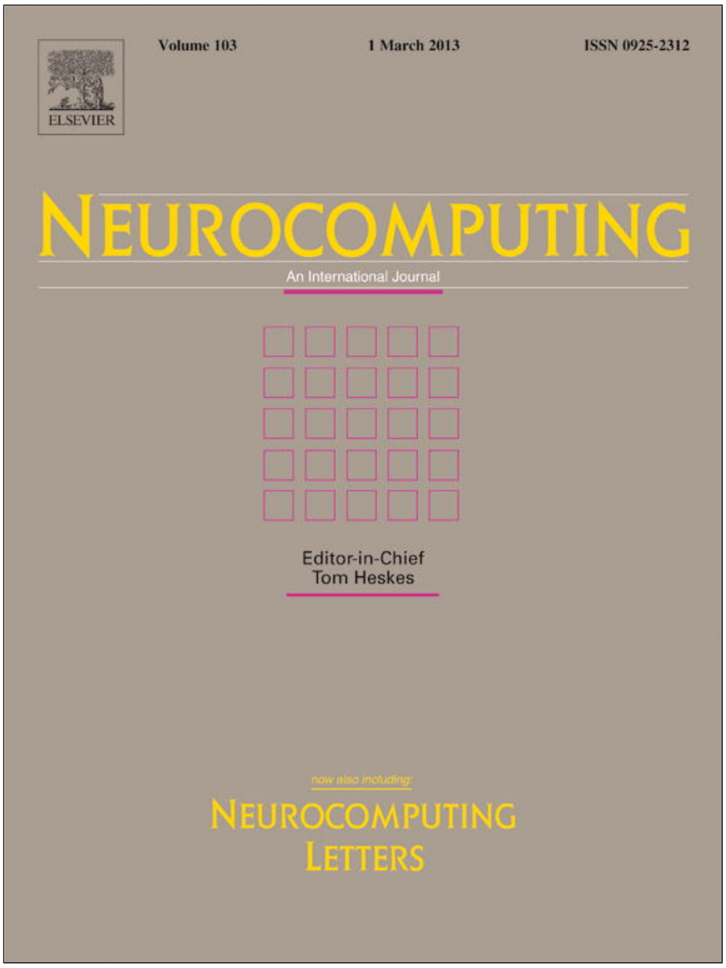
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Analysis on delay-dependent stability for neural networks with time-varying delays

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ABSTRACT

This paper considers the problem of delay-dependent stability criteria for neural networks with time-varying delays. First, by constructing a newly augmented Lyapunov–Krasovskii functional, a less conservative stability criterion is established in terms of linear matrix inequalities (LMIs). Second, by proposing a novel activation function condition which has not been considered, a further improved result is proposed. Finally, two numerical examples utilized in other literature are given to show the improvements over the existing ones and the effectiveness of the proposed idea.

1. Introduction

In the past few decades, neural networks have been widely applied to various fields such as load frequency control in power systems [1], pattern recognition [2], finance [3], associative memories [4], mechanics of structures and materials [5], smart antenna arrays [6], and other scientific areas [7–12]. Therefore, neural networks play important roles in many practical systems. Since the key feature of these applications with neural networks is that the equilibrium points of the designed network are stable, stability analysis of neural network is a prerequisite and an important work. In the implementation of neural networks, time delays naturally occur due to the finite switching speed of amplifies and may cause some sophisticated dynamical behaviors such as instability or oscillation of neural networks [13]. Therefore, delay-dependent stability analysis of neural networks with time-delays has been extensively investigated [14–30] since it is well known that delay-dependent stability criteria are generally less conservative than delay-independent ones when the sizes of time-delays are small.

The main aim of delay-dependent stability analysis is to get maximum delay bounds such that the designed networks are asymptotically stable for any delay less than maximum delay bounds. For the case of time-varying delays, maximum delay bounds for guaranteeing the asymptotic stability of the networks in [14–30] were investigated for different upper bounds of derivative of time-varying delays. Thus, how to construct Lyapunov–Krasovskii functional and estimate an upper bound of time-derivative of it play key roles to increase delay bounds. Recently, since a delay-partitioning idea was firstly proposed in [31], it is well recognized that the delay-partitioning approach can reduce the conservatism of stability criteria. One of the main advantages of this method can obtain more tighter upper bounds obtained by calculating the time-derivative of Lyapunov–Krasovskii functional, which leads to less conservative results. However, when the number of delay-partitioning number increases, the matrix formulation becomes more complex and the computational burden and time-consumption grow bigger.

In this regard, many researchers [24–30] have been studied the delay-partitioning method for delay-dependent stability criteria of neural networks with time-delays. In [24], by utilizing different free-weighting matrices in two delay subintervals, a piecewise delay method which is the same concept of two delay-partitioning approaches was proposed for the stability analysis of delayed neural networks. Xiao and Zhang [27] also studied the stability analysis for uncertain delayed neural networks by taking delay-partitioning number as two. Recently, in [28], the weighting-delay-based stability criteria for neural networks with time-varying delay were investigated by proposing the idea of dividing the delay interval by the weighted parameters. Very

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recently, new delay-derivative-dependent stability criteria for neural networks with unbounded distributed delay and discrete time-varying delays were presented in [29] by introducing an improved delay-partitioning technique and general convex combination. Further improved versions of the method [29] were introduced in [30]. However, in spite of such extensive researches mentioned above, there are still rooms for further improvements of the stability criteria.

In this paper, the problem of delay-dependent stability analysis for neural networks with time-varying delays is investigated. Unlike the method of [24–30], no delay-partitioning methods are utilized. Instead, by taking more information about states and activation functions as augmented vectors, an augmented Lyapunov–Krasovskii's functional is proposed. Then, inspired by the work of [32–34], a sufficient condition such that the considered neural networks are asymptotically stable is derived in terms of linear matrix inequalities (LMIs) which will be presented in Theorem 1. And, with the same Lyapunov–Krasovskii's functional considered, a new activation function condition which has not been considered yet in other literature is proposed and utilized in Theorem 2 to reduce the conservatism of stability criterion. Through two numerical examples utilized in other literature, it will be shown that the proposed stability criteria can provide larger delay bounds than the results of [24–30] in spite of not employing delay-partitioning approaches.

Notation: Throughout this paper, $\mathbb{R}^n$ denotes n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is the set of all $n \times m$ real matrices. For symmetric matrices X and Y, the notation $X>Y$ (respectively, $X \geq Y$) means that the matrix $X-Y$ is positive definite (respectively, nonnegative). $diag\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix. The subscript 'T' denotes the transpose of the matrix.

2. Problem statement

Consider the following neural networks with time-varying delays:

$$\dot{y}(t) = -Ay(t)+W_0 g(y(t))+W_1 g(y(t-h(t)))+b, \tag{1}$$

where $y(t)=[y_1(t),\ldots,y_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector, n denotes the number of neurons in a neural network, $g(y(t))=[g_1(y_1(t)),\ldots,g_n(y_n(t))]^T \in \mathbb{R}^n$ means the neuron activation function, $g(y(t-h(t)))=[g_1(y_1(t-h(t))),\ldots,g_n(y_n(t-h(t)))]^T \in \mathbb{R}^n$, $A=diag\{a_i\} \in \mathbb{R}^{n\times n}$ is a positive diagonal matrix, $W_0=(w_{ij}^0)_{n\times n} \in \mathbb{R}^{n\times n}$ and $W_1=(w_{ij}^1)_{n\times n} \in \mathbb{R}^{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons, and $b=[b_1,b_2,\ldots,b_n]^T \in \mathbb{R}^n$ represents a constant input vector.

The delay, $h(t)$, is a time-varying continuous function satisfying

$$0 \leq h(t) \leq h_U, \quad -\infty < \dot{h}(t) \leq h_D, \tag{2}$$

where $h_U>0$ and h_D are known constant scalar values.

The activation functions, $g_i(y_i(t))$, $i=1,\ldots,n$, are assumed to be bounded and hold the following condition:

$$k_i^- \leq \frac{g_i(u)-g_i(v)}{u-v} \leq k_i^+, \quad u,\ v \in \mathbb{R},\\ u \neq v, \quad i=1,\ldots,n, \tag{3}$$

where k_i^+ and k_i^- are constants.

For simplicity, in stability analysis of the neural networks (1), the equilibrium point $y^*=[y_1^*,\ldots,y_n^*]^T$ whose uniqueness has been reported in [8] is shifted to the origin by utilizing the transformation $x(\cdot)=y(\cdot)-y^*$, which leads the system (1) to the following form:

$$\dot{x}(t) = -Ax(t)+W_0 f(x(t))+W_1 f(x(t-h(t))) \tag{4}$$

where $x(t)=[x_1(t),\ldots,x_n(t)]^T \in \mathbb{R}^n$ is the state vector of the transformed system, $f(x(t))=[f_1(x_1(t)),\ldots,f_n(x_n(t))]^T$ and $f_j(x_j(t))=g_j(x_j(t)+y_j^*)-g_j(y_j^*)$ with $f_j(0)=0 (j=1,\ldots,n)$.

It should be noted that the activation functions $f_i(\cdot)$ $(i=1,\ldots,n)$ satisfy the following condition [9]:

$$k_i^- \leq \frac{f_i(u)-f_i(v)}{u-v} \leq k_i^+, \quad u,\ v \in \mathbb{R},\\ u \neq v, \quad i=1,\ldots,n. \tag{5}$$

If $v=0$ in (5), then we have

$$k_i^- \leq \frac{f_i(u)}{u} \leq k_i^+ \quad \forall\ u \neq 0, \quad i=1,\ldots,n, \tag{6}$$

which is equivalent to

$$[f_i(u)-k_i^- u][f_i(u)-k_i^+ u] \leq 0, \quad i=1,\ldots,n. \tag{7}$$

The objective of this paper is to investigate the delay dependent stability analysis of system (4) which will be conducted in Section 3.

Before deriving our main results, we state the following lemmas.

Lemma 1. *For any constant positive-definite matrix $M \in \mathbb{R}^{n\times n}$ and $\alpha \leq \beta$, the following inequalities hold*:

$$(\alpha-\beta)\int_\beta^\alpha \dot{x}^T(s)M\dot{x}(s)\,ds \geq \left(\int_\beta^\alpha \dot{x}(s)\,ds\right)^T M\left(\int_\beta^\alpha \dot{x}(s)\,ds\right), \tag{8}$$

$$\frac{(\alpha-\beta)^2}{2}\int_\beta^\alpha\int_s^\alpha \dot{x}^T(u)M\dot{x}(u)\,du\,ds \\ \geq \left(\int_\beta^\alpha\int_s^\alpha \dot{x}(u)\,du\,ds\right)^T M\left(\int_\beta^\alpha\int_s^\alpha \dot{x}(u)u\,ds\right). \tag{9}$$

Proof. According to Jensen's inequality in [35], one can obtain (8). Moreover, the following inequality holds:

$$(\alpha-s)\int_s^\alpha \dot{x}^T(u)M\dot{x}(u)\,du \geq \left(\int_s^\alpha \dot{x}(u)\,du\right)^T M\left(\int_s^\alpha \dot{x}(u)\,du\right). \tag{10}$$

By Schur Complements [36], Eq. (10) is equivalent to the following:

$$\begin{bmatrix} \int_s^\alpha \dot{x}^T(u)M\dot{x}(u)\,du & \int_s^\alpha \dot{x}^T(u)\,du \\ \int_s^\alpha \dot{x}(u)\,du & (\alpha-s)M^{-1} \end{bmatrix} \geq 0. \tag{11}$$

Integration of (11) from β to α yields

$$\begin{bmatrix} \int_\beta^\alpha\int_s^\alpha \dot{x}^T(u)M\dot{x}(u)\,du\,ds & \int_\beta^\alpha\int_s^\alpha \dot{x}^T(u)\,du\,ds \\ \int_\beta^\alpha\int_s^\alpha \dot{x}(u)\,du\,ds & \int_\beta^\alpha(\alpha-s)M^{-1}\,ds \end{bmatrix} \geq 0. \tag{12}$$

Therefore, the inequality (12) is equivalent to the inequality (9) according to Schur Complements. This completes the proof. □

Lemma 2 (*Skelton et al. [37]*). *Let $\zeta \in \mathbb{R}^n$, $\Phi=\Phi^T \in \mathbb{R}^{n\times n}$, and $B \in \mathbb{R}^{m\times n}$ such that rank$(B)<n$. Then, the following statements are equivalent*:

(1) $\zeta^T\Phi\zeta<0$, $B\zeta=0$, $\zeta \neq 0$,
(2) $(B^\perp)^T\Phi B^\perp<0$, *where $B^\perp$ is a right orthogonal complement of B.*

3. Main results

In this section, by use of augmented Lyapunov–Krasovskii functionals, new delay-dependent stability criteria for systems (4) will be proposed. For the sake of simplicity of matrix representation, e_i $(i=1,\ldots,12) \in \mathbb{R}^{12n\times n}$ are defined as block entry matrices. (For example, $e_3^T=[0,\ 0,\ I,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0,\ 0]$).

The notations for some matrices are defined as

$$\zeta(t)=\Big[x^T(t),\ x^T(t-h(t)),\ x^T(t-h_U),\ \dot{x}^T(t),\ \dot{x}^T(t-h_U),\ \int_{t-h(t)}^{t}x^T(s)\,ds,\ \int_{t-h_U}^{t-h(t)}x^T(s)\,ds,\ f^T(x(t)),\ f^T(x(t-h(t))),\ f^T(x(t-h_U)),\ \int_{t-h(t)}^{t}f^T(x(s))\,ds,\ \int_{t-h_U}^{t-h(t)}f^T(x(s))\,ds\Big]^T,$$

$$\alpha(t)=\left[x^T(t),\ \dot{x}^T(t),\ f^T(x(t))\right]^T,\ \beta(t)=\left[x^T(t),\ f^T(x(t))\right]^T,$$

$$\Pi_1=[e_1,\ e_3,\ e_6+e_7,\ e_{11}+e_{12}],$$
$$\Pi_2=[e_4,\ \ e_5,\ \ e_1-e_3,\ \ e_8-e_{10}],$$
$$\Pi_3=[e_1,\ e_4,\ e_8],\quad \Pi_4=[e_3,\ e_5,\ e_{10}],\quad \Pi_5=[e_1,\ e_8],$$
$$\Pi_6=[e_2,\ e_9],\quad \Pi_7=[e_6,\ e_1-e_2,\ e_{11},\ e_7,\ e_2-e_3\ \ e_{12}],$$
$$\Gamma=[-A,\ 0,\ 0,\ -I,\ 0,\ 0,\ 0,\ W_0,\ W_1,\ 0,\ 0,\ 0],$$
$$\Phi=e_8\Lambda e_4^T+e_4\Lambda e_8^T-e_1K_m\Lambda e_4^T-e_4\Lambda K_m e_1^T+e_1K_p\Delta e_4^T+e_4\Delta K_p e_1^T-e_8\Delta e_4^T-e_4\Delta e_8^T,$$
$$\Xi=(h_U^2/2)^2e_4Q_3e_4^T-(h_Ue_1-e_6-e_7)Q_3(h_Ue_1-e_6-e_7)^T,$$
$$\Psi=h_Ue_1Q_4e_1^T+h_Ue_4Q_5e_4^T+e_1P_1e_1^T+e_2(-P_1+P_2)e_2^T-e_3P_2e_3^T,$$
$$\begin{aligned}\Theta=&-2e_1K_mH_1K_pe_1^T+e_1(K_m+K_p)H_1e_8^T+e_8H_1(K_m+K_p)e_1^T\\&-2e_8H_1e_8^T-2e_2K_mH_2K_pe_2^T+e_2(K_m+K_p)H_2e_9^T\\&+e_9H_2(K_m+K_p)e_2^T-2e_9H_2e_9^T-2e_3K_mH_3K_pe_3^T\\&+e_3(K_m+K_p)H_3e_{10}^T+e_{10}H_3(K_m+K_p)e_3^T-2e_{10}H_3e_{10}^T,\end{aligned}$$
$$\begin{aligned}\Sigma_1=&\Phi+\Xi+\Psi+\Theta+\Pi_1\mathcal{R}\Pi_2^T+\Pi_2\mathcal{R}\Pi_1^T+\Pi_3\mathcal{N}\Pi_3^T-\Pi_4\mathcal{N}\Pi_4^T\\&+\Pi_5\mathcal{Q}\Pi_5^T-(1-h_D)\Pi_6\mathcal{Q}\Pi_6^T+h_U^2\Pi_3\mathcal{G}\Pi_3^T-\Pi_7\begin{bmatrix}\mathcal{G}&\mathcal{S}\\\star&\mathcal{G}\end{bmatrix}\Pi_7^T.\end{aligned}\quad(13)$$

Now, we have the following theorem.

Theorem 1. *For given scalars $h_U>0$ and h_D, diagonal matrices $K_p=diag\{k_1^+,\ldots,k_n^+\}$ and $K_m=diag\{k_1^-,\ldots,k_n^-\}$, the system (4) is asymptotically stable for $0\le h(t)\le h_U$ and $\dot{h}(t)\le h_D$ if there exist positive diagonal matrices $\Lambda=diag\{\lambda_1,\ldots,\lambda_n\}$, $\Delta=diag\{\delta_1,\ldots,\delta_n\}$, $H_i=diag\{h_{i1},\ldots,h_{in}\}$ $(i=1,2,3)$, positive definite matrices $\mathcal{R}=[R_{ij}]_{4\times4}\in\mathbb{R}^{4n\times4n}$, $\mathcal{N}=[N_{ij}]_{3\times3}\in\mathbb{R}^{3n\times3n}$, $\mathcal{Q}=[Q_{ij}]_{2\times2}\in\mathbb{R}^{2n\times2n}$, $\mathcal{G}=[G_{ij}]_{3\times3}\in\mathbb{R}^{3n\times3n}$, $Q_i(i=3,4,5)$, any symmetric matrices $P_i(i=1,2)$, and any matrix $\mathcal{S}=[S_{ij}]_{3\times3}\in\mathbb{R}^{3n\times3n}$, satisfying the following LMIs*:

$$(\Gamma^{\perp})^T\{\Sigma_1\}\Gamma^{\perp}<0,\quad(14)$$

$$\begin{bmatrix}\mathcal{G}&\mathcal{S}\\\star&\mathcal{G}\end{bmatrix}>0,\quad(15)$$

$$\begin{bmatrix}Q_4&P_1\\\star&Q_5\end{bmatrix}>0,\quad(16)$$

$$\begin{bmatrix}Q_4&P_2\\\star&Q_5\end{bmatrix}>0,\quad(17)$$

where Σ_1 and Γ are defined in (13), and $\Gamma^{\perp}$ is the right orthogonal complement of Γ.

Proof. For positive diagonal matrices Λ, Δ and positive definite matrices $\mathcal{R}$, $\mathcal{N}$, $\mathcal{Q}$, $\mathcal{G}$, Q_i $(i=3,4,5)$, let us consider the following Lyapunov–Krasovskii's functional candidate $V=\sum_{i=1}^{6}V_i$, where

$$V_1=\begin{bmatrix}x(t)\\x(t-h_U)\\\int_{t-h_U}^{t}x(s)\,ds\\\int_{t-h_U}^{t}f(x(s))\,ds\end{bmatrix}^T\mathcal{R}\begin{bmatrix}x(t)\\x(t-h_U)\\\int_{t-h_U}^{t}x(s)\,ds\\\int_{t-h_U}^{t}f(x(s))\,ds\end{bmatrix},$$

$$V_2=\int_{t-h_U}^{t}\alpha^T(s)\mathcal{N}\alpha(s)\,ds+2\sum_{i=1}^{n}\left(\lambda_i\int_0^{x_i(t)}(f_i(s)-k_i^-s)\,ds+\delta_i\int_0^{x_i(t)}(k_i^+s-f_i(s))\,ds\right),$$

$$V_3=\int_{t-h(t)}^{t}\beta(s)^T\mathcal{Q}\beta(s)\,ds,$$

$$V_4=h_U\int_{t-h_U}^{t}\int_s^t\alpha^T(u)\mathcal{G}\alpha(u)\,du\,ds,$$

$$V_5=(h_U^2/2)\int_{t-h_U}^{t}\int_s^t\int_u^t\dot{x}^T(v)Q_3\dot{x}(v)\,dv\,du\,ds,$$

$$V_6=\int_{t-h_U}^{t}\int_s^t x^T(u)Q_4x(u)\,du\,ds+\int_{t-h_U}^{t}\int_s^t\dot{x}^T(u)Q_5\dot{x}(u)\,du\,ds.\quad(18)$$

The time-derivative of V_1 is calculated as

$$\dot{V}_1=2\begin{bmatrix}x(t)\\x(t-h_U)\\\int_{t-h(t)}^{t}x(s)\,ds+\int_{t-h_U}^{t-h(t)}x(s)\,ds\\\int_{t-h(t)}^{t}f(x(s))\,ds+\int_{t-h_U}^{t-h(t)}f(x(s))\,ds\end{bmatrix}^T\mathcal{R}\begin{bmatrix}\dot{x}(t)\\\dot{x}(t-h_U)\\x(t)-x(t-h_U)\\f(x(t))-f(x(t-h_U))\end{bmatrix}$$
$$=\zeta^T(t)\,(\Pi_1\mathcal{R}\Pi_2^T+\Pi_2\mathcal{R}\Pi_1^T)\zeta(t).\quad(19)$$

By calculation of $\dot{V}_2$, we have

$$\begin{aligned}\dot{V}_2=&\alpha^T(t)\mathcal{N}\alpha(t)-\alpha^T(t-h_U)\mathcal{N}\alpha(t-h_U)+2\left[f(x(t))-K_mx(t)\right]^T\Lambda\dot{x}(t)\\&+2[K_px(t)-f(x(t))]^T\Delta\dot{x}(t)\\=&\zeta^T(t)(\Pi_3\mathcal{N}\Pi_3^T-\Pi_4\mathcal{N}\Pi_4^T+\Phi)\zeta(t).\end{aligned}\quad(20)$$

With the condition $\dot{h}(t)\le h_D$, an upper bound of V_3 is obtained as

$$\dot{V}_3\le\zeta^T(t)[\Pi_5\mathcal{Q}\Pi_5^T-(1-h_D)\Pi_6\mathcal{Q}\Pi_6^T]\zeta(t).\quad(21)$$

By use of Lemma 1 and Theorem 1 in [33], an estimation of $\dot{V}_4$ is

$$\begin{aligned}\dot{V}_4=&h_U^2\alpha^T(t)\mathcal{G}\alpha(t)-h_U\int_{t-h(t)}^{t}\alpha^T(s)\mathcal{G}\alpha(s)\,ds-h_U\int_{t-h_U}^{t-h(t)}\alpha^T(s)\mathcal{G}\alpha(s)\,ds\\\le&h_U^2\alpha^T(t)\mathcal{G}\alpha(t)-\left(\frac{h_U}{h(t)}\right)\left(\int_{t-h(t)}^{t}\alpha(s)\,ds\right)^T\mathcal{G}\left(\int_{t-h(t)}^{t}\alpha(s)\,ds\right)\\&-\left(\frac{h_U}{h_U-h(t)}\right)\left(\int_{t-h_U}^{t-h(t)}\alpha(s)\,ds\right)^T\mathcal{G}\left(\int_{t-h_U}^{t-h(t)}\alpha(s)\,ds\right)\\\le&h_U^2\alpha^T(t)\mathcal{G}\alpha(t)-\begin{bmatrix}\int_{t-h(t)}^{t}\alpha(s)\,ds\\\int_{t-h_U}^{t-h(t)}\alpha(s)\,ds\end{bmatrix}^T\begin{bmatrix}\mathcal{G}&\mathcal{S}\\\star&\mathcal{G}\end{bmatrix}\begin{bmatrix}\int_{t-h(t)}^{t}\alpha(s)\,ds\\\int_{t-h_U}^{t-h(t)}\alpha(s)\,ds\end{bmatrix}\\=&\zeta^T(t)\left\{h_U^2\Pi_3\mathcal{G}\Pi_3^T-\Pi_7\begin{bmatrix}\mathcal{G}&\mathcal{S}\\\star&\mathcal{G}\end{bmatrix}\Pi_7^T\right\}\zeta(t).\end{aligned}\quad(22)$$

For the detailed proof of Eq. (22), see [38].

By Lemma 1, $\dot{V}_5$ is bounded as

$$\begin{aligned}\dot{V}_5=&(h_U^2/2)^2\dot{x}^T(t)Q_3\dot{x}(t)-(h_U^2/2)\int_{t-h_U}^{t}\int_s^t\dot{x}^T(u)Q_3\dot{x}(u)\,du\,ds\\\le&(h_U^2/2)^2\dot{x}^T(t)Q_3\dot{x}(t)-\left(\int_{t-h_U}^{t}\int_s^t\dot{x}(u)\,du\,ds\right)^TQ_3\times\left(\int_{t-h_U}^{t}\int_s^t\dot{x}(u)\,du\,ds\right)\\=&\zeta^T(t)\Xi\zeta(t).\end{aligned}\quad(23)$$

Calculation of $\dot{V}_6$ leads to

$$\begin{aligned}\dot{V}_6=&h_Ux^T(t)Q_4x(t)-\int_{t-h_U}^{t}x^T(s)Q_4x(s)\,ds\\&+h_U\dot{x}^T(t)Q_5\dot{x}(t)-\int_{t-h_U}^{t}\dot{x}^T(s)Q_5\dot{x}(s)\,ds.\end{aligned}\quad(24)$$

Inspired by the work of [34], the following two zero equalities with any symmetric matrices P_1 and P_2 are considered:

$$0 = x^T(t)P_1x(t) - x^T(t-h(t))P_1x(t-h(t)) - 2\int_{t-h(t)}^{t} x^T(s)P_1\dot{x}(s)\,ds,$$
$$0 = x^T(t-h(t))P_2x(t-h(t)) - x^T(t-h_U)P_2x(t-h_U) - 2\int_{t-h_U}^{t-h(t)} x^T(s)P_2\dot{x}(s)\,ds. \quad (25)$$

With the above two zero equalities, an upper bound of $\dot{V}_6$ is

$$\dot{V}_6 \le \zeta^T(t)\Psi\zeta(t) - \int_{t-h(t)}^{t}\begin{bmatrix} x(s)\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix} Q_4 & P_1\\ \star & Q_5\end{bmatrix}\begin{bmatrix} x(s)\\ \dot{x}(s)\end{bmatrix}ds - \int_{t-h_U}^{t-h(t)}\begin{bmatrix} x(s)\\ \dot{x}(s)\end{bmatrix}^T\begin{bmatrix} Q_4 & P_2\\ \star & Q_5\end{bmatrix}\begin{bmatrix} x(s)\\ \dot{x}(s)\end{bmatrix}ds. \quad (26)$$

From (7), for any positive diagonal matrices $H_1 = diag\{h_{11},\dots,h_{1n}\}$, $H_2 = diag\{h_{21},\dots,h_{2n}\}$, and $H_3 = diag\{h_{31},\dots,h_{3n}\}$, the following inequality holds:

$$\begin{aligned}0 \le &-2\sum_{i=1}^{n} h_{1i}[f_i(x_i(t)) - k_i^- x_i(t)][f_i(x_i(t)) - k_i^+ x_i(t)]\\ &-2\sum_{i=1}^{n} h_{2i}[f_i(x_i(t-h(t))) - k_i^- x_i(t-h(t))][f_i(x_i(t-h(t)))\\ &-k_i^+ x_i(t-h(t))]\\ &-2\sum_{i=1}^{n} h_{3i}[f_i(x_i(t-h_U)) - k_i^- x_i(t-h_U)][f_i(x_i(t-h_U)) - k_i^+ x_i(t-h_U)]\\ &= \zeta^T(t)\Theta\zeta(t).\end{aligned} \quad (27)$$

From Eqs. (18)–(27) and by application of S-procedure [36], if Eqs. (16) and (17) hold, then an upper bound of $\dot{V}$ is

$$\dot{V} \le \zeta^T(t)\Sigma_1\zeta(t), \quad (28)$$

where Σ_1 are defined in (13).

By Lemma 2, $\zeta^T(t)\Sigma_1\zeta(t) < 0$ with $0 = \Gamma\zeta(t)$ is equivalent to $(\Gamma^\perp)^T\Sigma_1\Gamma^\perp < 0$. Therefore, if LMIs (14)–(17) hold, then the neural networks (4) is asymptotically stable. This completes the proof. □

Remark 1. In Theorem 1, the augmented vector $\zeta(t)$ has integrating terms of activation function $f(x(t))$ which are $\int_{t-h(t)}^{t} f(x(s))\,ds$ and $\int_{t-h_U}^{t-h(t)} f(x(s))\,ds$. By these terms, more past history of $f(x(t))$ can be utilized, which may lead less conservative results.

Remark 2. Recently, the reciprocally convex optimization technique to reduce the conservatism of stability criteria for systems with time-varying delays was proposed in [33]. Motivated by this work, the proposed method of [33] was utilized in Eq. (22). However, an augmented vector with $\int_{t-h(t)}^{t} x(s)\,ds$, $\int_{t-h_U}^{t-h(t)} x(s)\,ds$, $\int_{t-h(t)}^{t} f(x(s))\,ds$, $\int_{t-h_U}^{t-h(t)} f(x(s))\,ds$ was used, which is different from the method of [33].

Next, based on the results of Theorem 1, a further improved stability criterion for system (1) will be introduced as Theorem 2 by utilizing new activation condition which has not been proposed yet.

Theorem 2. *For given scalars $h_U > 0$ and h_D, diagonal matrices $K_p = diag\{k_1^+,\dots,k_n^+\}$ and $K_m = diag\{k_1^-,\dots,k_n^-\}$, the system (4) is asymptotically stable for $0 \le h(t) \le h_U$ and $\dot{h}(t) \le h_D$ if there exist positive diagonal matrices $\Lambda = diag\{\lambda_1,\dots,\lambda_n\}$, $\Delta = diag\{\delta_1,\dots,\delta_n\}$, $H_i = diag\{h_{i1},\dots,h_{in}\}$ $(i = 1,\dots,5)$, positive definite matrices $\mathcal{R} = [R_{ij}]_{4\times4} \in \mathbb{R}^{4n\times4n}$, $\mathcal{N} = [N_{ij}]_{3\times3} \in \mathbb{R}^{3n\times3n}$, $\mathcal{Q} = [Q_{ij}]_{2\times2} \in \mathbb{R}^{2n\times2n}$, $\mathcal{G} = [G_{ij}]_{3\times3} \in \mathbb{R}^{3n\times3n}$, Q_i $(i = 3,4,5)$, any symmetric matrices P_i $(i = 1,2)$, and any matrix $\mathcal{S} = [S_{ij}]_{3\times3} \in \mathbb{R}^{3n\times3n}$, satisfying the following LMIs*:

$$(\Gamma^\perp)^T\{\Sigma_1 + \Omega\}\Gamma^\perp < 0, \quad (29)$$

$$\begin{bmatrix}\mathcal{G} & \mathcal{S}\\ \star & \mathcal{G}\end{bmatrix} > 0, \quad (30)$$

$$\begin{bmatrix}Q_4 & P_1\\ \star & Q_5\end{bmatrix} > 0, \quad (31)$$

$$\begin{bmatrix}Q_4 & P_2\\ \star & Q_5\end{bmatrix} > 0, \quad (32)$$

where Σ_1, Γ are defined in (13), $\Gamma^\perp$ is the right orthogonal complement of Γ, and Ω is defined as

$$\begin{aligned}\Omega = &-[e_8 - e_9 - (e_1 - e_2)K_m]H_4[e_8 - e_9 - (e_1 - e_2)K_p]^T\\ &-[e_8 - e_9 - (e_1 - e_2)K_p]H_4[e_8 - e_9 - (e_1 - e_2)K_m]^T\\ &-[e_9 - e_{10} - (e_2 - e_3)K_m]H_5[e_9 - e_{10} - (e_2 - e_3)K_p]^T\\ &-[e_9 - e_{10} - (e_2 - e_3)(K_p)]H_5[e_9 - e_{10} - (e_2 - e_3)K_m]^T.\end{aligned} \quad (33)$$

Proof. From (5), the following conditions hold:

$$k_i^- \le \frac{f_i(x_i(t)) - f_i(x_i(t-h(t)))}{x_i(t) - x_i(t-h(t))} \le k_i^+,$$
$$k_i^- \le \frac{f_i(x_i(t-h(t))) - f_j(x_i(t-h_U))}{x_i(t-h(t)) - x_i(t-h_U)} \le k_i^+,$$
$$i = 1,\dots,n. \quad (34)$$

For $i = 1,\dots,n$, the above two conditions are equivalent to

$$[f_i(x_i(t)) - f_i(x_i(t-h(t))) - k_i^-(x_i(t) - x_i(t-h(t)))] \times [f_i(x_i(t)) - f_i(x_i(t-h(t))) - k_i^+(x_i(t) - x_i(t-h(t)))] \le 0, \quad (35)$$

$$[f_i(x_i(t-h(t))) - f_i(x_i(t-h_U)) - k_i^-(x_i(t-h(t)) - x_i(t-h_U))] \times [f_i(x_i(t-h(t))) - f_i(x_i(t-h_U)) - k_i^+(x_i(t-h(t)) - x_i(t-h_U))] \le 0. \quad (36)$$

Therefore, for any positive diagonal matrices $H_4 = diag\{h_{4i},\dots,h_{4n}\}$ and $H_5 = diag\{h_{5i},\dots,h_{5n}\}$, the following inequality holds

$$\begin{aligned}0 \le &-2\sum_{i=1}^{n}\{h_{4i}[f_i(x_i(t)) - f_i(x_i(t-h(t))) - k_i^-(x_i(t) - x_i(t-h(t)))]\\ &\times[f_i(x_i(t)) - f_i(x_i(t-h(t))) - k_i^+(x_i(t) - x_i(t-h(t)))]\}\\ &-2\sum_{i=1}^{n}\{h_{5i}[f_i(x_i(t-h(t))) - f_i(x_i(t-h_U)) - k_i^-(x_i(t-h(t)) - x_i(t-h_U))]\\ &\times[f_i(x_i(t-h(t))) - f_i(x_i(t-h_U)) - k_i^+(x_i(t-h(t)) - x_i(t-h_U))]\}\\ &= \zeta^T(t)\Omega\zeta(t).\end{aligned} \quad (37)$$

By the consideration of Eq. (37) and the same Lyapunov–Krasovskii's functional (18), the other procedure is straightforward from the proof of Theorem 1, so we omit it. □

Remark 3. In many works [16–30], the condition of (6) was utilized to reduce the conservatism of stability criteria. However, the condition of Eq. (37) in Theorem 2 is proposed for the first time in this work. Through two numerical examples in checking the conservatism of delay-dependent stability criteria for system (4), it will be shown that the newly proposed activation condition significantly improves the feasible region of stability criteria by comparing maximum delay bounds which are one of important index for checking the conservatism of stability criteria.

Remark 4. When the information of an upper bound of $\dot{h}(t)$ is unknown or larger than one, Theorems 1 and 2 also provide delay-dependent stability criteria for (4) by letting $\mathcal{Q} = 0$.

4. Numerical examples

In this section, two numerical examples which utilized in other works to check the conservatism of stability criteria are utilized to show the improvements on the feasible regions of the proposed stability criteria. To do this, MATLAB, YALMIP 3.0 and SeDuMi 1.3 are used to solve LMI problems.

Example 1. Consider the neural networks (4) with the parameters

$$A=\begin{bmatrix}2 & 0\\ 0 & 2\end{bmatrix},\quad W_0=\begin{bmatrix}1 & 1\\ -1 & -1\end{bmatrix},\quad W_1=\begin{bmatrix}0.88 & 1\\ 1 & 1\end{bmatrix},$$
$$K_p=diag\{0.4,0.8\},\quad K_m=diag\{0,0\}. \tag{38}$$

For this system, by dividing delay interval into two and employing different free-weighting matrices at each interval, improved maximum delay bounds were obtained in [24,26] when h_D is 0.8, 0.9, and unknown (or larger than one). In [29,30], delay-partitioning techniques when delay-partitioning number is two were applied to obtain maximum delay bounds for the above system. By application of Theorems 1 and 2, our delay bounds are obtained and the detail comparison of our results with existing ones [24,26,29,30] is given in Table 1. From Table 1, one can see that Theorem 1 enhances the feasible region of stability criteria in spite of not utilizing the delay-partitioning technique. Furthermore, Theorem 2 provides larger delay bounds than that of Theorem 1, which shows the effectiveness of the proposed idea in Theorem 2 to reduce the conservatism of stability criteria. To confirm the obtained result, h_U=2.8222 for unknown h_D, a simulation result when $x(0)=[1,-1]^T$, $f(x(t))=[0.4\tanh(x_1(t)),0.8\tanh(x_2(t))]^T$, and $h(t)=2.8222|\sin(t)|$ is shown in Fig. 1. From Fig. 1, one can see that the state responses converge to zero as time gets larger.

Example 2. Consider the neural networks (4) where

$$A=\begin{bmatrix}1.2769 & 0 & 0 & 0\\ 0 & 0.6231 & 0 & 0\\ 0 & 0 & 0.9230 & 0\\ 0 & 0 & 0 & 0.4480\end{bmatrix},$$
$$W_0=\begin{bmatrix}-0.0373 & 0.4852 & -0.3351 & 0.2336\\ -1.6033 & 0.5988 & -0.3224 & 1.2352\\ 0.3394 & -0.0860 & -0.3824 & -0.5785\\ -0.1311 & 0.3253 & -0.9534 & -0.5015\end{bmatrix},$$
$$W_1=\begin{bmatrix}0.8674 & -1.2405 & -0.5325 & 0.0220\\ 0.0474 & -0.9164 & 0.0360 & 0.9816\\ 1.8495 & 2.6117 & -0.3788 & 0.8428\\ -2.0413 & 0.5179 & 1.1734 & -0.2775\end{bmatrix},$$
$$K_p=diag\{0.1137,\ 0.1279,\ 0.7994,\ 0.2368\},$$
$$K_m=diag\{0,\ 0,\ 0,\ 0\}. \tag{39}$$

Table 1
Delay bounds h_U with different h_D (Example 1).

h_D	0.8	0.9	Unknown (or $h_D\geq 1$)
[24] (m=2)[a]	2.8634	1.9508	1.7809
[26] (m=2)[a]	2.8854	1.9631	1.7810
[29] (m=2)[a]	3.1150	2.1153	1.3189
[30] (m=2)[a]	3.2113	2.2172	1.3718
Theorem 1	3.7174	2.6871	2.2975
Theorem 2	3.7174	2.8339	2.8222

[a] m is a delay-partitioning number.

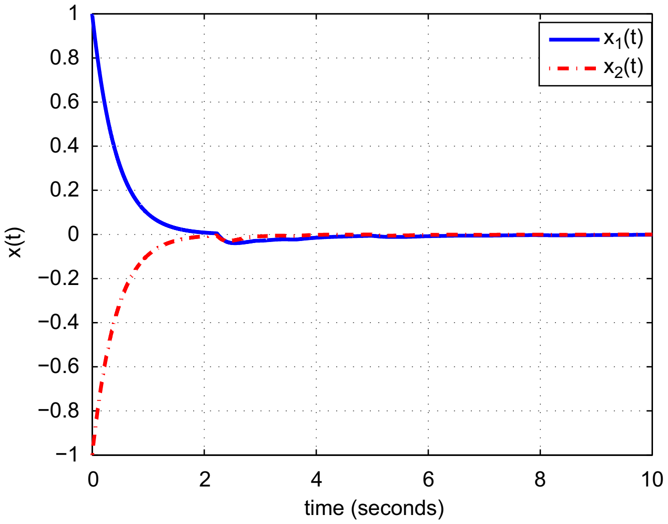

Fig. 1. State responses of system considered in Example 1.

Table 2
Delay bounds h_U with different h_D (Example 2).

h_D	0.1	0.5	0.9	Unknown (or $h_D\geq 1$)
[28] ($\rho=0.6$)	3.3574	2.5915	2.1306	2.0779
[27] (m=2)[a]	3.5546	2.6438	2.1349	–
[26] (m=2)[a]	3.7525	2.7353	2.2760	2.1326
Theorem 1	3.7024	2.8589	2.3473	2.2106
Theorem 2	3.7857	3.0546	2.6703	2.6575

[a] m is a delay-partitioning number.

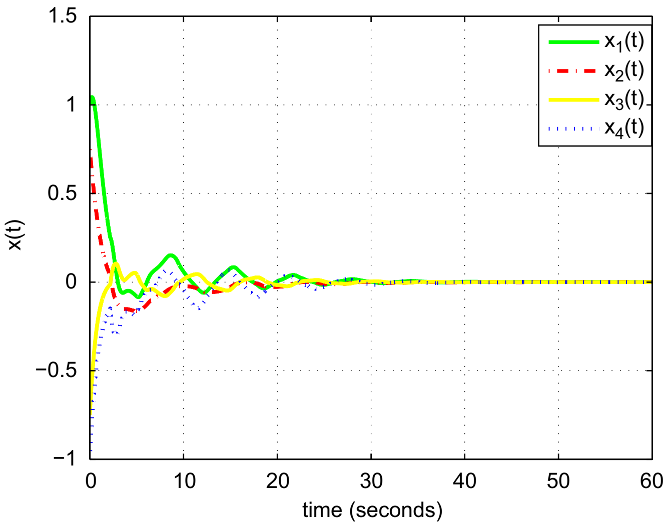

Fig. 2. State responses of system considered in Example 2.

Table 2 gives the comparison results on the maximum delay bound allowed via the methods in recent works and our new study. From Table 2, it can be seen that Theorem 2 gives larger delay bounds than very recent results in [26–28]. To confirm the obtained result, h_U=2.6575 for unknown h_D, a simulation result when $x(0)=[1,0.75,-0.75,-1]^T$, $f(x(t))=[0.1137\tanh(x_1(t)),\ 0.1279\tanh(x_2(t)),\ 0.7994\tanh(x_3(t)),\ 0.2368\tanh(x_4(t))]^T$, and $h(t)=2.6575|\sin(t)|$ is shown in Fig. 2. From Fig. 2, one can see that the state responses also converge to zero as time gets larger.

5. Conclusions

In this paper, two delay-dependent stability criteria for neural networks with time-varying delays have been proposed by the use of the Lyapunov method and the LMI framework. In Theorem 1, by construction of the augmented Lyapunov–Krasovskii functional, the improved delay-dependent stability criterion has been proposed without use of delay-partitioning techniques. And, with new inequalities of activation functions, the further improved stability criterion was proposed in Theorem 2. Through two numerical examples, the improvement of the proposed stability criteria has been successfully verified.

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