our main result.

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