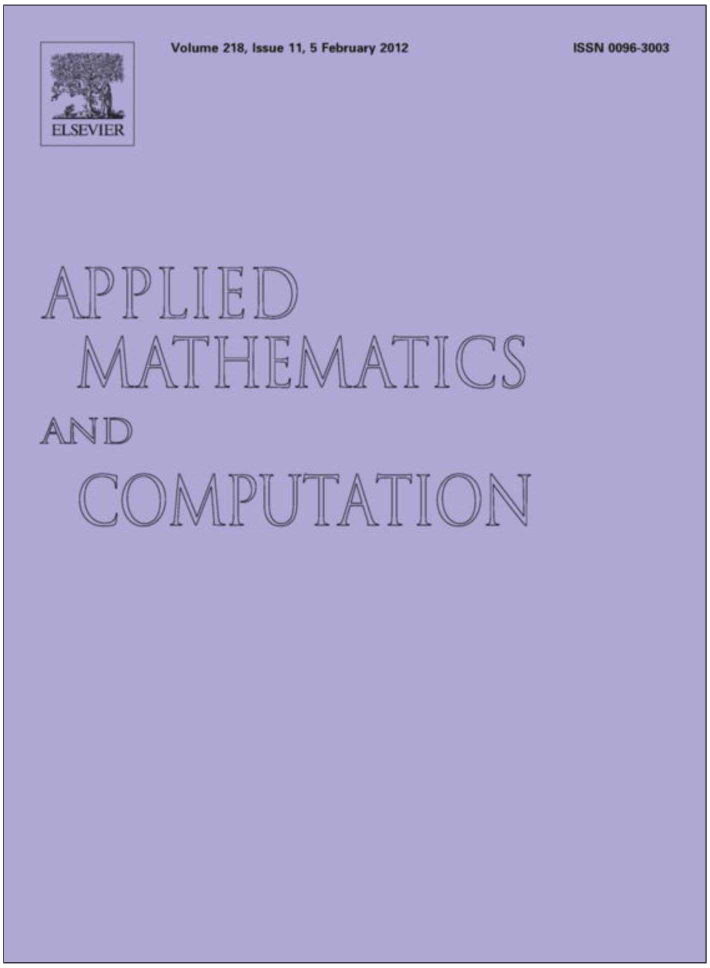
Volume 218, Issue 11, 5 February 2012
ISSN 0096-3003
ELSEVIER
APPLIED MATHEMATICS AND COMPUTATION

Applied Mathematics and Computation 218 (2012) 6469–6481

Contents lists available at SciVerse ScienceDirect

Applied Mathematics and Computation

journal homepage: www.elsevier.com/locate/amc

Guaranteed cost synchronization of a complex dynamical network via dynamic feedback control

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ARTICLE INFO

Keywords:
Complex dynamical network
Synchronization
Guaranteed cost control
Dynamic feedback controller

ABSTRACT

In this paper, the problem of guaranteed cost synchronization for a complex network is investigated. In order to achieve the synchronization, two types of guaranteed cost dynamic feedback controller are designed. Based on Lyapunov stability theory, a linear matrix inequality (LMI) convex optimization problem is formulated to find the controller which guarantees the asymptotic stability and minimizes the upper bound of a given quadratic cost function. Finally, a numerical example is given to illustrate the proposed method.

1. Introduction

Complex dynamical networks have received a great deal of attention since they are shown to widely exist in various fields of real world [1–4] such as the Internet, the World Wide Web (WWW), food chain, electricity distribution networks, relationship networks, disease transmission networks, and so on. A complex network is a set of interconnected nodes, in which a node is a basic unit with specific contents or dynamics. Many of these networks exhibit complexity in the overall topological properties and dynamical properties of the network nodes and the coupled units. The complex nature of the networks results in a series of important research problems. In particular, one significant and interesting phenomenon is the synchronization of all its dynamics. Therefore, many researchers have focused on this topic and have developed several efficient synchronization techniques for complex dynamical networks [5–16]. Synchronization of complex dynamical networks can be divided into two points of view. One is the synchronization of a complex network that is called 'inner synchronization' [7–11]. It means that all the nodes in a complex network eventually approach to trajectory of a target node. Another is called 'outer synchronization' [13–15] which considers the synchronization between two or more complex networks regardless of synchronization of inner network. Especially in inner synchronization of a complex network, there is another way to divide study of synchronization of a complex network according to existence of control input. Sometimes in a complex network, synchronization can be achieved without controller under the several conditions. This kind of research is just to analyze existing phenomena. On the contrary, there is a problem of designing controller to compensate the undesirable factors. As categories mentioned above, a control problem for inner synchronization will be investigated in this paper.

Until now, in order to treat the synchronization problem for a complex network, several control schemes such as linear sate feedback [8], state observer based control [12], impulsive control [16], adaptive control [9,14,15] and pinning control [10–13] are applied. However, to the best of the authors' knowledge, the synchronization problem via dynamic feedback

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doi:10.1016/j.amc.2011.11.112

controller for complex networks has not been investigated to date. In some real control situations, there is a strong need to construct a dynamic feedback controller instead of a static feedback controller in order to obtain a better performance and dynamical behavior of the state response. The dynamic controller will provide more flexibility compared to the static controller and the apparent advantage of this type of controller is that it provides more free parameters for selection [24]. So it is very worth to consider the design problem of dynamic controller for synchronization in a complex network.

On the other hand, when controlling a real plant, it is also desirable to design a control system which is not only stable but also guarantees an adequate level of performance. Chang and Peng [17] has first introduced one way for this problem that is called guaranteed cost control approach [17–23]. The approach has the advantage of providing an upper bound on a given linear quadratic cost function. Up to date, unfortunately, there are a few paper about the topic of guaranteed cost control for complex network [10].

In this paper, we consider two types of dynamic feedback control for the inner synchronization of a complex network. The dynamical property of two controllers is basically same, but they have different dimensions of control gains according to topological structures. Moreover, designing feedback controllers which consider both stability and performance of the system is difficult job, but it needs as an another way to solve real world control problems. Unfortunately, the guaranteed cost control problem for synchronization of a complex network has been received very little attention until now. Therefore, applying guaranteed cost control scheme to synchronization of a complex network deserves more attention and consideration. The existence condition of such controller is derived in terms of LMIs which can be easily solved by standard convex optimization algorithms [25].

This paper is organized as follows. A problem statement is described in Section 2. Section 3 provides the design method of a stabilizing controller for synchronization of a complex network. A numerical example is given in Section 4 to show the effectiveness of the derived results. Conclusions are drawn in Section 5.

Notation. $\mathbb{R}^n$ is the n-dimensional Euclidean space, $\mathbb{R}^{m\times n}$ denotes the set of $m \times n$ real matrix. $X > 0$ (respectively, $X \geqslant 0$) means that the matrix X is a real symmetric positive definite matrix (respectively, positive semi-definite). $A(i:n, j:m)$ (respectively, $A(i:n)$) denotes the matrix (respectively, vector) which consisted of i to n rows and j to m columns of the matrix A. I_n denotes the n-dimensional identity matrix. $\otimes$ stands for the notation of Kronecker product.

2. Problem statement and preliminaries

Consider a complex dynamical network consisting of N linearly coupled identical nodes described by

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} c_{ij}x_j(t) + u_i(t) \quad i = 1, \ldots, N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \ldots, x_{in})^T \in \mathbb{R}^n$ is the state vector of the ith node, $f: \mathbb{R}^n \to \mathbb{R}^n$ is a smooth nonlinear vector field, $u_i(t)$ is the control input of ith node, and c_{ij} is the coupling configuration parameter representing the coupling strength and the topological structure of the network, in which c_{ij} is nonzero if there is a connection from node i to node $j(i \neq j)$, and is zero, otherwise.

For simplicity, let us define

$$C = (c_{ij})_{N\times N} \tag{2}$$

and the diagonal elements of the matrix C are assumed that

$$c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij}, \quad i = 1, \ldots, N. \tag{3}$$

Also, the smooth nonlinear function $f(\cdot)$ is satisfied following Lipshitz condition:

$$\|f(a) - f(b)\| \leqslant l\|a - b\|. \tag{4}$$

Definition 1. A complex network is said to achieve asymptotical inner synchronization, if

$$x_1(t) = x_2(t) = \cdots = x_N(t) = s(t) \quad \text{as } t \to \infty,$$

where $s(t) \in \mathbb{R}^n$ is a solution of a target node, satisfying

$$\dot{s}(t) = f(s(t)). \tag{5}$$

For our synchronization scheme, let us define error vectors as follows:

$$e_i(t) = s(t) - x_i(t). \tag{6}$$

From Eq. (6), the error dynamics is given to

$$\dot{e}_i(t) = \bar{f}(e_i(t)) - \sum_{j=i}^{N} c_{ij}e_j(t) - u_i(t), \quad i = 1, \ldots, N \tag{7}$$

where $\bar{f}(e_i(t)) = f(s(t)) - f(x_i(t))$.

Then, Eq. (7) can be rewritten as a vector–matrix form:

$$\dot{e}(t) = -C \otimes I_n e(t) + F(t) - u(t), \tag{8}$$

where $F(t) = \left[\bar{f}^T(e_1(t)), \bar{f}^T(e_2(t)), \ldots, \bar{f}^T(e_N(t))\right]^T$, $e(t) = \left[e_1^T(t), e_2^T(t), \ldots, e_N^T(t)\right]^T$, and $u(t) = \left[u_1^T(t), u_2^T, \ldots, u_N^T(t)\right]^T$.

Here, the aim of this paper is to stabilize the error system (8) via dynamic feedback controller. In this regard, we propose the following two types of dynamic feedback controllers:

- Controller I : $\begin{cases} \dot{\zeta}_1(t) = A_{c1} \otimes I_n \zeta_1(t) + B_{c1} \otimes I_n e(t), \\ U_1(t) = C_{c1} \otimes I_n \zeta_1(t), \quad \zeta_1(0) = 0, \end{cases}$ (9)

- Controller II : $\begin{cases} \dot{\zeta}_2(t) = A_{c2}\zeta_2(t) + B_{c2}e(t), \\ U_2(t) = C_{c2}\zeta_2(t), \quad \zeta_2(0) = 0, \end{cases}$ (10)

where $\zeta_1(t) \in \mathbb{R}^{nN}$ and $\zeta_2(t) \in \mathbb{R}^{nN}$ are the controller state vectors, A_{c1}, B_{c1}, C_{c1} are constant gain matrices of $N \times N$ dimensions and A_{c2}, B_{c2}, C_{c2} are also constant gain matrices of $nN \times nN$ dimensions. These two control laws $(U_1(t), U_2(t))$ given in Eqs. (9) and (10) will be applied as control input $u(t)$ for achieving synchronization of the complex network (1).

In order to treat network performance, we are going to consider guaranteed cost synchronization of a complex network. For this, following quadratic cost function is defined.

$$J = \int_0^\infty (e^T(t)Q \otimes I_n e(t) + u^T(t)R \otimes I_n u(t))dt, \tag{11}$$

where Q and $R \in \mathbb{R}^{N\times N}$ are given positive-definite matrices.

Before proceeding further, a well-known fact is given, below.

Fact 1 (Schur complements). *Given constant symmetric matrices Σ_1, Σ_2, Σ_3 where $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then $\Sigma_1 + \Sigma_3^T\Sigma_2^{-1}\Sigma_3 < 0$ if and only if*

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0, \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

3. Main results

In this section, two types of guaranteed cost dynamic feedback controllers will be designed to achieve our synchronization goal as mentioned before.

First, let us consider the dynamic feedback controller (9). Applying this controller $U_1(t)$ in (9) to $u(t)$ in error system (8) results in the following closed-loop system

$$\dot{z}_1(t) = \overline{H}_1 \otimes I_n z_1(t) + \begin{bmatrix} F(t) \\ 0 \end{bmatrix}, \tag{12}$$

where

$$z_1(t) = \begin{bmatrix} e(t) \\ \zeta_1(t) \end{bmatrix} \in \mathbb{R}^{2nN}, \quad \overline{H}_1 = \begin{bmatrix} -C & -C_{c1} \\ B_{c1} & A_{c1} \end{bmatrix} \in \mathbb{R}^{2N\times 2N}.$$

The corresponding closed-loop cost function is

$$J = \int_0^\infty z_1^T(t)\overline{Q}_1 \otimes I_n z_1(t)dt. \tag{13}$$

where $\overline{Q}_1 = \begin{bmatrix} Q & 0 \\ 0 & C_{c1}^T R C_{c1} \end{bmatrix}$.

Definition 2. Consider error system (8). If there exist a controller $u(t)$ and a positive constant J^* such that the closed-loop system (12) is asymptotically stable and the closed-loop value of the cost function (11) satisfies $J \leqslant J^*$, then J^* is said to be a guaranteed cost and the controller $u(t)$ is said to be guaranteed cost synchronization controller.

Here, the first objective is to find a stabilizing dynamic feedback controller $U_1(t)$ such that the resulting closed-loop system is asymptotically stable and the closed loop value of the cost function (11) satisfies $J \leqslant J^*$. The following is the result:

Theorem 1. *For given Q > 0 and R > 0 and a given Lipshitz constant $L_1 = lI_N$, if there exist positive-definite matrices $S_1, Y_1 \in \mathbb{R}^{N\times N}$ and matrices $X_{11}, X_{12}, X_{13} \in \mathbb{R}^{N\times N}$ satisfying the following LMIs:*

$$\begin{bmatrix} \Omega_{11} & Y_1Q & X_{11}R & \Omega_{12} \\ \star & -Q & 0 & 0 \\ \star & 0 & -R & 0 \\ \star & 0 & 0 & \Omega_{13} \end{bmatrix} < 0, \tag{14}$$

$$\begin{bmatrix} Y_1 & I \\ I & S_1 \end{bmatrix} > 0, \tag{15}$$

where

$$\begin{aligned}\Omega_{11} &= Y_1L_1 + L_1Y_1 - Y_1C^T - CY_1 - X_{11} - X_{11}^T,\\ \Omega_{12} &= L_1 - C + X_{13} + Y_1Q,\\ \Omega_{13} &= L_1S_1 + S_1L_1 - C^TS_1 - S_1C + X_{12} + X_{12}^T + Q,\end{aligned}$$

then, the dynamic control law (9) is the guaranteed cost synchronization controller of the complex network (1) and the upper bound of cost function is

$$J \leqslant e^T(0)S_1 \otimes I_n e(0) \triangleq J^*. \tag{16}$$

Proof. Let us consider the following Lyapunov function:

$$V(t) = z_1^T(t)P_1 \otimes I_n z_1(t), \tag{17}$$

where $P_1 \in \mathbb{R}^{2N\times 2N} > 0$.

By use of Lipshitz condition, the time derivative of Lyapunov function (17) is

$$\begin{aligned}\dot{V}(t) &= z_1^T(t)\left(\overline{H}_1^TP_1 + P_1\overline{H}_1\right) \otimes I_n z_1(t) + 2z_1^T(t)\begin{bmatrix} F(t) \\ 0 \end{bmatrix} \leqslant z_1^T(t)\left(H_1^TP_1 + P_1H_1 + \overline{Q}_1\right) \otimes I_n z_1(t) - z_1^T(t)\overline{Q}_1 \otimes I_n z_1(t)\\ &= z_1^T(t)\Sigma_1 \otimes I_n z_1(t) - z_1^T(t)\overline{Q}_1 \otimes I_n z_1(t),\end{aligned} \tag{18}$$

where $H_1 = \begin{bmatrix} L_1 - C & -C_{c1} \\ B_{c1} & A_{c1} \end{bmatrix}$ and $\Sigma_1 = H_1^TP_1 + P_1H_1 + \overline{Q}_1$.

Therefore, if $\Sigma_1 < 0$, there exist a positive scalar γ_1 such that

$$\dot{V} \leqslant -z_1^T(t)\overline{Q}_1 \otimes I_n z_1(t) \leqslant -\gamma_1\|e(t)\|^2, \tag{19}$$

which guarantees the asymptotic stability of the system by Lyapunov stability theory.

It should be noted that in the matrix Σ_1, the matrix $P_1 > 0$ and the controller parameters A_{c1}, B_{c1} and C_{c1}, which included in the matrix H_1, are unknown and occur in nonlinear fashion. Hence, the inequality $\Sigma_1 < 0$ cannot be considered as an LMI problem. In the following, we will use a method of changing variables such that the inequality can be solved as convex optimization algorithm [26].

First, partition the matrix P_1 and its inverse as

$$P_1 = \begin{bmatrix} S_1 & D_1 \\ D_1^T & T_1 \end{bmatrix}, \quad P_1^{-1} = \begin{bmatrix} Y_1 & M_1 \\ M_1^T & W_1 \end{bmatrix}, \tag{20}$$

where S_1, Y_1 are positive-definite matrices, and $M_1, D_1 \in \mathbb{R}^{N\times N}$ are invertible matrices. It should be pointed out that the equality $P_1^{-1}P_1 = I$ gives that

$$M_1D_1^T = I - Y_1S_1. \tag{21}$$

Define two matrices as

$$E_1 = \begin{bmatrix} Y_1 & I \\ M_1^T & 0 \end{bmatrix}, \quad F_1 = \begin{bmatrix} I & S_1 \\ 0 & D_1^T \end{bmatrix}. \tag{22}$$

Then, it follows that

$$P_1E_1 = F_1, \quad E_1^TP_1E_1 = E_1^TF_1 = \begin{bmatrix} Y_1 & I \\ I & S_1 \end{bmatrix} > 0. \tag{23}$$

Now, postmultiplying and premultiplying the matrix inequality, $\Sigma_1 < 0$, by the matrix E_1^T and by its transpose, respectively, gives

$$F_1^T H_1 E_1 + E_1^T H_1^T F_1 + E_1^T \overline{Q}_1 E_1 < 0. \tag{24}$$

By the relation Eqs. (20)–(23), it can be easily obtained that Eq. (24) is equivalent to

$$\begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \star & \Gamma_{13} \end{bmatrix} < 0, \tag{25}$$

where

$$\begin{aligned} \Gamma_{11} &= Y_1 L_1 + L_1 Y_1 - Y_1 C^T - C Y_1 - M_1 C_{c1}^T - C_{c1} M_1^T + Y_1 Q Y_1 + M_1 C_{c1}^T R C_{c1} M_1^T, \\ \Gamma_{12} &= L_1 - C - Y_1 C^T S_1 + Y_1 B_{c1}^T D_1^T - M_1 C_{c1}^T S_1 + M_1 A_{c1}^T D_1^T + Y_1 Q, \\ \Gamma_{13} &= L_1 S_1 + S_1 L_1 - C^T S_1 - S_1 C + D_1 B_{c1} + B_{c1}^T D_1^T + Q. \end{aligned} \tag{26}$$

By defining a new set of variables as follows:

$$\begin{aligned} X_{11} &= M_1 C_{c1}^T, \\ X_{12} &= D_1 B_{c1}, \\ X_{13} &= Y_1 L_1 S_1 - Y_1 C^T S_1 + Y_1 X_{12}^T - X_{11} S_1 + M_1 A_{c1}^T D_1^T, \end{aligned} \tag{27}$$

then, Eq. (25) is simplified to following inequality:

$$\begin{bmatrix} \Omega_{11} + Y_1 Q Y_1 + M_1 C_{c1}^T R C_{c1} M_1^T & \Omega_{12} \\ \star & \Omega_{13} \end{bmatrix} < 0, \tag{28}$$

where Ω_{11}, Ω_{12} and Ω_{13} are defined in Theorem 1.

Then, by Fact 1, the inequality (28) is equivalent to the LMI (14).

On the other hand, from Eq. (19) we have

$$\dot{V} \leqslant -z_1^T(t) \overline{Q}_1 \otimes I_n z_1(t). \tag{29}$$

Integrating both sides of the above inequality from 0 to T_f leads to

$$\int_0^{T_f} z_1^T(t) \overline{Q}_1 \otimes I_n z_1(t) < V(0) - V(T_f). \tag{30}$$

Since the asymptotic stability of the system has already been established, we conclude that $V(T_f) \to 0$ as $t \to \infty$. Hence we have

$$J \leqslant z_1^T(0) \overline{Q}_1 \otimes I_n z_1(0) = e^T(0) S_1 \otimes I_n e(0) = J^*. \tag{31}$$

This completes the proof. □

Theorem 1 presents a method of designing a dynamic feedback controller for synchronization of a complex network (1). In the following, we will present a method of selecting the optimal guaranteed cost controller minimizing the upper bound of the guaranteed cost (16).

Theorem 2. *Consider the error system* (8) *with cost function* (11). *If the following LMI optimization problem,*

$$\min_{\beta, S_1, Y_1, X_{11}, X_{12}, X_{13}} \beta \tag{32}$$

$$\text{subject to} \quad \text{(i) LMIs (14) and (15)}, \tag{33}$$

$$\text{(ii)} \begin{bmatrix} -\beta & e^T(0) S_1 \otimes I_n \\ \star & -S_1 \otimes I_n \end{bmatrix} < 0 \tag{34}$$

has the solution set ($\beta, S_1, Y_1, X_{11}, X_{12}, X_{13}$), *the controller* (9) *is the optimal dynamic guaranteed cost synchronization controller which ensures the minimization of the guaranteed cost* (16) *of the system. The optimal cost is* $J^* = \beta$.

Proof. By Theorem 1, (33) is clear, and from Fact 1, the LMI (34) is equivalent to $e^T(0) S_1 \otimes I_n e(0) < \beta$. So, it follows from (16). Thus, the minimization of β implies the minimization of the guaranteed cost (16). It is well-known that the convexity of the LMI optimization problem ensures that a global optimum, when it exists, is reachable. This completes the proof. □

Remark 1. Given any solution of the LMIs given in Eqs. (14) and (15) in Theorem 1, a corresponding controller of the form Eq. (9) will be constructed as follows:

- Compute the invertible matrices M_1 and D_1 satisfying Eq. (21) using matrix algebra.
- Utilizing the matrices M_1 and D_1 obtained above, solve the system of Eq. (27) for B_{c1}, C_{c1} and A_{c1} (in this order).

In Theorem 1 and 2, control parameters, A_{c1}, B_{c1}, C_{c1}, have been calculated with $N \times N$ dimensions. After getting the control parameters A_{c1}, B_{c1}, C_{c1}, control input is applied to the system by using Kronecker product. On the other side, there is another way that is to consider control parameters A_{c2}, B_{c2}, C_{c2} with $nN \times nN$ dimensions. In order to handle this method, let's apply control input, $U_2(t)$, given in Eq. (10) to $u(t)$ in error system (8).

Then we obtain following closed-loop system

$$\dot{z}_2(t) = \overline{H}_2 z_2(t) + \begin{bmatrix} F(t) \\ 0 \end{bmatrix}, \tag{35}$$

where

$$z_2(t) = \begin{bmatrix} e(t) \\ \zeta_2(t) \end{bmatrix} \in \mathbb{R}^{2nN}, \quad \overline{H}_2 = \begin{bmatrix} -C \otimes I_n & -C_{c2} \\ B_{c2} & A_{c2} \end{bmatrix} \in \mathbb{R}^{2nN \times 2nN}.$$

The corresponding closed-loop cost function is

$$J = \int_0^{\infty} z_2^T(t) \begin{bmatrix} Q_2 & 0 \\ 0 & C_{c2}^T R_2 C_{c2} \end{bmatrix} z_2(t)dt \equiv \int_0^{\infty} z_2^T(t)\overline{Q}_2 z_2(t)dt, \tag{36}$$

where $Q_2 = Q \otimes I_n$ and $R_2 = R \otimes I_n$.

Then we have following theorem.

Theorem 3. *For given $Q > 0$ and $R > 0$ and a given Lipshitz constant $L_2 = lI_{nN}$, if there exist positive-definite matrices $S_2, Y_2 \in \mathbb{R}^{nN \times nN}$ and matrices $X_{21}, X_{22}, X_{23} \in \mathbb{R}^{nN \times nN}$ satisfying the following LMIs:*

$$\begin{bmatrix} \Omega_{21} & Y_2 Q_2 & X_{21} R_2 & \Omega_{22} \\ \star & -Q_2 & 0 & 0 \\ \star & 0 & -R_2 & 0 \\ \star & 0 & 0 & \Omega_{23} \end{bmatrix} < 0, \tag{37}$$

$$\begin{bmatrix} Y_2 & I \\ I & S_2 \end{bmatrix} > 0, \tag{38}$$

where

$$\Omega_{21} = Y_2 L_2 + L_2 Y_2 - Y_2 C^T - C Y_2 - X_{21} - X_{21}^T,$$
$$\Omega_{22} = L_2 - C + X_{23} + Y_2 Q_2,$$
$$\Omega_{23} = L_2 S_2 + S_2 L_2 - C^T S_2 - S_2 C + X_{22} + X_{22}^T + Q_2,$$

then, the dynamic control law (10) is the guaranteed cost synchronization controller of the complex network Eq. *(1) and the upper bound of cost function is*

$$J \leqslant e^T(0) S_2 e(0) \triangleq J^*. \tag{39}$$

Proof. Let us consider the following Lyapunov function:

$$V(t) = z_2^T(t) P_2 z_2(t), \tag{40}$$

where $P_2 \in \mathbb{R}^{2nN \times 2nN} > 0$.

Then the rest of proof for Theorem 3 is straightforward from the proof of Theorem 1, so, it is omitted. □

The optimal control law minimizing upper bound of the guaranteed cost (39) can get from the following theorem.

Theorem 4. *Consider the error system (8) with cost function (11). If the following LMI optimization problem,*

$$\min_{\beta, S_2, Y_2, X_{21}, X_{22}, X_{23}} \beta \tag{41}$$

subject to (i) *LMIs* (37) *and* (38) (42)

$$\text{(ii)} \begin{bmatrix} -\beta & e^T(0) S_2 \\ \star & -S_2 \end{bmatrix} < 0 \tag{43}$$

has the solution set (β, S_2, Y_2, X_{21}, X_{22}, X_{23}), the controller (10) is the optimal dynamic feedback controller which ensures the minimization of the guaranteed cost (39) of the system. The optimal cost is $J^* = \beta$.

Proof. The proof of Theorem 4 is also same to proof of Theorem 2, so, it is omitted. □

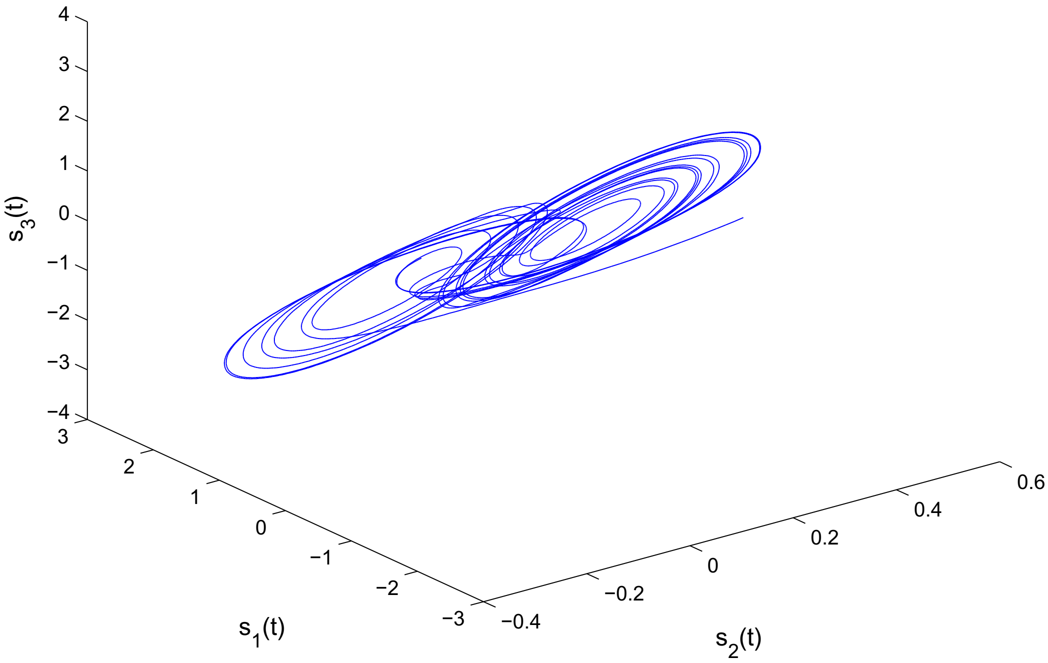

Fig. 1. The chaotic behavior of Chua's circuit.

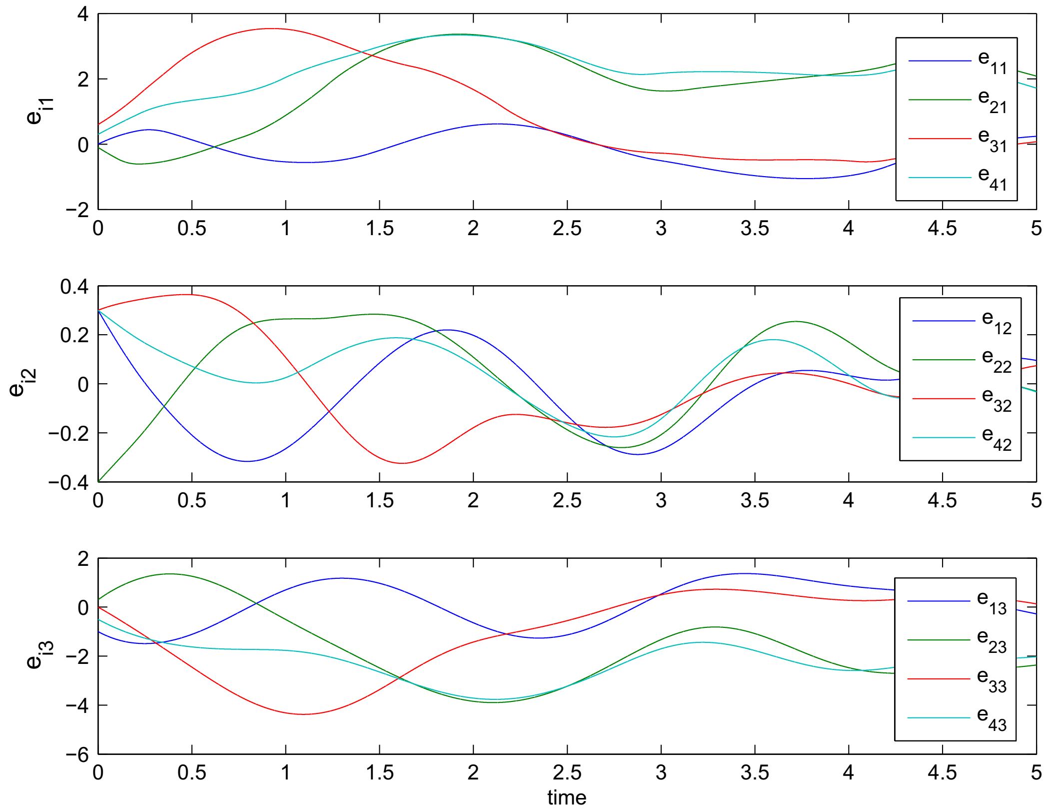

Fig. 2. Error signals in case of no controller.

Remark 2. The difference between the controller (9) and (10) is the dimensions of control gains. In case of the controller (9), control parameters, A_{c1}, B_{c1}, C_{c1}, are calculated from $N \times N$ dimensional LMI variables. It means that controller (9) has just N control gain at one node regardless of dimensions of a node. On the other hand, the control parameters of the controller (10), A_{c2}, B_{c2}, C_{c2}, are obtained from $nN \times nN$ dimensional LMI variables. So, controller (10) has $n \times nN$ control gain associated with each nodes. For instance, 1st node of the system is affected from control state $\zeta_1(1:n)$ or $\zeta_2(1:n)$ by multiplying $C_{c1}(1,1)$ or $C_{c2}(1:n,1:n)$, respectively. So, in order to find feasible LMI solution set and control parameters in the same system, the case of the controller (10) take more time but is more flexible method than the case of the controller (9).

4. Numerical example

In order to show the effectiveness of the proposed method, we present a numerical example which is inner synchronization of a complex network with four identical nodes. Each nodes are Chua's chaotic circuit [27] and its chaotic behavior is displayed in Fig. 1. Thus, the complex network system consisting of four nodes is described by:

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} c_{ij}x_j(t) + u_i(t), \quad i = 1, \ldots, 4, \tag{44}$$

where

$$f(x_i(t)) = \begin{bmatrix} a(x_{i2} - h(x_{i1})) \\ x_{i1} - x_{i2} + x_{i3} \\ -bx_{i2} \end{bmatrix},$$

$$h(x_{i1}) = m_1 x_{i1} + \frac{1}{2}(m_0 - m_1)(|x_{i1} + c| - |x_{i1} - c|),$$

with the parameters $a = 9$, $b = 14.28$, $c = 1$, $m_0 = -1/7$, $m_1 = 2/7$.

Here, the Chua's circuit is also chosen as a target node. In this example, initial conditions of each nodes are chosen: $x_1(0) = [-0.9\ -1.5\ -3.7]$, $x_2(0) = [-0.1\ -0.4\ 0.3]$, $x_3(0) = [0.6\ -1.5\ 0]$, $x_4(0) = [2.1\ -1.5\ 1.3]$, $s(0) = [0.1\ 0.5\ -\ 0.7]$. It is noted that the Lipshitz constant of Chua's circuit is $l = 5$ and coupling matrix, C, is given by

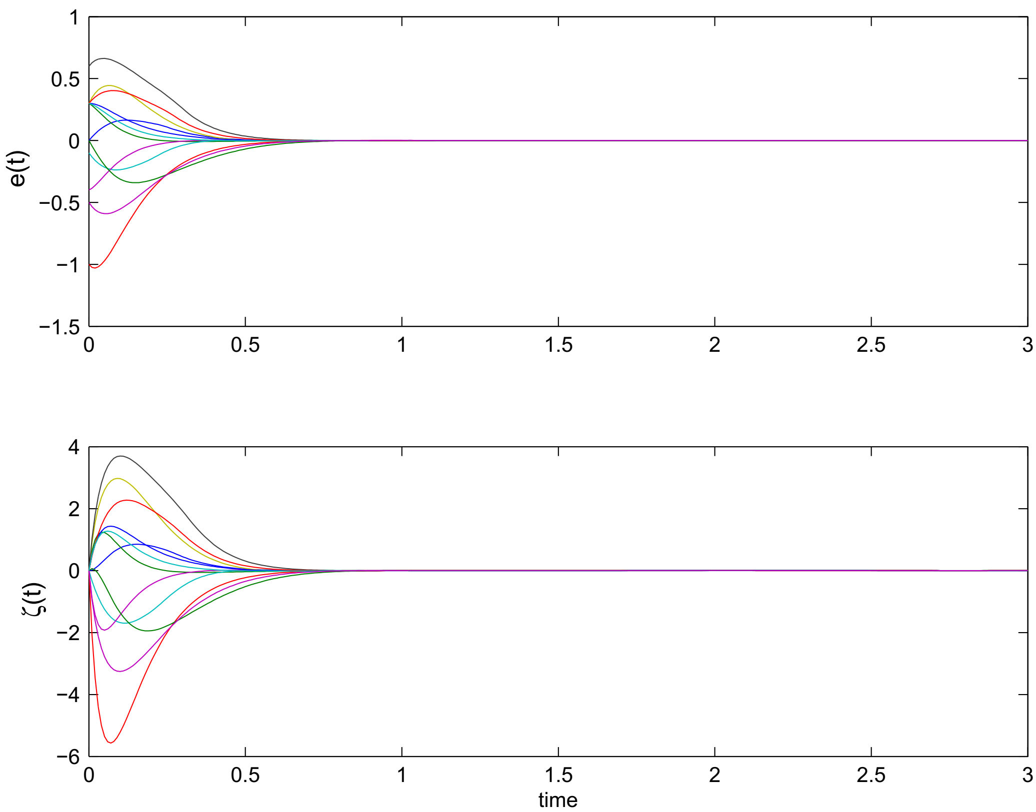

Fig. 3. Error signals by Theorem 1.

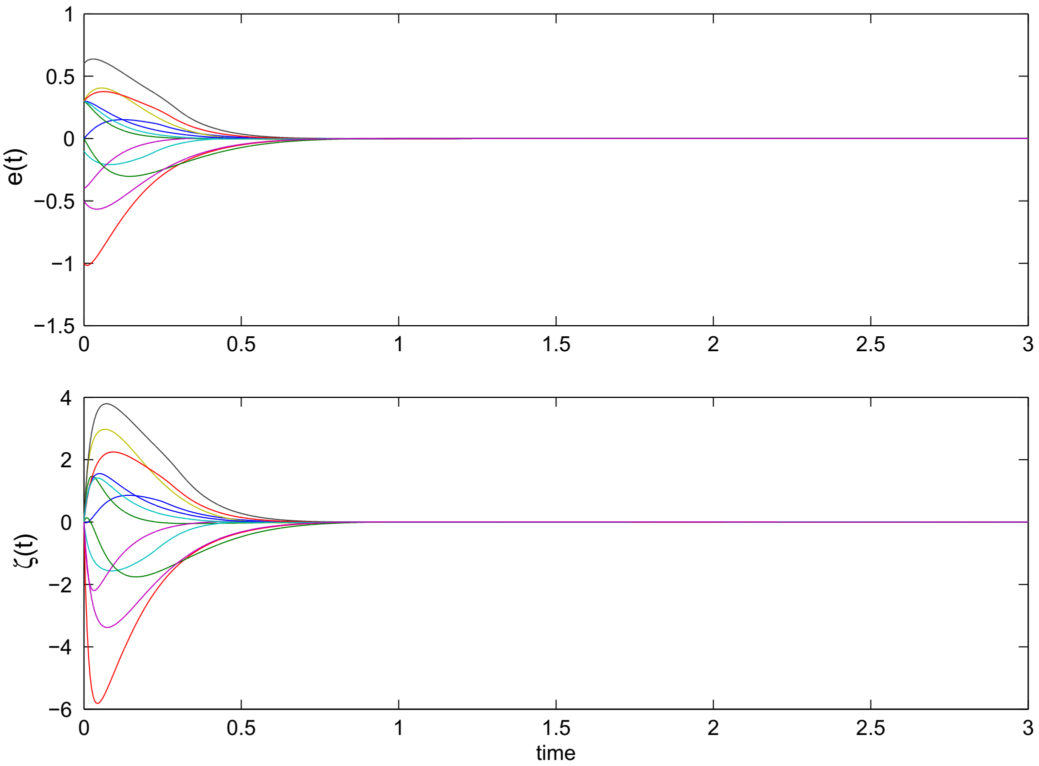

Fig. 4. Error signals by Theorem 3.

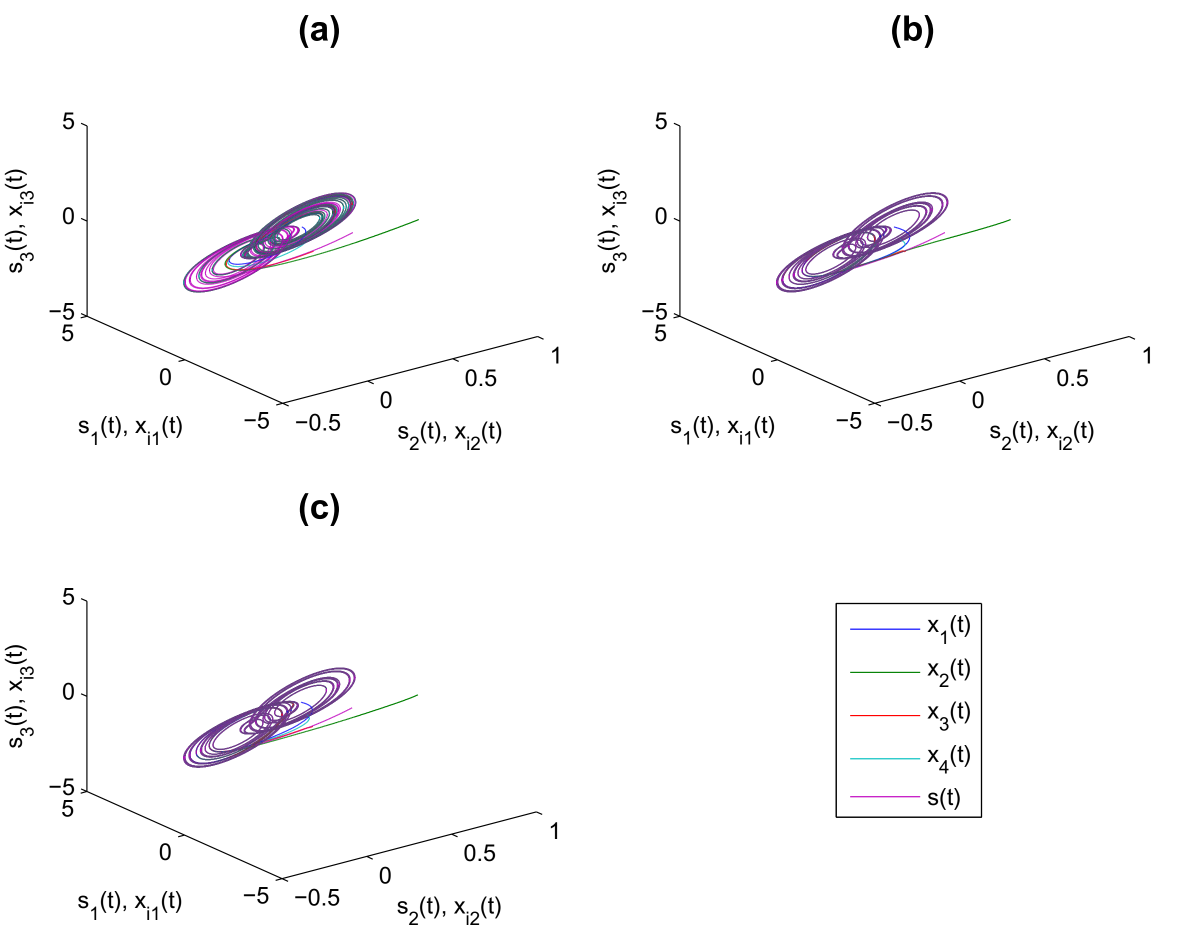

Fig. 5. The dynamic behaviors of system (44) by application of (a) no controller (b) Controller I (c) Controller II.

Table 1
The cost value of each method.

	β
Theorem 1	429.6246
Theorem 2	15.6089
Theorem 3	205.6637
Theorem 4	15.6089

$$C = 0.2 \times \begin{bmatrix} -3 & 1 & 1 & 1 \\ 1 & -3 & 1 & 1 \\ 0 & 1 & -2 & 1 \\ 0 & 1 & 1 & -2 \end{bmatrix}. \tag{45}$$

The parameters associated with cost function are chosen as $Q = I_N$ and $R = I_N$. In order to show original behavior of the complex network (44), the trajectories of the error signals without controller is depicted in Fig. 2. Now, by solving LMI problem (14) and (15) in Theorem 1 and (37) and (38) in Theorem 3, we can calculate feasible solution set from Eq. (27). Then, we found gain matrices of the controller (9) and (10) (see Appendix A).

The simulation results with control input which are calculated by Theorem 1 and Theorem 3 are presented in Figs. 3 and 4. As seen in the figures, the trajectories of error systems are indeed well stabilized and also the state orbits of controller approach to zero. It can be concluded that our proposed dynamic controller guarantees asymptotic synchronization of the complex network (44) under some value of performance index. Additionally, Fig. 5 shows dynamic behavior of the system (44) in case of without control input, Controller I (9) and Controller II (10), respectively.

Finally, by solving the optimization problems (32) and (41) in Theorem 2 and Theorem 4 respectively, the optimal guaranteed cost of closed-loop system are obtained and listed in Table 1. As seen in the table, it should be pointed out that the optimal costs by two control methods given in (9) and (10) are same.

5. Conclusions

In this paper, the design problem of guaranteed cost dynamic feedback controller for inner synchronization of a complex dynamical network has been studied by Lyapunov method and LMI framework. Two types of dynamic controller have been proposed and existence conditions of the controllers have been derived in terms of LMIs. Finally, one numerical example was illustrated to show the effectiveness of the designed controllers.

Acknowledgements

The work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (2010-0009373) and 2010 Yeungnam University Research Grant.

Appendix A

- The parameters of controller by Theorem 1:

$$A_{c1} = \begin{bmatrix} -32.0682 & -1.4215 & -0.1990 & 1.2881 \\ -1.2559 & -36.8397 & 0.2318 & 0.0967 \\ -0.0177 & 0.2060 & -41.5051 & 1.5273 \\ 1.0586 & 0.0781 & 1.1378 & -32.9178 \end{bmatrix},$$

$$B_{c1} = \begin{bmatrix} -241.0386 & -306.2214 & -223.2174 & -273.7313 \\ -43.0955 & -13.2824 & 458.5424 & -418.0740 \\ 541.7097 & -447.7295 & 24.3317 & -3.1062 \\ 213.8543 & 317.7898 & -215.8701 & -264.0778 \end{bmatrix},$$

$$C_{c1} = \begin{bmatrix} -0.1479 & -0.0175 & 0.2963 & 0.1800 \\ -0.1873 & -0.0014 & -0.2315 & 0.2337 \\ -0.1645 & 0.2769 & 0.0085 & -0.1843 \\ -0.1770 & -0.2406 & -0.0105 & -0.2267 \end{bmatrix}.$$

- The parameters of controller by Theorem 3:

$$A_{c2} = \left[\begin{matrix} -26.9264 & 0.0000 & 0.0000 & 0.2995 & -0.5947 & -0.0000 \\ -0.0000 & -26.9264 & -0.0000 & 0.0000 & 0.0000 & -0.6658 \\ 0.0000 & -0.0000 & -26.9264 & 0.5947 & 0.2995 & 0.0000 \\ 0.2804 & 0.0000 & 0.5568 & -30.0254 & 0.0000 & -0.0000 \\ -0.5568 & 0.0000 & 0.2804 & 0.0000 & -30.0254 & -0.0000 \\ -0.0000 & -0.6234 & -0.0000 & -0.0000 & -0.0000 & -30.0254 \\ -0.0040 & -0.0000 & -0.0596 & 0.1059 & 0.0447 & -0.0000 \\ -0.0000 & 0.0597 & -0.0000 & -0.0000 & -0.0000 & 0.1149 \\ 0.0596 & -0.0000 & -0.0040 & -0.0447 & 0.1059 & 0.0000 \\ 0.0000 & 0.6935 & 0.0000 & -0.0000 & 0.0000 & -0.0155 \\ 0.4500 & -0.0000 & 0.5276 & 0.0151 & -0.0037 & -0.0000 \\ -0.5276 & 0.0000 & 0.4500 & 0.0037 & 0.0151 & 0.0000 \end{matrix}\right.$$

$$\left.\begin{matrix} 0.0075 & 0.0000 & -0.1108 & -0.0000 & 0.4304 & -0.5045 \\ -0.0000 & -0.1110 & -0.0000 & 0.6631 & -0.0000 & 0.0000 \\ 0.1108 & 0.0000 & 0.0075 & -0.0000 & 0.5045 & 0.4304 \\ 0.1273 & -0.0000 & -0.0537 & 0.0000 & 0.0176 & 0.0043 \\ 0.0537 & 0.0000 & 0.1273 & -0.0000 & -0.0043 & 0.0176 \\ 0.0000 & 0.1382 & 0.0000 & -0.0181 & -0.0000 & 0.0000 \\ -32.0453 & 0.0000 & -0.0000 & 0.0000 & -0.7449 & -0.5533 \\ 0.0000 & -32.0453 & 0.0000 & 0.9279 & -0.0000 & -0.0000 \\ -0.0000 & -0.0000 & -32.0453 & 0.0000 & 0.5533 & -0.7449 \\ 0.0000 & 0.7063 & -0.0000 & -28.0766 & -0.0000 & 0.0000 \\ -0.5670 & -0.0000 & 0.4211 & 0.0000 & -28.0766 & -0.0000 \\ -0.4211 & 0.0000 & -0.5670 & -0.0000 & 0.0000 & -28.0766 \end{matrix}\right],$$

$$B_{c2} = \left[\begin{matrix} 0.0000 & -39.5489 & -239.7719 & -0.0000 & -50.6054 & -306.8040 \\ 243.0117 & -0.0000 & 0.0000 & 310.9495 & -0.0000 & -0.0000 \\ 0.0000 & -239.7719 & 39.5489 & 0.0000 & -306.8040 & 50.6054 \\ -0.0000 & 45.3105 & 14.1685 & -0.0000 & -4.5741 & -1.4303 \\ -0.0000 & 14.1685 & -45.3105 & -0.0000 & -1.4303 & 4.5741 \\ 47.4740 & 0.0000 & -0.0000 & -4.7925 & 0.0000 & 0.0000 \\ 0.0000 & -545.2575 & 52.5509 & 0.0000 & 424.9733 & -40.9581 \\ -547.7840 & 0.0000 & -0.0000 & 426.9425 & 0.0000 & 0.0000 \\ 0.0000 & 52.5509 & 545.2575 & 0.0000 & -40.9581 & -424.9733 \\ -229.5594 & 0.0000 & 0.0000 & -365.4544 & -0.0000 & -0.0000 \\ -0.0000 & 196.5670 & 118.5705 & 0.0000 & 312.9310 & 188.7621 \\ -0.0000 & 118.5705 & -196.5670 & 0.0000 & 188.7621 & -312.9310 \end{matrix}\right.$$

$$\left.\begin{matrix} -0.0000 & -38.7802 & -235.1113 & 0.0000 & -48.5096 & -294.0980 \\ 238.2881 & -0.0000 & 0.0000 & 298.0718 & -0.0000 & 0.0000 \\ -0.0000 & -235.1113 & 38.7802 & 0.0000 & -294.0980 & 48.5096 \\ 0.0000 & -461.2316 & -144.2260 & -0.0000 & 399.7919 & 125.0140 \\ 0.0000 & -144.2260 & 461.2316 & -0.0000 & 125.0140 & -399.7919 \\ -483.2554 & 0.0000 & -0.0000 & 418.8819 & 0.0000 & -0.0000 \\ -0.0000 & -28.0781 & 2.7061 & 0.0000 & 23.9353 & -2.3068 \\ -28.2082 & 0.0000 & -0.0000 & 24.0463 & 0.0000 & -0.0000 \\ -0.0000 & 2.7061 & 28.0781 & 0.0000 & -2.3068 & -23.9353 \\ 236.2700 & -0.0000 & 0.0000 & 296.0551 & -0.0000 & 0.0000 \\ -0.0000 & -202.3131 & -122.0366 & 0.0000 & -253.5058 & -152.9164 \\ 0.0000 & -122.0366 & 202.3131 & -0.0000 & -152.9164 & 253.5058 \end{matrix}\right],$$

$$
C_{c2} = \left[\begin{array}{cccccccccccc}
-0.0000 & 0.1139 & 0.0000 & 0.0000 & -0.0000 & 0.0170 & 0.0000 & -0.2325 & 0.0000 & -0.1273 & 0.0000 & 0.0000 \\
-0.0185 & -0.0000 & -0.1124 & 0.0163 & 0.0051 & 0.0000 & -0.2314 & 0.0000 & 0.0223 & -0.0000 & 0.1090 & 0.0657 \\
-0.1124 & 0.0000 & 0.0185 & 0.0051 & -0.0163 & -0.0000 & 0.0223 & -0.0000 & 0.2314 & -0.0000 & 0.0657 & -0.1090 \\
-0.0000 & 0.1461 & 0.0000 & 0.0000 & -0.0000 & -0.0050 & 0.0000 & 0.1722 & 0.0000 & -0.1809 & 0.0000 & 0.0000 \\
-0.0238 & -0.0000 & -0.1441 & -0.0048 & -0.0015 & 0.0000 & 0.1714 & 0.0000 & -0.0165 & -0.0000 & 0.1549 & 0.0934 \\
-0.1441 & -0.0000 & 0.0238 & -0.0015 & 0.0048 & -0.0000 & -0.0165 & -0.0000 & -0.1714 & -0.0000 & 0.0934 & -0.1549 \\
-0.0000 & 0.1280 & -0.0000 & 0.0000 & 0.0000 & -0.2192 & -0.0000 & -0.0084 & -0.0000 & 0.1313 & -0.0000 & 0.0000 \\
-0.0208 & -0.0000 & -0.1263 & -0.2092 & -0.0654 & 0.0000 & -0.0084 & 0.0000 & 0.0008 & -0.0000 & -0.1124 & -0.0678 \\
-0.1263 & 0.0000 & 0.0208 & -0.0654 & 0.2092 & -0.0000 & 0.0008 & -0.0000 & 0.0084 & 0.0000 & -0.0678 & 0.1124 \\
0.0000 & 0.1467 & 0.0000 & -0.0000 & -0.0000 & 0.1827 & 0.0000 & 0.0164 & 0.0000 & 0.1644 & 0.0000 & -0.0000 \\
-0.0239 & -0.0000 & -0.1448 & 0.1744 & 0.0545 & 0.0000 & 0.0163 & 0.0000 & -0.0016 & -0.0000 & -0.1408 & -0.0849 \\
-0.1448 & 0.0000 & 0.0239 & 0.0545 & -0.1744 & -0.0000 & -0.0016 & -0.0000 & -0.0163 & 0.0000 & -0.0849 & 0.1408
\end{array}\right].
$$

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