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Adaptive synchronization for uncertain chaotic neural networks with mixed time delays using fuzzy disturbance observer

S.C. Jeong [a], D.H. Ji [b], Ju H. Park [c,*], S.C. Won [a,*]

[a] Department of Electronic and Electrical Engineering, Pohang University of Science and Technology, San 31, Hyoja-Dong, Pohang 790-784, Republic of Korea
[b] Mobile communication Division, Digital Media and Communications, Samsung Electronics, Co. Ltd., Maetan-dong, Suwon 416-2, Republic of Korea
[c] Nonlinear Dynamics Group, Department of Electrical Engineering, Yeungnam University 214-1 Dae-dong, Kyongsan 712-749, Republic of Korea

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ABSTRACT

This paper proposes a robust adaptive control method for synchronization of uncertain chaotic neural networks with mixed delays. Uncertainty and disturbance in the networks are estimated by fuzzy disturbance observer without any prior information about them. The proposed control scheme with adaptive laws is derived based on Lyapunov–Krasovskii stability theory to guarantee the globally asymptotical synchronization between the networks. An example is illustrated to show the effectiveness of the proposed method.

1. Introduction

In the past few decades, there has been considerable attention in the study of neural networks due to their potential applications in various areas, such as signal processing pattern recognition, static image processing, associative memory and combinatorial optimization [1–4]. It has been shown that artificial neural network models can exhibit some chaotic behaviors [5–8]. Since the pioneering works of Pecora and Carroll [9], Synchronization of chaotic neural networks has been intensively investigated in many fields [10–13]. In the implementation of the neural networks, time delays between neurons in the networks often arise in the processing of information storage and transmission, which may lead to instability, oscillation, and bifurcation of the neural network model [8,14]. Many studies have been developed for the synchronization problem of delayed chaotic neural networks. Some have considered the networks with time-varying delays [15–19]. However, there exist various chaotic neural networks with both time-varying delays and distributed delays in realistic network models. Therefore, it is worth taking into account the chaotic neural networks with the mixed time delays including time-varying and distributed delays [20–29]. A control method with two sufficient conditions to ensure the globally exponential stability for the error system has been proposed based on the drive-response concept [20,21]. In [22], a synchronization problem of the networks with mixed delays has been discussed by using an adaptive feedback control technique. Sufficient conditions for asymptotical or exponential synchronization are derived in terms of Linear matrix inequalities (LMIs) by constructing proper Lyapunov–Krasovskii functional [23–25]. Sliding mode control technique is proposed to synchronize nonidentical chaotic neural networks with mixed delays [26,27]. The synchronization problems of stochastic perturbed chaotic neural networks with mixed delays have been investigated in [28,29].

It is known that the uncertainty and disturbance are unavoidable factors in many practical situations and they can destroy the network stability or can make the synchronization more difficult. Some works for uncertain neural networks have been developed to overcome their effects [15–32]. They often require some prior information of the uncertain factors, such as its structure or upper bound. However, the information may not be available due to physical limitations in practical cases.

* Corresponding authors.
E-mail addresses: somunza@postech.ac.kr (S.C. Jeong), captainzone@gmail.com (D.H. Ji), jessie@ynu.ac.kr (J.H. Park), won@postech.ac.kr (S.C. Won).

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Fuzzy logic system can be a good solution to be used in the situations because it can provide an estimator for a unknown function or value. Fuzzy disturbance observer (FDO) has been proposed to estimate uncertainty and disturbance without requiring any prior information about them [33]. The estimated values have been used to compensate the uncertain factors via state feedback controller. In [34], a robust tracking control approach using a discrete-time FDO has been proposed for nonlinear sampled systems. Recently, a more precise FDO has been constructed by modifying the law used to update the parameter vector and the modified FDO showed better performances, compared with the conventional one [35]. Even though the FDO presented good performances to overcome the unknown factor, applications of the existing research are still limited. Especially, there has been still no research using the technique for uncertain chaotic neural networks with mixed time delays.

In this paper, we propose a robust adaptive synchronization method for uncertain chaotic neural networks with time-varying delays and distributed delays. The uncertain factors including uncertainties and disturbances are estimated by the FDO without requiring any prior knowledge about the factors. The estimated values are used to compensate the factors in the proposed method. Based on Lyapunov–Krasovskii stability theory, the control scheme with adaptive laws is derived and guarantees the globally asymptotical synchronization between the networks. An example is illustrated to show the effectiveness of the proposed method.

2. Problem statement

Consider the following chaotic neural network with time-varying delay and distributed delay:

$$\dot{x}(t) = -Cx(t) + Af(x(t)) + Bg(x(t-\tau(t))) + D\int_{t-\sigma(t)}^{t} h(x(s))ds + I, \tag{1}$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector and $C = diag(c_1, c_2, \ldots, c_n)$ is a positive diagonal matrix. $A = (a_{ij})_{n\times n}, B = (b_{ij})_{n\times n}$, and $D = (d_{ij})_{n\times n}$ are the connection weight matrix, the time varying delayed connection weight matrix and distributively delayed connection weight matrix, respectively. $I = [I_1, I_2, \ldots, I_n]^T \in \mathbb{R}^n$ is an external input vector, $\tau(t)$ is the time-varying delay, the positive constant $0 \leqslant \sigma(t) \leqslant \bar{\sigma}$ is the distributed time-delay. The initial conditions are given by $x_i(t) = \psi_{xi}(t) \in C([-r, 0], \mathbb{R})$, where $C([-r, 0], \mathbb{R})$ denotes the set of all continuous functions from $[r, 0]$ to $\mathbb{R}$ and $r = \max\{\tau(t), \sigma(t)\}$. $f(x(t)), g(x(t-\tau(t)))$, and $h(x(t))$ are the activation functions of the neurons and described as

$$\begin{aligned}
f(x(t)) &= [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^T, \\
g(x(t-\tau(t))) &= [g_1(x_1(t-\tau(t))), g_2(x_2(t-\tau(t))), \ldots, g_n(x_n(t-\tau(t)))]^T, \\
h(x(t)) &= [h_1(x_1(t)), h_2(x_2(t)), \ldots, h_n(x_n(t))]^T.
\end{aligned}$$

We consider the network (1) as the drive system. The response system having the uncertainty and disturbance is established as follows:

$$\begin{aligned}
\dot{y}(t) &= -(C+\Delta C)y(t) + (A+\Delta A)f(y(t)) + (B+\Delta B)g(y(t-\tau(t))) + (D+\Delta D)\int_{t-\sigma(t)}^{t} h(y(s))ds + I + d(t) + u(t) \\
&= -Cy(t) + Af(y(t)) + Bg(y(t-\tau(t))) + D\int_{t-\sigma(t)}^{t} h(y(s)) + \Omega(t) + I + u(t),
\end{aligned} \tag{2}$$

where $y(t) = [y_1(t), y_2(t), \ldots, y_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector of the response system. C, A, B, and D are matrices which are the same as in (1). $f(y(t)), g(y(t-\tau(t)))$, and $h(y(t))$ are the activation functions of the response system neurons which are defined in the same manner with the drive system (1). The initial conditions are given by $y_i(t) = \psi_{yi}(t) \in C([-r, 0], \mathbb{R})$. $\Delta Cy(t), \Delta Af(y(t))$, and $\Delta B\int_{t-\sigma(t)}^{t} h(y(s))ds$ are the uncertainties and $d(t)$ is the disturbance. The overall disturbance is defined as $\Omega(t) = [\omega_1(t), \omega_2(t), \ldots, \omega_n(t)] = \Delta Cy(t) + \Delta Af(y(t)) + \Delta B\int_{t-\sigma(t)}^{t} h(y(s))ds + d(t)$.

Define the synchronization error as $e(t) = y(t) - x(t) \in \mathbb{R}^n$. Subtracting the drive system (1) from the response system (2) yields the dynamical system

$$\dot{e}(t) = -Ce(t) + Af(e(t)) + Bg(e(t-\tau(t))) + D\int_{t-\sigma(t)}^{t} h(e(s))ds + \Omega(t) + u(t), \tag{3}$$

where $f(e(t)) = f(y(t)) - f(x(t)), g(e(t)) = g(y(t-\tau(t))) - g(x(t-\tau(t))), h(e(t)) = h(y(t)) - h(x(t))$. Then, our goal is to design the controller $u(t)$ which makes the error dynamical system (3) stabilized, that is,

$$\lim_{t\to\infty}\|e(t)\| = \lim_{t\to\infty}\|y(t) - x(t)\| = 0. \tag{4}$$

This means that the response system (2) is synchronized with the drive system (1).

Throughout this paper, the activation functions $f(\cdot), g(\cdot)$, and $h(\cdot)$ and the delay $\tau(t)$ satisfy the following assumptions.

Assumption 1. The activation functions $f(\cdot), g(\cdot)$, and $h(\cdot)$ satisfy the Lipschitz condition with positive constants $\lambda_{fi}, \lambda_{gi}$, and an $n \times n$ constant matrix L, respectively, i.e., for $i = 1, 2, \ldots, n$

$$|f_i(\alpha) - f_i(\beta)| \leqslant \lambda_{fi}|\alpha - \beta|, \quad |g_i(\alpha) - g_i(\beta)| \leqslant \lambda_{gi}|\alpha - \beta|, \quad |h_i(\bar{\alpha}) - h_i(\bar{\beta})| \leqslant L|\bar{\alpha} - \bar{\beta}|, \tag{5}$$

where $\alpha, \beta \in \mathbb{R}$ and $\bar{\alpha}, \bar{\beta} \in \mathbb{R}^n$.

Assumption 2. The time delay $\tau(t)$ is a bounded and continuously differentiable function such that $0 \leqslant \tau(t) \leqslant \bar{\tau}$ and $0 \leqslant \dot{\tau}(t) \leqslant \mu < 1$.

The following lemmas are essential in establishing our result.

Lemma 1 [36]. *Given any vector x, y of appropriate dimensions and a positive definite matrix $P > 0$ with compatible dimensions, then the following inequality holds,*

$$2x^T y \leqslant x^T P x + y^T P^{-1} y. \tag{6}$$

Lemma 2 [37]. *For any positive definite matrix $W^T = W \in \mathbb{R}^{m\times m}$, scalar $h > 0$, and vector function $\omega : [0, h] \to \mathbb{R}^m$, such that the integrations concerned are well defined, the following inequality holds*

$$\left(\int_0^h \omega(s)ds\right)^T W \left(\int_0^h \omega(s)ds\right) \leqslant h\int_0^h \omega^T(s)W\omega(s)ds. \tag{7}$$

3. Adaptive synchronization using fuzzy disturbance observer

In this section, we propose an adaptive synchronization method for the uncertain chaotic neural networks (1) and (2). The first step for the synchronization is how well we can overcome the overall disturbance $\Omega(t)$. We will use the fuzzy logic system (FLS) to accomplish that [38]. First, let us briefly describe the basic configuration of the FLS used in this paper. The FLS performs a mapping from a compact set $X = X_1 \times \ldots \times X_n \subset \mathbb{R}^n$ to a compact set $V \subset \mathbb{R}$. The fuzzy rule base consists of a collection of M fuzzy If–Then rules:

$$\begin{aligned} R^{(l)} : &\text{If } x_1 \text{ is } A_1^l, \text{ and} \ldots \text{ and}, x_n \text{ is } A_n^l, \\ &\text{Then } y \text{ is } G^l, \end{aligned} \tag{8}$$

where $x = [x_1, \ldots, x_n]^T \in X$ is the input of the FLS and $y \in V$ is its output, where A_i^l and G^l are labels of fuzzy sets in X_i and R for $l = 1, 2, \ldots, M$. By using a product inference engine, a center-average defuzzifier, and a singleton fuzzifier, the output of the fuzzy system can be expressed as

$$y(x) = \frac{\sum_{l=1}^M y_l\left(\prod_{i=1}^n \mu_{A_i^l}(x_i)\right)}{\sum_{l=1}^M\left(\prod_{i=1}^n \mu_{A_i^l}(x_i)\right)} = \theta^T\xi(x), \tag{9}$$

where $\mu_{A_i^l}(x_i)$ is membership function value of the fuzzy variable x_i, M is the number of fuzzy rules, $\theta = [y_1, y_2, \ldots, y_M]^T$ is an adjustable parameter vector, and $\xi(x) = [\xi_1(x), \xi_2(x), \ldots, \xi_M(x)]^T$ is a regressive vector defined as

$$\xi_l(x) = \frac{\prod_{i=1}^n \mu_{A_i^l}(x_i)}{\sum\limits_{l=1}^M\left(\prod_{i=1}^n \mu_{A_i^l}(x_i)\right)}, \tag{10}$$

which are called fuzzy basis functions (FBFs).

It is well known that the fuzzy system (9) can estimate unknown function with an arbitrarily small error based on 'universal approximation theorem' [38]. This characteristic provides that the overall disturbance $\Omega(t)$ including uncertainties and disturbances of the neural network (2) can be estimated by the FLS.

Remark 1. In the existing studies [15,27,30–32], they need some information about the uncertainties or disturbances, such as the upper bound and the structure of them. However, the information may be not available in many practical cases. The approach using the FLS can be a good way to estimate the disturbance in such cases.

Remark 2. Takagi–Sugeno (T-S) fuzzy modeling is one of main methods using FLS. Many studies have proposed novel stability criteria for time-delayed uncertain neural networks modeled by T–S fuzzy system [39,40]. In this paper, we show that the FLS can be used as an estimator of unknown factors in the system.

Now, we present the FDO design procedure to obtain the estimated value of the overall disturbance $\Omega(t)$. Consider the following observer system

$$\dot{\hat{y}}(t) = -Cy(t) + Af(y(t)) + Bg(y(t-\tau(t))) + D\int_{t-\sigma(t)}^{t} h(y(s))ds + \hat{\Omega}(t) + u(t) + P(y(t) - \hat{y}(t)), \tag{11}$$

where $P = diag(p_1, p_2, \ldots, p_n) > 0, \hat{y}(t) = [\hat{y}_1(t), \hat{y}_2(t), \ldots, \hat{y}_n(t)]^T$ is the state of the observer system, $\hat{\Omega}(t) = [\hat{\omega}_1(t), \hat{\omega}_2(t), \ldots, \hat{\omega}_n(t)]^T$ is the estimation for $\Omega(t)$ with $\hat{\omega}_i(t) = \theta_i^T(t)\xi_i(y(t), y(t-\tau(t)))$ for $i = 1, 2, \ldots, n$. $\theta_i \in \mathbb{R}^M$ is the fuzzy parameter vector. $\xi_i(y(t), y(t-\tau(t))) \in \mathbb{R}^M$ is the fuzzy basis function vector. Then, by the universal approximation theorem [38], an FLS $\hat{\omega}_i(t)$ exists such that

$$|\omega_i(t) - \hat{\omega}_i(t)| < \bar{\varepsilon}_i, \tag{12}$$

where $\bar{\varepsilon}_i \in \mathbb{R}$ is the upper bound of fuzzy approximation error. Hence, we can obtain the estimator $\hat{\Omega}(t)$ for $\Omega(t)$ with arbitrarily small error bounds $|\varepsilon_i| = |\omega_i(t) - \hat{\omega}_i(t)| \leqslant \bar{\varepsilon}_i \in \mathbb{R}$.

We define the *observation error* as $\varphi(t) = y(t) - \hat{y}(t)$. Then, from (2) and (11), we have the error dynamics

$$\dot{\varphi}(t) = \dot{y}(t) - \dot{\hat{y}}(t) = \Omega(t) - \hat{\Omega}(t) - P(y(t) - \hat{y}(t)) = \varepsilon(t) - P\varphi(t), \tag{13}$$

where $\varepsilon(t) = [\varepsilon_1(t), \varepsilon_2(t), \ldots, \varepsilon_n(t)]^T = \Omega(t) - \hat{\Omega}(t) = \dot{\varphi}(t) + P\varphi(t)$.

The *disturbance reconstruction error* $\varepsilon(t)$ can be rewritten as

$$\varepsilon(t) = \Omega(t) - \hat{\Omega}(t) = \Omega(t) - \hat{\Omega}^*(t) + \hat{\Omega}^*(t) - \hat{\Omega}(t) = l(t) + m(t), \tag{14}$$

where

$$l(t) = [l_1(t), l_2(t), \ldots, l_n(t)]^T = \Omega(t) - \hat{\Omega}^*(t), \tag{15}$$

$$\begin{aligned} m(t) &= [m_1(t), m_2(t), \ldots, m_n(t)]^T = \hat{\Omega}^*(t) - \hat{\Omega}(t), \\ &= \left[\tilde{\theta}_1^T(t)\xi_1(y(t), y(t-\tau(t))), \tilde{\theta}_2^T(t)\xi_2(y(t), y(t-\tau(t))), \ldots, \tilde{\theta}_n^T(t)\xi_n(y(t), y(t-\tau(t)))\right]^T, \end{aligned} \tag{16}$$

$$\tilde{\theta}_i(t) = \theta_i^*(t) - \theta_i(t), \tag{17}$$

$$\hat{\Omega}^*(t) = \left[\hat{\omega}_1^*(t), \hat{\omega}_2^*(t), \ldots, \hat{\omega}_n^*(t)\right]^T, \tag{18}$$

$$\hat{\omega}_i^*(t) = \hat{\omega}_i(y(t), y(t-\tau(t)|\theta_i^*(t)) = \theta_i^{*T}(t)\xi_i(y(t), y(t-\tau(t)), \tag{19}$$

$$\theta_i^*(t) = \arg\min_{\theta_i(t)}\left[\sup_{y(t), y(t-\tau(t))} |\hat{\omega}_i(y(t), y(t-\tau(t))|\theta_i) - \omega_i(y(t), y(t-\tau(t)))|\right].$$

We propose an adaptation law for $\theta_i(t)$ of the estimator $\hat{\omega}_i(t)$ to estimate $\omega_i(t)$ in the following theorem.

Theorem 1. *Consider the chaotic neural network (2) and the observer system (11). If the adaptation law for the parameter vector $\theta_i(t)$ for $\hat{\omega}_i(y(t), y(t-\tau(t)|\theta_i(t))$ is chosen as*

$$\dot{\theta}_i(t) = \gamma_1\xi_i(y(t), y(t-\tau(t)))(\varphi_i(t) + \gamma_0\varepsilon_i(t)), \tag{20}$$

where γ_0 and γ_1 are positive constants, then the unknown factors $\omega_i(t)$ are estimated by $\hat{\omega}_i(y(t), y(t-\tau(t)|\theta_i(t)) = \theta_i^T(t)\xi_i(y(t), y(t-\tau(t)))$ guaranteeing the following robust performance as follows:

$$\sum_{i=1}^{n}\left[\int_0^T p_i\varphi_i^2(t)dt + \int_0^T \gamma_{i0}m_i^2(t)dt\right] \leqslant \sum_{i=1}^{n}\left[\varphi_i^2(0) + \frac{1}{\gamma_{i1}}\tilde{\theta}_i^T(0)\tilde{\theta}_i(0) + \int_0^T\left(\gamma_{i0} + \frac{1}{p_i}\right)l_i^2(t)dt\right]. \tag{21}$$

Proof. Choose the following Lyapunov function candidate:

$$V_F(t) = \frac{1}{2}\varphi^T(t)\varphi(t) + \frac{1}{2\gamma_1}\sum_{i=1}^{n}\tilde{\theta}_i^T(t)\tilde{\theta}_i(t), \tag{22}$$

where γ_1 is a pre-designed positive constant. By differentiating $V_F(t)$ along (13) and using the adaptive law (20), we can obtain

$$\begin{aligned}\dot{V}_F(t) &= \varphi^T(t)\dot{\varphi}(t) + \frac{1}{\gamma_1}\sum_{i=1}^{n}\tilde{\theta}_i^T(t)\dot{\tilde{\theta}}_i(t), = \varphi^T(t)(-P\varphi(t) + \Omega(t) - \hat{\Omega}(t)) + \frac{1}{\gamma_1}\sum_{i=1}^{n}\tilde{\theta}_i^T(t)\dot{\tilde{\theta}}_i(t),\\ &= \varphi^T(t)(-P\varphi(t) + \Omega(t) - \hat{\Omega}^*(t) + \hat{\Omega}^*(t) - \hat{\Omega}(t)) + \frac{1}{\gamma_1}\sum_{i=1}^{n}\tilde{\theta}_i^T(t)\dot{\tilde{\theta}}_i(t),\\ &= -\varphi^T(t)P\varphi(t) + \varphi^T(t)l(t) + \varphi^T(t)(\hat{\Omega}^*(t) - \hat{\Omega}(t)) - \sum_{i=1}^{n}\tilde{\theta}_i^T(t)\xi_i(y(t), y(t-\tau(t)))(\varphi_i(t) + \gamma_0\varepsilon_i(t)),\\ &= \sum_{i=1}^{n}\Big[-p_i\varphi_i^2(t) + \varphi_i(t)l_i(t) + \tilde{\theta}_i^T(t)\xi_i(y(t), y(t-\tau(t)))\varphi_i(t) - \tilde{\theta}_i^T(t)\xi_i(y(t), y(t-\tau(t)))(\varphi_i(t) + \gamma_0\varepsilon_i(t))\Big],\\ &= \sum_{j=1}^{n}\left[-p_i\varphi_i^2(t) + \varphi_i(t)l_i(t) - \gamma_0 m_i^2(t) - \gamma_0 m_i(t)l_i(t)\right]. \end{aligned} \tag{23}$$

By applying the following inequalities

$$\varphi_i(t)l_i(t) \leqslant \frac{1}{2}p_i\varphi_i^2(t) + \frac{1}{2p_i}l_i^2(t) \text{ and } -m_i(t)l_i(t) \leqslant \frac{1}{2}m_i^2(t) + \frac{1}{2}l_i^2(t), \tag{24}$$

we can rewrite (23) as follows:

$$\begin{aligned}\dot{V}_F &\leqslant \sum_{i=1}^{n}\left[-p_i\varphi_i^2(t) - \gamma_0 m_i^2(t) + \frac{1}{2}\gamma_0 m_i^2(t) + \frac{1}{2}\gamma_{i0}l_i^2(t) + \frac{1}{2}p_i\varphi_i^2(t) + \frac{1}{2p_i}l_i^2(t)\right]\\ &= \sum_{i=1}^{n}\left[-\frac{1}{2}p_i\varphi_i^2(t) - \frac{1}{2}\gamma_0 m_i^2(t) + \frac{1}{2}\left(\gamma_0 + \frac{1}{p_i}\right)l_i^2(t)\right]. \end{aligned} \tag{25}$$

Integrating both sides of (25) from 0 to T yields

$$\frac{1}{2}\sum_{i=1}^{n}\left[\int_0^T p_i\varphi_i^2(t)dt + \int_0^T \gamma_0 m_i^2(t)dt\right] \leqslant V_F(0) - V_F(T) + \frac{1}{2}\sum_{i=1}^{n}\int_0^T\left(\gamma_0 + \frac{1}{p_i}\right)l_i^2(t)dt. \tag{26}$$

Inequality (26) is equivalent to (21) in *Theorem 1*, because $V_F(T) > 0$. This completes the proof. □

Based on Barbalat's lemma [41], the robust performance inequality (21) can be explained. If $l_i(t) \in \mathcal{L}_2$, i.e., $\int_0^\infty l_i^2(t)dt < \infty$, then $\varphi_i \in \mathcal{L}_2$ and $m_i \in \mathcal{L}_2$. This means $\lim_{t\to\infty}\|\varphi_i(t)\| = 0$ and $\lim_{t\to\infty}\|m_i(t)\| = 0$. Even though $l_i \notin \mathcal{L}_2$, one can say $\varphi_i^2(t)$ is bounded by $l_i^2(t)$. Hence, we can reduce the observation error $\varphi_i(t)$ to an arbitrarily small value by adjusting the pre-determined positive weighting matrix $\gamma_0 + \frac{1}{p_i}$. Therefore, we can conclude that $\hat{\Omega}(t)$ can estimate $\Omega(t)$ with arbitrarily small error.

Remark 3. The FLS has been widely applied in much literature since the work of [38]. They have used the approach to estimate the system nonlinear function. The estimated value may cause the singularity problem because it is utilized as denominator of the controller. By using the technique to estimate the overall disturbance, the problem can be prevented. Moreover, we can design the FDO and the controller, independently.

From Theorem 1, we can see that the FDO with the adaptation law (20) for the parameter vector can estimate the overall disturbance $\Omega(t)$ in (2). This means that the value can be used to compensate the overall disturbance Ω_i. However, fuzzy approximation error $\varepsilon(t)$ still exists. To eliminate the remaining error and achieve the synchronization between (1) and (2), we propose a robust adaptive controller design method in the following theorem.

Theorem 2. *Consider the drive system (1) and the response system (2). The systems are globally asymptotically synchronized, if a robust adaptive controller and adaptation laws are chosen as*

$$u(t) = -K_1e(t) - K_2\frac{e(t)}{\|e(t)\|} - \hat{\Omega}(t), \tag{27}$$

$$\dot{k}_{1i}(t) = \alpha_i e_i^2(t), \tag{28}$$

$$\dot{k}_{2i}(t) = \beta_i\frac{e_i^2(t)}{\|e_i(t)\|}, \tag{29}$$

where $K_1 = diag(k_{11}, k_{12}, \ldots, k_{1n}) > 0, K_2 = diag(k_{21}, k_{22}, \ldots, k_{2n}) > 0$, $\hat{\Omega}(t)$ *is the output of FDO of which the parameter vector is adjusted by (20), and* α_i, β_i *are positive constants.*

Proof. Choose the following Lyapunov–Krasovskii function candidate:

$$V(t) = \frac{1}{2}e^T(t)e(t) + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\alpha_i}\tilde{k}_{1i}^2(t) + \frac{1}{2}\sum_{i=1}^{n}\frac{1}{\beta_i}\tilde{k}_{2i}^2(t) + \frac{1}{2(1-\mu)}\int_{t-\tau(t)}^{t}g^T(e(r))g(e(r))dr + \frac{1}{2}\int_{-\bar{\sigma}}^{0}\int_{t+s}^{t}e^T(\eta)Qe(\eta)d\eta ds, \tag{30}$$

where $\tilde{k}_{1i} = k_{1i}(t) - k^*_{1i},\ \tilde{k}_{2i} = k_{2i}(t) - k^*_{2i}$ with constants $k^*_{1i},\ k^*_{2i}$ which will be designed and $Q = \delta\bar{\sigma}L^TL$ with a constant $\delta > 0$. By differentiating V_1 along the error dynamics (3), we can obtain

$$\begin{aligned}\dot{V}(t) &= e^T(t)\dot{e}(t) + \sum_{i=1}^{n}\frac{1}{\alpha_i}\tilde{k}_{1i}(t)\dot{\tilde{k}}_{1i}(t) + \sum_{i=1}^{n}\frac{1}{\beta_i}\tilde{k}_{2i}(t)\dot{\tilde{k}}_{2i}(t) + \frac{1}{2(1-\mu)}g^T(e(t))g(e(t)) \\ &\quad - \frac{1-\dot{\tau}(t)}{2(1-\mu)}g^T(e(t-\tau(t)))g(e(-\tau(t))) + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) - \frac{1}{2}\int_{t-\bar{\sigma}}^{t} e^T(s)Qe(s)ds, \\ &= e^T(t)\left[-Ce(t) + Af(e(t)) + Bg(e(t-\tau(t))) + D\int_{t-\sigma(t)}^{t} h(e(s))ds + \Omega(t) + u(t)\right] \\ &\quad + \sum_{i=1}^{n}\frac{1}{\alpha_i}\tilde{k}_{1i}(t)\dot{\tilde{k}}_{1i}(t) + \sum_{i=1}^{n}\frac{1}{\beta_i}\tilde{k}_{2i}(t)\dot{\tilde{k}}_{2i}(t) + \frac{1}{2(1-\mu)}g^T(e(t))g(e(t)) - \frac{1-\dot{\tau}(t)}{2(1-\mu)}g^T(e(t-\tau(t)))g(e(-\tau(t))) \\ &\quad + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) - \frac{1}{2}\int_{t-\bar{\sigma}}^{t} e^T(s)Qe(s)ds. \end{aligned} \tag{31}$$

By using the control laws (27)–(29) and the definition of $\varepsilon(t)$ (14), we have

$$\begin{aligned}\dot{V}(t) &= e^T(t)\left[-Ce(t) + Af(e(t)) + Bg(e(t-\tau(t))) + D\int_{t-\sigma(t)}^{t} h(e(s))ds + \Omega(t) - K_1e(t) - K_2\frac{e(t)}{\|e(t)\|} - \hat{\Omega}(t)\right] \\ &\quad + \sum_{i=1}^{n}\frac{1}{\alpha_i}\tilde{k}_{1i}(t)\dot{\tilde{k}}_{1i}(t) + \sum_{i=1}^{n}\frac{1}{\beta_i}\tilde{k}_{2i}(t)\dot{\tilde{k}}_{2i}(t) + \frac{1}{2(1-\mu)}g^T(e(t))g(e(t)) - \frac{1-\dot{\tau}(t)}{2(1-\mu)}g^T(e(t-\tau(t)))g(e(-\tau(t))) \\ &\quad + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) - \frac{1}{2}\int_{t-\bar{\sigma}}^{t} e^T(s)Qe(s)ds, \\ &= e^T(t)\left[-Ce(t) + Af(e(t)) + Bg(e(t-\tau(t))) + D\int_{t-\sigma(t)}^{t} h(e(s))ds + \varepsilon(t) - K_1^*e(t) - K_2^*\frac{e(t)}{\|e(t)\|}\right] \\ &\quad + \frac{1}{2(1-\mu)}g^T(e(t))g(e(t)) - \frac{1-\dot{\tau}(t)}{2(1-\mu)}g^T(e(t-\tau(t)))g(e(-\tau(t))) + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) - \frac{1}{2}\int_{t-\bar{\sigma}}^{t} e^T(s)Qe(s)ds, \end{aligned} \tag{32}$$

where $K_1^* = diag(k^*_{11}, k^*_{12}, \ldots, k^*_{1n})$ and $K_2^* = diag(k^*_{21}, k^*_{22}, \ldots, k^*_{2n})$. Using Lemma 1, we have the following two inequalities

$$e^T(t)Af(e(t)) = \left[A^Te(t)\right]^T f(e(t)) \leqslant \frac{1}{2}e^T(t)AA^Te(t) + \frac{1}{2}f^T(e(t))f(e(t)), \tag{33}$$

$$e^T(t)Bg(e(t-\tau(t))) = \left[B^Te(t)\right]^T g(e(t-\tau(t))) \leqslant \frac{1}{2}e^T(t)BB^Te(t) + \frac{1}{2}g^T(e(t-\tau(t)))g(e(t-\tau(t))). \tag{34}$$

By the inequalities (33), (34) and $\frac{1-\dot{\tau}(t)}{1-\mu} \geqslant 1$ derived from Assumption 2, the following inequality is obtained

$$\begin{aligned}\dot{V}(t) &\leqslant -e^T(t)Ce(t) + \frac{1}{2}e^T(t)AA^Te(t) + \frac{1}{2}f^T(e(t))f(e(t)) + \frac{1}{2}e^T(t)BB^Te(t) + \frac{1}{2}g^T(e(t-\tau(t)))g(e(t-\tau(t))) \\ &\quad + e^T(t)D\int_{t-\sigma(t)}^{t} h(e(s))ds + e^T(t)\varepsilon(t) - e^T(t)K_1^*e(t) - \frac{e^T(t)K_2^*e(t)}{\|e(t)\|} + \frac{1}{2(1-\mu)}g^T(e(t))g(e(t)) \\ &\quad - \frac{1-\dot{\tau}(t)}{2(1-\mu)}g^T(e(t-\tau(t)))g(e(-\tau(t))) + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) - \frac{1}{2}\int_{t-\bar{\sigma}}^{t} e^T(s)Qe(s)ds \leqslant -e^T(t)Ce(t) \\ &\quad + \frac{1}{2}e^T(t)AA^Te(t) + \frac{1}{2}f^T(e(t))f(e(t)) + \frac{1}{2}e^T(t)BB^Te(t) + e^T(t)D\int_{t-\sigma(t)}^{t} h(e(s))ds + e^T(t)\varepsilon(t) \\ &\quad - e^T(t)K_1^*e(t) - \frac{e^T(t)K_2^*e(t)}{\|e(t)\|} + \frac{1}{2(1-\mu)}g^T(e(t))g(e(t)) + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) - \frac{1}{2}\int_{t-\bar{\sigma}}^{t} e^T(s)Qe(s)ds. \end{aligned} \tag{35}$$

We can obtain the following inequalities from Assumption 1

$$|f_i(e_i(t))| = |f_i(y_i(t)) - f_i(x_i(t))| \leqslant \lambda_{fi}|e_i(t)|, \quad |g_i(e_i(t))| = |g_i(y_i(t)) - g_i(x_i(t))| \leqslant \lambda_{gi}|e_i(t)|, \tag{36}$$

Then, they are rewritten as

$$f^T(e(t))f(e(t)) = \sum_{i=1}^{n} f_i^2(e_i(t)) \leqslant \sum_{i=1}^{n}\lambda_{fi}^2 e_i^2(t) = e^T(t)\Lambda_f e(t), \tag{37}$$

$$g^T(e(t))g(e(t)) = \sum_{i=1}^{n} g_i^2(e_i(t)) \leqslant \sum_{i=1}^{n}\lambda_{gi}^2 e_i^2(t) = e^T(t)\Lambda_g e(t), \tag{38}$$

where $\Lambda_f = diag(\lambda_{f1}^2, \lambda_{f2}^2, \ldots, \lambda_{fn}^2)$ and $\Lambda_g = diag(\lambda_{g1}^2, \lambda_{g2}^2, \ldots, \lambda_{gn}^2)$. Hence, we have

$$\begin{aligned}\dot{V}(t) &\leqslant -e^T(t)Ce(t) + \frac{1}{2}e^T(t)AA^Te(t) + \frac{1}{2}e^T(t)BB^Te(t) + e^T(t)\varepsilon(t) - \frac{e^T(t)K_2^*e(t)}{\|e(t)\|} \\ &\quad + e^T(t)\left[\frac{1}{2}\Lambda_f + \frac{1}{2(1-\mu)}\Lambda_g - K_1^*\right]e(t) + e^T(t)D\int_{t-\sigma(t)}^t h(e(s))ds + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) - \frac{1}{2}\int_{t-\bar{\sigma}}^t e^T(s)Qe(s)ds. \end{aligned} \tag{39}$$

Let us denote a variable Φ with a positive scalar δ

$$\Phi = \delta^{\frac{1}{2}}\int_{t-\sigma(t)}^t h(e(s))ds - \delta^{-\frac{1}{2}}D^Te(t) \in \mathbb{R}^n. \tag{40}$$

It follows from the matrix inequality $\Phi^T\Phi \geqslant 0$ as follows:

$$\begin{aligned}\Phi^T\Phi &= \left[\delta^{\frac{1}{2}}\int_{t-\sigma(t)}^t h^T(e(s))ds - \delta^{-\frac{1}{2}}e^T(t)D\right]\left[\delta^{\frac{1}{2}}\int_{t-\sigma(t)}^t h(e(s))ds - \delta^{-\frac{1}{2}}D^Te(t)\right], \\ &= \delta\int_{t-\sigma(t)}^t h^T(e(s))ds\int_{t-\sigma(t)}^t h(e(s))ds - \int_{t-\sigma(t)}^t h^T(e(s))dsD^Te(t) - e^T(t)D\int_{t-\sigma(t)}^t h(e(s))ds + \delta^{-1}e^T(t)DD^Te(t) \\ &\geqslant 0. \end{aligned} \tag{41}$$

Thus, we can obtain the following inequality

$$\frac{1}{2}\int_{t-\sigma(t)}^t h^T(e(s))dsD^Te(t) + \frac{1}{2}e^T(t)D\int_{t-\sigma(t)}^t h(e(s))ds \leqslant \frac{\delta}{2}\int_{t-\sigma(t)}^t h^T(e(s))\int_{t-\sigma(t)}^t h(e(s))ds + \frac{\delta^{-1}}{2}e^T(t)DD^Te(t). \tag{42}$$

Using the inequality (42) yields

$$\begin{aligned}\dot{V}(t) &\leqslant -e^T(t)Ce(t) + \frac{1}{2}e^T(t)AA^Te(t) + \frac{1}{2}e^T(t)BB^Te(t) + e^T(t)\varepsilon(t) - \frac{e^T(t)K_2^*e(t)}{\|e(t)\|} \\ &\quad + e^T(t)\left[\frac{1}{2}\Lambda_f + \frac{1}{2(1-\mu)}\Lambda_g - K_1^*\right]e(t) + \frac{1}{2}\int_{t-\sigma(t)}^t h^T(e(s))dsD^Te(t) + \frac{1}{2}e^T(t)D\int_{t-\sigma(t)}^t h(e(s))ds \\ &\quad + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) - \frac{1}{2}\int_{t-\bar{\sigma}}^t e^T(s)Qe(s)ds \\ &\leqslant -e^T(t)Ce(t) + \frac{1}{2}e^T(t)AA^Te(t) + \frac{1}{2}e^T(t)BB^Te(t) + e^T(t)\varepsilon(t) - \frac{e^T(t)K_2^*e(t)}{\|e(t)\|} \\ &\quad + e^T(t)\left[\frac{1}{2}\Lambda_f + \frac{1}{2(1-\mu)}\Lambda_g - K_1^*\right]e(t) + \frac{\delta}{2}\int_{t-\sigma(t)}^t h^T(e(s))ds\int_{t-\sigma(t)}^t h(e(s))ds + \frac{\delta^{-1}}{2}e^T(t)DD^Te(t) \\ &\quad + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) - \frac{1}{2}\int_{t-\bar{\sigma}}^t e^T(s)Qe(s)ds. \end{aligned} \tag{43}$$

From Assumption 1 and Lemma 2, we further have

$$\begin{aligned}\frac{\delta}{2}\int_{t-\sigma(t)}^t h^T(e(s))ds\int_{t-\sigma(t)}^t h(e(s))ds &\leqslant \frac{\delta}{2}\sigma\int_{t-\sigma}^t h^T(e(s))h(e(s))ds \leqslant \frac{\delta}{2}\bar{\sigma}\int_{t-\bar{\sigma}}^t h^T(e(s))h(e(s))ds \\ &\leqslant \frac{\delta}{2}\bar{\sigma}\int_{t-\bar{\sigma}}^t e^T(s)L^TLe(s)ds = \frac{1}{2}\int_{t-\bar{\sigma}}^t e^T(s)Qe(s)ds. \end{aligned} \tag{44}$$

Therefore,

$$\begin{aligned}\dot{V}(t) &\leqslant -e^T(t)Ce(t) + \frac{1}{2}e^T(t)AA^Te(t) + \frac{1}{2}e^T(t)BB^Te(t) + e^T(t)\varepsilon(t) - \frac{e^T(t)K_2^*e(t)}{\|e(t)\|} \\ &\quad + e^T(t)\left[\frac{1}{2}\Lambda_f + \frac{1}{2(1-\mu)}\Lambda_g - K_1^*\right]e(t) + \frac{\delta^{-1}}{2}e^T(t)DD^Te(t) + \frac{1}{2}\bar{\sigma}e^T(t)Qe(t) \\ &\leqslant e^T(t)\left[-C + \frac{1}{2}AA^T + \frac{1}{2}BB^T + \frac{1}{2}\Lambda_f + \frac{1}{2(1-\mu)}\Lambda_g - K_1^* + \frac{\delta^{-1}}{2}DD^T + \frac{1}{2}\bar{\sigma}Q\right]e(t) + \frac{e^T(t)\left[\bar{\varepsilon}I_n - K_2^*\right]e(t)}{\|e(t)\|}, \end{aligned} \tag{45}$$

where $I_n \in \mathbb{R}^{n\times n}$ is identity matrix and, by the universal approximation theorem, $\bar{\varepsilon}$ is the upper bound of $\|\varepsilon\|$, i.e., $\|\varepsilon\| \leqslant \bar{\varepsilon}$. Taking appropriate positive parameters k_{1i} and k_{2i} for $i = 1, 2, \ldots, n$ such that

$$\Psi = -C + \frac{1}{2}AA^T + \frac{1}{2}BB^T + \frac{1}{2}\Lambda_f + \frac{1}{2(1-\mu)}\Lambda_g - K_1^* + \frac{\delta^{-1}}{2}DD^T + \frac{1}{2}\bar{\sigma}Q < 0, \Xi = \bar{\varepsilon}I_n - K_2^* < 0, \tag{46}$$

yields the following inequality:

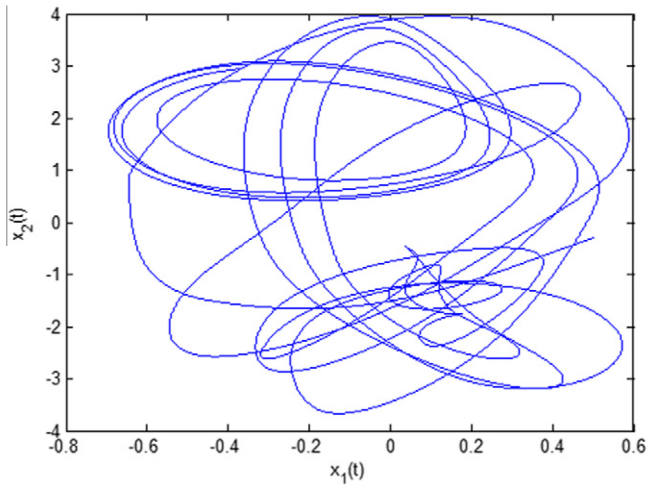


Fig. 1. Chaotic behavior of the drive system.

$$\dot{V}(t) \leqslant e^T(t)\Psi e(t) + \frac{e^T(t)\Xi e(t)}{\|e(t)\|} \leqslant 0. \tag{47}$$

From (47), it is obvious that $\dot{V}(t) \leqslant 0$ for all $e(t)$. Moreover, the positive differentiable Lyapunov function $V(t)$ is radially unbounded and the set $S = \{e(t) \in \mathbb{R}^n | \dot{V}(t) = 0\} = \{e(t) \in \mathbb{R}^n | e(t) = 0\}$ contains no solutions other than the trivial solution $e(t) = 0$. According to Lasalle's invariance principle [42], one can conclude that the synchronization error $e(t)$ is globally asymptotically stable, i.e. $\lim_{t\to\infty}\|e(t)\| = 0$. Therefore, this means that the response system (2) having both uncertainties and disturbances is globally asymptotically synchronized with the drive system (1) by the control law (27) and adaptation laws (28), (29). This completes the proof. □

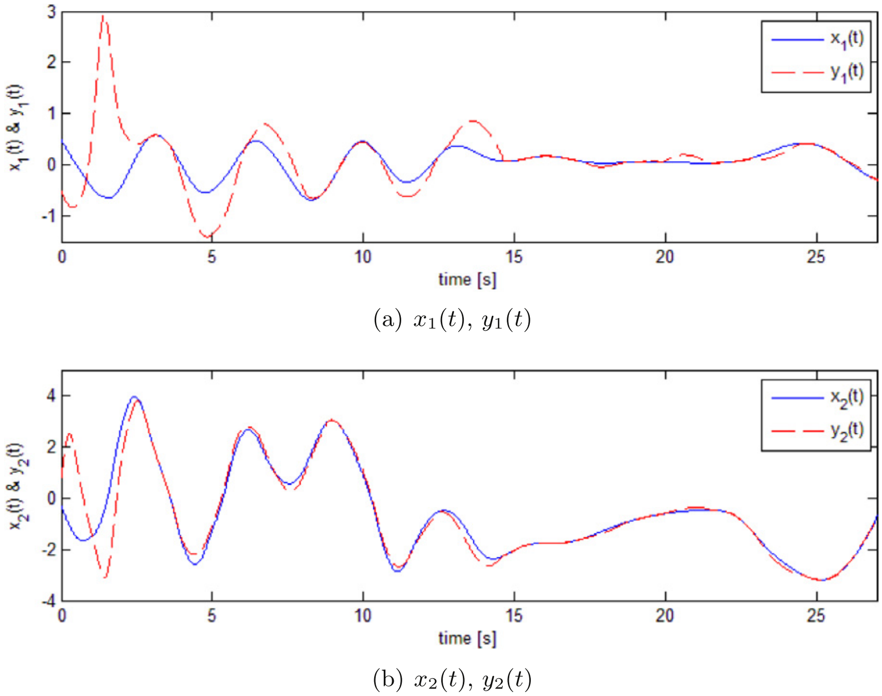
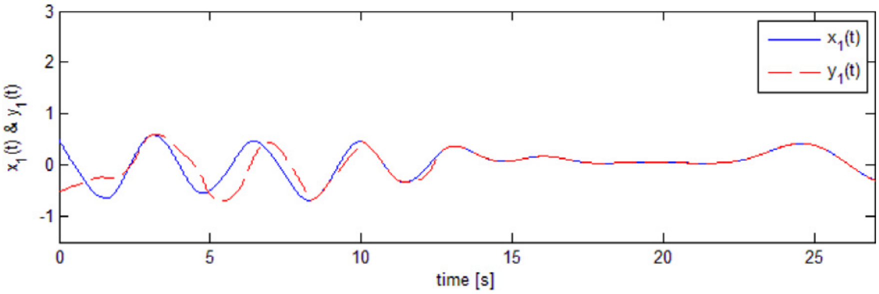
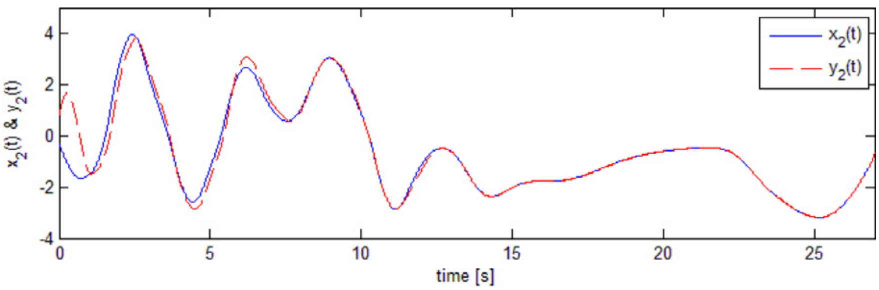


Fig. 2. Trajectories of the drive system and response system when the FDO is not applied.

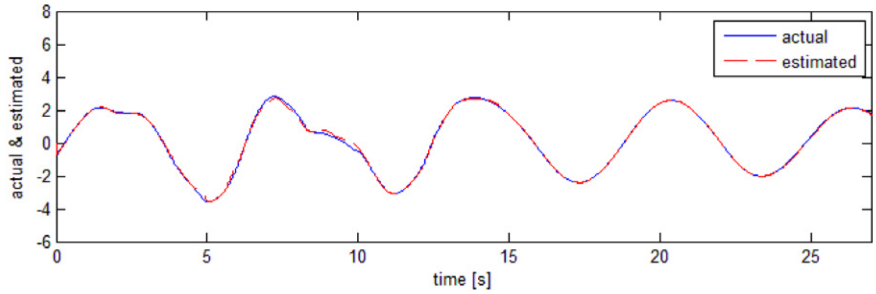


(a) $x_1(t)$, $y_1(t)$

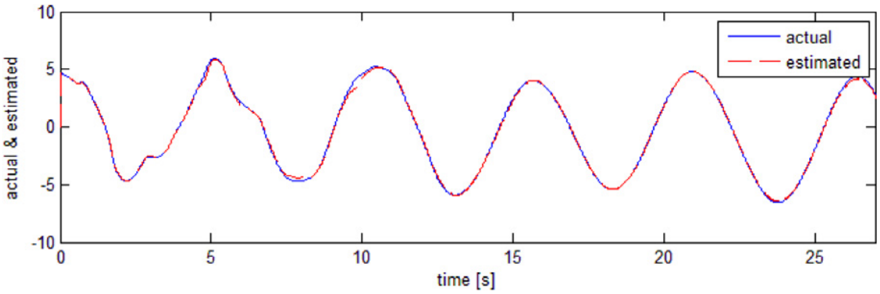


(b) $x_2(t)$, $y_2(t)$

Fig. 3. Trajectories of the drive system and response system when the proposed method is applied.



(a) Actual $\Omega_1(t)$, estimated $\Omega_1(t)$

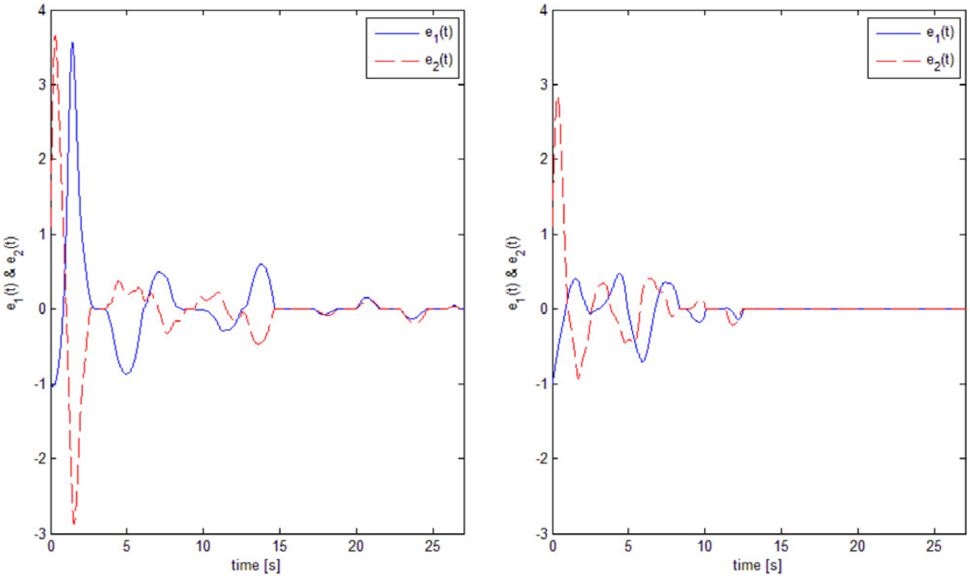


(b) Actual $\Omega_2(t)$, estimated $\hat{\Omega}_2(t)$

Fig. 4. Actual and estimated values of the overall disturbance $\Omega(t)$.

4. Numerical examples

In this section, a numerical example is presented to illustrate the effectiveness of our scheme proposed in the previous sections. The simulations are conducted in Simulink (MATLAB) using a fixed-step fourth order Runge–Kutta solver with sam-



(a) Error without FDO (b) Error by the proposed method

Fig. 5. Synchronization error $e(t)$ for two cases.

ple period $T_s = 0.001$s. We consider a two-dimensional chaotic neural network with the mixed delay as the drive system (1), which is described with

$$C = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \quad A = \begin{bmatrix} 1.8 & -0.15 \\ -5.2 & 3.5 \end{bmatrix}, \quad B = \begin{bmatrix} -1.7 & -0.12 \\ -0.26 & -2.5 \end{bmatrix}, \quad D = \begin{bmatrix} 0.6 & 0.15 \\ -2 & -0.12 \end{bmatrix}, \quad I = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

$$f(x(t)) = g(x(t)) = h(x(t)) = \begin{bmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \end{bmatrix}, \quad \tau(t) = 0.1\sin(t) + 1, \quad \sigma(t) = \frac{2e^t}{1+e^t}.$$

The initial condition associated with the drive system is given as $x_1(s) = 0.5,\ x_2(s) = -0.3$ for all $s \in [-2, 0]$. Fig. 1 shows the chaotic behavior of the drive system. The response system (2) is affected by uncertainties and disturbances as follows:

$$\Delta C = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad \Delta A = \begin{bmatrix} 0.5 & 0 \\ -1 & -1 \end{bmatrix}, \quad \Delta B = \begin{bmatrix} -0.5 & 0 \\ 0 & 0.6 \end{bmatrix},$$

$$\Delta D = \begin{bmatrix} -0.3 & 0 \\ 0.1 & 0.03 \end{bmatrix}, \quad d(t) = \begin{bmatrix} 2\sin(t) + 1.6y_1(t) \\ 2\cos(1.5t) + 0.5y_1(t)y_2(t) \end{bmatrix}.$$

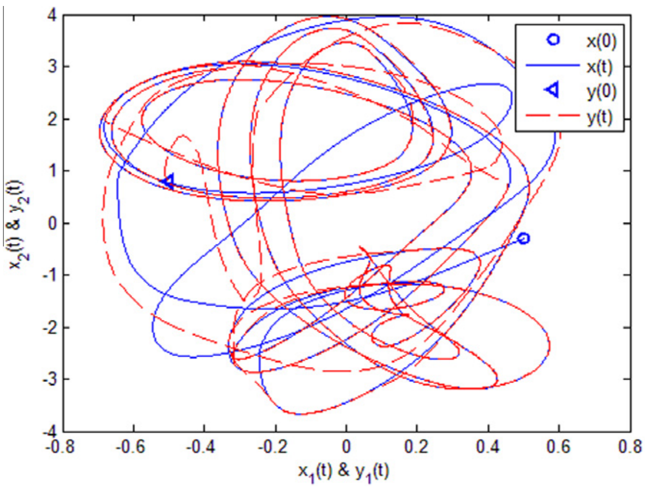


Fig. 6. State trajectories of the drive system $x(t)$ and response system $y(t)$.

The initial condition of the response system is given as $y_1(s) = -0.5, y_2(s) = 0.8$ for all $s \in [-2, 0]$. The output of the FDO, $\hat{\Omega}(t) = \left[\,\theta_1^T(t)\xi_1(y(t), y(t-\tau(t))) \quad \theta_2^T(t)\xi_2(y(t), y(t-\tau(t)))\,\right]^T$, is applied to the control input (27). In order to construct the FDO (11), we use the input vector of the FLS as $Z = [\,z_1(t) \quad z_2(t) \quad z_3(t) \quad z_4(t)\,]^T = [\,y_1(t) \quad y_2(t) \quad y_1(t-\tau(t)) \quad y_2(t-\tau(t))\,]^T$ where $z_1(t), z_3(t) \in [-0.8, 0.8]$ and $z_2(t), z_4(t) \in [-5, 5]$. We choose three centers of the Gaussian membership function $\mu_l(t) = exp\left[-(z_l(t) - c_{ml})^2/\sigma_l^2\right]$ for $l = 1,2,3,4$ where $\sigma_1 = \sigma_3 = 0.5$ and $\sigma_2 = \sigma_4 = 3$, i.e. $C_m = [\,c_{m1} \quad c_{m2} \quad c_{m3}\,]$ for $m = 1, 2, \ldots, 81$ with uniform distance. The parameters and initial values in the FDO are $\gamma_0 = \gamma_1 = 25, p_i = 5, \hat{y}(0) = 0$, and $\theta(0) = 0$. Ones used in the proposed control scheme (27)–(29) are chosen as $\alpha_i = \beta_i = 1$ and $k_{i1}(0) = k_{i2}(0) = 0$ for $i = 1, 2$.

We compare the simulation results to ones without the FDO to present its effectiveness. Fig. 2 shows the state trajectories of the drive and response system when the FDO is not applied. One can see that the errors between the systems still remain. On the other hand, we can remove the remaining error by using the proposed method with the FDO (Fig. 3). This is why the estimated values for the overall disturbance $\Omega(t)$ by the FDO effectively compensate the actual one. The values $\Omega(t)$ and $\hat{\Omega}(t)$ are shown in Fig. 4. The synchronization error $e(t) = y(t) - x(t)$ is presented to compare two cases in Fig. 5. Fig. 6 presents the synchronization between the chaotic neural networks. Therefore, from these results we conclude that the response system (2) is successfully synchronized with the drive system (1) by the proposed method.

5. Conclusion

We have proposed a robust adaptive synchronization method for uncertain chaotic neural networks with both time-varying and distributed delays. By using the FDO, the uncertain factors including uncertainties and disturbances have been estimated without requiring any prior information about the factors. The estimated values have been used to compensate the factors. Based on Lyapunov–Krasovskii stability theory, the control scheme with adaptive laws has been derived, guaranteeing the globally asymptotical synchronization between the neural networks. An example has shown the effectiveness of the proposed method.

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