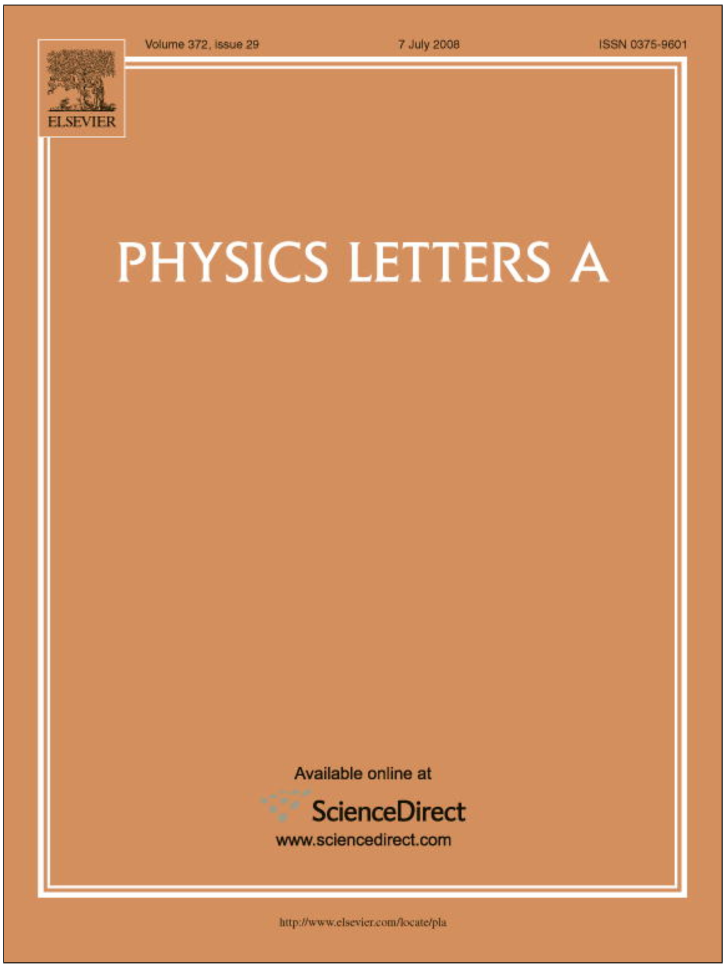
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$\mathcal{H}_\infty$ synchronization of chaotic systems via dynamic feedback approach

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ABSTRACT

This Letter considers $\mathcal{H}_\infty$ synchronization of a general class of chaotic systems with external disturbance. Based on Lyapunov theory and linear matrix inequality (LMI) formulation, the novel feedback controller is established to not only guarantee stable synchronization of both master and slave systems but also reduce the effect of external disturbance to an $\mathcal{H}_\infty$ norm constraint. A dynamic feedback control scheme is proposed for $\mathcal{H}_\infty$ synchronization in chaotic systems for the first time. Then, a criterion for existence of the controller is given in terms of LMIs. Finally, a numerical simulation is presented to show the effectiveness of the proposed chaos synchronization scheme.

1. Introduction

Chaos is very interesting nonlinear phenomenon and has extensive applications in many areas. In particular, chaos synchronization, first proposed by Fujisaka and Yamada in 1983 [1], did not received great attention until 1990 [2]. From then on, chaos synchronization has been developed extensively due to its various applications such as biology, economics, signal generator design, secure communication, and so on [3–5].

The idea of synchronization is to use the output of the master system to control the slave system so that the output of the response system follows the output of the master system asymptotically. Based on control theory, a number of synchronization schemes such as variable structure control [6,7], observer-based control [8], time-delay feedback approach [9], back-stepping design technique [10], active control [11], parameters adaptive control [12,13], nonlinear control [14–17] have been proposed in the literature.

On the other hand, some noise or disturbances always exist in real systems that may cause instability and poor performance. Therefore, the effect of the noises or disturbances must be also reduced in synchronization process for chaotic systems. In this regards, recently, Y.-Y. Hou et al. [18] firstly adopted the $\mathcal{H}_\infty$ control concept [19,20] to reduce the effect of the disturbance for chaotic synchronization problem of a general class of chaotic systems based on Lyapunov theory and linear matrix inequality (LMI) optimization technique under linear matrix equality (LME) formulation. In their work, a design method for static output feedback controller was proposed to guarantee $\mathcal{H}_\infty$ synchronization between the master and slave systems.

In this Letter, the problem of $\mathcal{H}_\infty$ chaos synchronization to general chaotic system with disturbance is considered. A new stabilizing controller for the synchronization between master and slave systems is designed. The controller consists of two parts. One is the linear dynamic feedback controller, the other is the nonlinear static feedback controller. By the control scheme, the closed-loop error system is asymptotically stable and the $\mathcal{H}_\infty$-norm from the disturbance to controlled output is reduced to a prescribed level. Based on the Lyapunov method and LMI framework, an existence criterion for such controller is represented in terms of LMIs. The LMIs can be easily solved by various convex optimization algorithms developed recently [21].

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The organization of this Letter is as follows. In Sections 2 and 3, the problem statement and drive-response synchronization scheme are presented for a class of general chaotic system with external disturbance. In Section 4, a numerical example is given to demonstrate the effectiveness of the proposed method. Finally, a conclusion is given.

Notation. $\mathcal{R}^n$ denotes the n-dimensional Euclidean space, and $\mathcal{R}^{m\times n}$ is the set of $m \times n$ real matrix. $|\cdot|$ represents the absolute value. $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \geqslant Y$) means that the matrix $X - Y$ is positive definite, (respectively, nonnegative). diag$\{\cdots\}$ denotes the block diagonal matrix. $\star$ represents the elements below the main diagonal of a symmetric matrix. I denotes the identity matrix with appropriate dimension. A^T means the transpose of the matrix A. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the largest and smallest eigenvalue of A, respectively.

2. Problem statement and preliminary

Consider a class of chaotic systems described by the nonlinear differential equation as follows:

$$\dot{x}(t) = Ax(t) + \bar{A}f\big(x(t)\big), \qquad y(t) = Cx(t) \tag{1}$$

where $x \in \mathcal{R}^n$ is the state variable, $y \in \mathcal{R}^q$ is the output, the matrices $A, \bar{A} \in \mathcal{R}^{n\times n}$ and $C \in \mathcal{R}^{q\times n}$ are known constant matrices, and $f(x(t)) \in \mathcal{R}^n$ is a nonlinear function vector satisfying the global Lipschitz condition: i.e.,

$$\big\|f(x_1) - f(x_2)\big\| \leqslant \delta\|x_1 - x_2\| \quad \forall x_1, x_2 \in \mathcal{R}^n \tag{2}$$

for some positive scalar δ.

The synchronization problem of system (1) is considered using the drive-response configuration. This is, if the system (1) is regarded as the drive system, a suitable response system with control input should be constructed to synchronize the drive system. According to the above drive-response concept, unidirectionally coupled chaotic systems can be described by the following equations:

$$\dot{x}_m(t) = Ax_m(t) + \bar{A}f\big(x_m(t)\big), \qquad y_m(t) = Cx_m(t) \tag{3}$$

and

$$\dot{x}_s(t) = Ax_s(t) + \bar{A}f\big(x_s(t)\big) + Bu(t) + Dw(t) + \alpha(t), \qquad y_s(t) = Cx_s(t) \tag{4}$$

where $x_m(t)$, $x_s(t) \in \mathcal{R}^n$ are the state vectors of master system and slave system, respectively, B, C and D are constant matrices with appropriate dimensions, $w(t) \in \mathcal{R}^l$ is the disturbance, $y_m(t)$ and $y_s(t)$ are the outputs of the master and the slave system, respectively, $u(t)$ is a unidirectionally coupled term, which is regarded as the main control input and will be appropriately designed such that the specific control objective is achieved, and $\alpha(t)$ is secondary control signal.

Define the synchronization error as

$$e(t) = x_s(t) - x_m(t). \tag{5}$$

Then, the dynamics of synchronization error between the master and slave systems given in Eqs. (3)–(4) can be described by

$$\dot{e}(t) = Ae(t) + \bar{A}\big(f\big(x_s(t)\big) - f\big(x_m(t)\big)\big) + Bu(t) + Dw(t), \qquad y_e(t) = Ce(t) \tag{6}$$

where $y_e(t) = y_s(t) - y_m(t)$.

Next, in order to $\mathcal{H}_\infty$ synchronize between drive system (3) and response one (4), let us consider the following main dynamic feedback controller:

$$\dot{\zeta}(t) = A_c\zeta(t) + B_ce(t), \qquad u(t) = C_c\zeta(t), \qquad \zeta(0) = 0, \tag{7}$$

where $\zeta(t) \in \mathcal{R}^n$ is the controller state, and A_c, B_c and C_c are gain matrices with appropriate dimensions to be determined later. Applying this controller (7) to system (6) results in the closed-loop system

$$\dot{\bar{x}}(t) = \Sigma_1\bar{x}(t) + \Sigma_2 + \bar{D}w(t) + \bar{\alpha}(t) \tag{8}$$

where

$$\bar{x}(t) = \begin{bmatrix} e(t) \\ \zeta(t) \end{bmatrix}, \qquad \Sigma_1 = \begin{bmatrix} A & BC_c \\ B_c & A_c \end{bmatrix}, \qquad \Sigma_2 = \begin{bmatrix} \bar{A}(f(x_s(t)) - f(x_m(t))) \\ 0 \end{bmatrix},$$
$$\bar{\alpha}(t) = \begin{bmatrix} \alpha(t) \\ 0 \end{bmatrix}, \qquad \bar{D} = \begin{bmatrix} D \\ 0 \end{bmatrix}. \tag{9}$$

Definition (*$\mathcal{H}_\infty$ synchronization*). (See [19].) The synchronization error systems (3) is $\mathcal{H}_\infty$ synchronization with the disturbance attenuation γ if the following conditions are satisfied:

- With zero disturbance, the synchronization error systems (3) with control input $u(t)$ is exponentially stable.
- With zero initial condition and a given constant $\gamma > 0$, the following condition holds:

$$J = \int_0^\infty \big[y_e^T(t)y_e(t) - \gamma^2 w^T(t)w(t)\big]\,dt \leqslant 0 \quad \left(\text{i.e.} \sup_{w\neq 0,\, w\in L_2[0,\infty]} \frac{\|y_e(t)\|_2}{\|w(t)\|_2} \leqslant \gamma\right). \tag{10}$$

Then, the controller $u(t) + \alpha(t)$ is said to be the $\mathcal{H}_\infty$ synchronization controller with the disturbance attenuation γ. The parameter γ is called the $\mathcal{H}_\infty$-norm bound of this controller.

3. Main result

First, let us design the secondary feedback controller $\alpha(t)$ before deriving main result for $\mathcal{H}_\infty$ synchronization.

In order to design the control input $\alpha(t)$, define a positive-definite matrix $P \in \mathcal{R}^{2n\times 2n}$ and its inverse, which plays an important role in our analysis:

$$P = \begin{bmatrix} S & N \\ N^T & U \end{bmatrix}, \qquad P^{-1} = \begin{bmatrix} Y & M \\ M^T & W \end{bmatrix} \tag{11}$$

where $S \in \mathcal{R}^{n\times n}$ and $Y \in \mathcal{R}^{n\times n}$ are positive definite matrices, and $M \in \mathcal{R}^{n\times n}$ and $N \in \mathcal{R}^{n\times n}$ are invertible matrices.

Now, we propose the control signal $\alpha(t)$ as

$$\alpha(t) = -\frac{\delta^2\lambda_{\max}^{1/2}(\bar{A}^T\bar{A})(Se(t)+N\zeta(t))|e(t)|}{\|Se(t)+N\zeta(t)\|}, \tag{12}$$

where δ is the Lipschitz condition parameter defined in Eq. (2).

The main result for achieving $\mathcal{H}_\infty$ synchronization is stated in the following theorem.

Theorem 1. *For given η and γ, there exist a main dynamic controller (7) and secondary control law (12) for the error system (6) if there exist any matrices $\hat{A}$, $\hat{B}$, $\hat{C}$, and positive-definite matrices S and Y satisfying the following LMIs*:

$$\Phi = \begin{bmatrix} \Phi_1 & \Phi_2 & D & Y^TC^T \\ \star & \Phi_3 & S^TD & C^T \\ \star & \star & -\gamma^2 I & 0 \\ \star & \star & \star & -I \end{bmatrix} < 0 \tag{13}$$

and

$$\begin{bmatrix} Y & I \\ I & S \end{bmatrix} > 0 \tag{14}$$

where

$$\begin{aligned} \Phi_1 &= AY + YA^T + B\hat{C} + \hat{C}^TB^T + \eta Y, \\ \Phi_2 &= A + \hat{A}^T + \eta I, \\ \Phi_3 &= SA + A^TS + \hat{B} + \hat{B}^T + \eta S. \end{aligned} \tag{15}$$

Then, the $\mathcal{H}_\infty$ synchronization with the disturbance attenuation γ is obtained by control laws (7) and (12).

Proof. Let us consider the following Lyapunov function:

$$V = \bar{x}^T(t)P\bar{x}(t). \tag{16}$$

Taking the time derivative of V along the solution of (8), we have

$$\begin{aligned} \dot{V} &= \dot{\bar{x}}^T(t)P\bar{x}(t) + \bar{x}^T(t)P\dot{\bar{x}}(t) \\ &= \bar{x}^T(t)\big(\Sigma_1^TP + P\Sigma_1\big)\bar{x}(t) + 2\bar{x}^T(t)P\bar{D}w(t) + 2\bar{x}^T(t)P\Sigma_2 + 2\bar{x}^T(t)P\bar{\alpha}(t) \\ &= \bar{x}^T(t)\big(\Sigma_1^TP + P\Sigma_1\big)\bar{x}(t) + 2\bar{x}^T(t)P\bar{D}w(t) + 2\big(Se(t)+N\zeta(t)\big)^T\bar{A}\big(f\big(x_s(t)\big) - f\big(x_m(t)\big)\big) + 2\big(Se(t)+N\zeta(t)\big)^T\alpha(t) \\ &\leqslant \bar{x}^T(t)\big(\Sigma_1^TP + P\Sigma_1\big)\bar{x}(t) + 2\bar{x}^T(t)P\bar{D}w(t) \\ &\quad + 2\big\|\big(Se(t)+N\zeta(t)\big)^T\big\| \cdot \big\|\bar{A}\big(f\big(x_s(t)\big) - f\big(x_m(t)\big)\big)\big\| + 2\big(Se(t)+N\zeta(t)\big)^T\alpha(t). \end{aligned} \tag{17}$$

Using global Lipschitz condition (2) and the definition of matrix norm gives that

$$\dot{V} \leqslant \bar{x}^T(t)\big(\Sigma_1^TP + P\Sigma_1\big)\bar{x}(t) + 2\bar{x}^T(t)P\bar{D}w(t) + 2\delta^2\lambda_{\max}^{1/2}\big(\bar{A}^T\bar{A}\big) \cdot \big\|\big(Se(t)+N\zeta(t)\big)^T\big\| \cdot \big|e(t)\big| + 2\big(Se(t)+N\zeta(t)\big)^T\alpha(t). \tag{18}$$

Then, substituting the control law (12) into Eq. (18) gives that

$$\begin{aligned} \dot{V} &\leqslant \bar{x}^T(t)\big(\Sigma_1^TP + P\Sigma_1\big)\bar{x}(t) + 2\bar{x}^T(t)P\bar{D}w(t) + 2\delta^2\lambda_{\max}^{1/2}\big(\bar{A}^T\bar{A}\big)\big\|\big(Se(t)+N\zeta(t)\big)^T\big\| \cdot \big|e(t)\big| \\ &\quad - 2\big(Se(t)+N\zeta(t)\big)^T\frac{\delta^2\lambda_{\max}^{1/2}(\bar{A}^T\bar{A}) \cdot (Se(t)+N\zeta(t))|e(t)|}{\|Se(t)+N\zeta(t)\|} \\ &= \bar{x}^T(t)\big(\Sigma_1^TP + P\Sigma_1\big)\bar{x}(t) + 2\bar{x}^T(t)P\bar{D}w(t). \end{aligned} \tag{19}$$

Thus, if the inequality, $\bar{x}^T(t)(\Sigma_1^TP + P\Sigma_1)\bar{x}(t) + 2\bar{x}^T(t)P\bar{D}w(t) \leqslant 0$, holds, i.e.,

$$\begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \Sigma_1^TP + P\Sigma_1 & P\bar{D} \\ \bar{D}^TP & 0 \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix} \leqslant 0, \tag{20}$$

we have $\dot{V} \leqslant 0$.

Define a function $J(\bar{x}(t), w(t))$ as follows:

$$J(\bar{x}(t), w(t)) = \dot{V} + y_e^T(t)y_e(t) - \gamma^2 w^T(t)w(t). \tag{21}$$

Substituting (19) into (21) yields

$$J(\bar{x}(t), w(t)) = \dot{V} + y_e^T(t)y_e(t) - \gamma^2 w^T(t)w(t) \leqslant \begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix}^T \Theta \begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix} \tag{22}$$

where

$$\Theta = \begin{bmatrix} \Sigma_1^T P + P\Sigma_1 + \bar{C} & P\bar{D} \\ \bar{D}^T P & -\gamma^2 I \end{bmatrix}, \qquad \bar{C} = \begin{bmatrix} C^T C & 0 \\ 0 & 0 \end{bmatrix}. \tag{23}$$

If the matrix $P^T = P > 0$ and a constant $\eta > 0$ satisfy the following condition:

$$\bar{\Theta} = \Theta + \begin{bmatrix} \eta P & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \Sigma_1^T P + P\Sigma_1 + \bar{C} + \eta P & P\bar{D} \\ \bar{D}^T P & -\gamma^2 I \end{bmatrix} < 0, \tag{24}$$

then, we have

$$J(\bar{x}(t), w(t)) \leqslant \begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix}^T \Theta \begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix} = \begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix}^T \left(\bar{\Theta} - \begin{bmatrix} \eta P & 0 \\ 0 & 0 \end{bmatrix}\right) \begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix} < -\begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix}^T \begin{bmatrix} \eta P & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \bar{x}(t) \\ w(t) \end{bmatrix}. \tag{25}$$

From Eq. (25), we can easily obtain that

$$\dot{V}|_{w(t)=0} < -\eta\lambda_{\min}(P)\left\|\bar{x}(t)\right\|^2 < 0 \quad \text{for all } \bar{x}(t) \neq 0. \tag{26}$$

Based on Lyapunov stability theory, the synchronization error system (6) with the dynamic controller $u(t)$ and secondary controller $\alpha(t)$ is exponentially stable for $w(t) = 0$.

Integrating the function in (25) from 0 to ∞, we have

$$V(\infty) - V(0) + \int_0^\infty \left(\|y_e(t)\|_2^2 - \gamma^2\|w(t)\|_2^2\right) dt \leqslant 0. \tag{27}$$

With zero initial condition, we have

$$\int_0^\infty \left(\|y_e(t)\|_2^2 - \gamma^2\|w(t)\|_2^2\right) dt \leqslant 0. \tag{28}$$

By the definition, $\mathcal{H}_\infty$ synchronization with the disturbance attenuation γ is obtained by the controller $u(t) + \alpha(t)$.

However, it is not easy to solve the sufficient condition (24) and find the control parameter since it is not a standard LMI form. In order to find the controller parameters A_c, B_c and C_c, which included in the matrix Σ_1, are unknown and occur in nonlinear fashion. Thus, we will use a method of changing variables such that the inequality can be solved as convex optimization algorithms. First, from the relationship (11), the equality $P^{-1}P = I$ gives that

$$MN^T = I - YS. \tag{29}$$

Define

$$\Psi_1 = \begin{bmatrix} Y & I \\ M^T & 0 \end{bmatrix}, \qquad \Psi_2 = \begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix}. \tag{30}$$

Then, it follows that

$$P\Psi_1 = \Psi_2, \qquad \Psi_1^T P\Psi_1 = \Psi_1^T\Psi_2 = \begin{bmatrix} Y & I \\ I & S \end{bmatrix} > 0. \tag{31}$$

Now, to use the concept of congruence transformation, the inequality (24) by postmultiplying and premultiplying the matrix diag$[\Psi_1^T, I]$ and by its transpose, respectively, is equivalent to

$$\begin{bmatrix} \Psi_2^T\Sigma_1\Psi_1 + \Psi_1^T\Sigma_1\Psi_2 + \eta\Psi_2^T\Psi_1 + \Psi_1^T\bar{C}\Psi_1 & \Psi_2^T\bar{D} \\ \bar{D}^T\Psi_2 & -\gamma^2 I \end{bmatrix} < 0. \tag{32}$$

By matrix computation and Schur's complement [21], it is obvious that Eq. (32) is equivalent to

$$\tilde{\Theta} = \begin{bmatrix} \Gamma_{11} & \Gamma_{21} & D & Y^T C^T \\ \star & \Gamma_{22} & S^T D & C^T \\ \star & \star & -\gamma^2 I & 0 \\ \star & \star & \star & -I \end{bmatrix} < 0, \tag{33}$$

where

$$\Gamma_{11} = AY + YA^T + BC_cM^T + MC_c^TB^T + \eta Y,$$
$$\Gamma_{21} = A + \left(SAY + SBC_cM^T + NB_cY + NA_cM^T\right)^T + \eta I,$$
$$\Gamma_{22} = SA + A^TS + NB_c + B_c^TN^T + \eta S. \tag{34}$$

For simplification of representation given in Eq. (33), let us define a new set of variables as follows:

$$\hat{A} = SAY + SB\hat{C} + \hat{B}Y + NA_cM^T, \quad \hat{B} = NB_c, \quad \hat{C} = C_cM^T. \tag{35}$$

Then, the matrix inequality (33) is equivalent to the LMI (13). Also, the LMI (14) guarantees the positiveness of the matrix P by (11) and (29). This completes the proof. □

Remark 1. In order to design feedback controller for $\mathcal{H}_\infty$ synchronization in this work, the error signal $e(t)$ is used instead of error output $y_e(t)$. Without loss of generality, one can use the signal $y_e(t)$ for output feedback control scheme [18].

Remark 2. Given any solution of the LMIs given in Theorem 1, a corresponding dynamic controller of the form (7) will be constructed as follows:

- Compute the invertible matrices M and N satisfying (29) using matrix algebra.
- Utilizing the matrices M and N obtained above, solve the system of Eqs. (35) for B_c, C_c and A_c (in order).

4. Numerical example

In this section, to verify and demonstrate the effectiveness of the proposed method, we discuss the simulation result for four-dimensional Hopfield neural network [3]. Master (3) and slave (4) chaotic Hopfield-neural networks are given as following parameters:

$$A = -\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad \bar{A} = \begin{bmatrix} 0.85 & -2 & -0.5 & 0.5 \\ 1.8 & 1.15 & 0.6 & 0.3 \\ 1.1 & 1.21 & 2.5 & 0.05 \\ 0.1 & -0.4 & -1.5 & 1.45 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$
$$D = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \qquad C = [\,1 \quad 1 \quad 0 \quad 0\,], \qquad f\left(x(t)\right) = \begin{bmatrix} \tanh(x_1(t)) \\ \tanh(x_2(t)) \\ \tanh(x_3(t)) \\ \tanh(x_4(t)) \end{bmatrix}.$$

In the numerical simulations, the fourth-order Runge-Kutta method is used to solve the systems with time step size 0.001. For the simulation, the following initial conditions are used:

$$\left(x_{m1}(0), x_{m2}(0), x_{m3}(0), x_{m4}(0)\right) = (0.1, -0.5, 0.2, -0.3),$$
$$\left(x_{s1}(0), x_{s2}(0), x_{s3}(0), x_{s4}(0)\right) = (-0.2, 1, 0.1, 1).$$

The Lipschitz's constant of $f(x(t))$ is $\delta = 1$ and a Gaussian noise with mean 0 and variance 1 is imposed on the slave system.

Now, in order to make synchronization of the systems (3) and (4) via dynamic control law (7) and secondary controller (12), let us solve the problem given in Theorem 1 with a constant $\eta = 0.31$ and the disturbance attenuation $\gamma = 0.2$.

By using MATLAB's LMI Control Toolbox [21], we found a possible solution set of the LMIs given in Theorem 1:

$$S = \begin{bmatrix} 964.2066 & -82.6739 & -438.9412 & -438.9412 \\ -82.6739 & 992.9059 & -453.0410 & -453.0410 \\ -438.9412 & -453.0410 & 976.9885 & -81.7546 \\ -438.9412 & -453.0410 & -81.7546 & 976.9885 \end{bmatrix},$$
$$Y = 10^3 \times \begin{bmatrix} 0.8792 & -0.8594 & 0.0037 & 0.0037 \\ -0.8594 & 0.8758 & -0.0038 & -0.0038 \\ 0.0037 & -0.0038 & 1.0759 & 0.0171 \\ 0.0037 & -0.0038 & 0.0171 & 1.0759 \end{bmatrix},$$
$$\hat{A} = \begin{bmatrix} -66.7478 & -65.3976 & -56.1309 & -56.1309 \\ -63.9459 & -61.6994 & -54.9902 & -54.9902 \\ -35.6171 & -37.0348 & -43.6086 & -44.2986 \\ -35.6171 & -37.0348 & -44.2986 & -43.6086 \end{bmatrix},$$
$$\hat{B} = \begin{bmatrix} -134.1134 \\ -116.4931 \\ -970.6761 \\ -970.6761 \end{bmatrix}, \qquad \hat{C} = 10^3 \times [\,-0.1937 \quad -1.7859 \quad 0.0009 \quad 0.0009\,].$$

After further calculation for N and M, a stabilizing dynamic feedback controller (7) is obtained as follows:

$$A_c = \begin{bmatrix} -1.2068 & -0.1064 & 0.0000 & -2.0896 \\ 0.1816 & -0.9066 & -0.0000 & 1.8144 \\ 0.1106 & 0.0568 & -1.0000 & 0.9486 \\ -173.0838 & -88.2313 & 0.0000 & -362.3924 \end{bmatrix},$$

$$B_c = \begin{bmatrix} 0.0001 \\ -0.0003 \\ -0.0011 \\ 8.5324 \end{bmatrix}, \qquad C_c = [\,-193.70 \quad -99.700 \quad 0.0000 \quad -1979.1\,],$$

which implies that the synchronization of two Hopfield neural networks can be achieved.

First, without disturbance signal and by applying the dynamic controller (7) with the parameters obtained above and secondary controller (12), the synchronization error between drive and response systems is given in Fig. 1. It shows that the synchronization error converges to zero exponentially. In this case, the control inputs $u(t)$ and $\alpha(t)$ is illustrated at Figs. 2 and 3.

To observe the $\mathcal{H}_\infty$ performance with disturbance attenuation, the response of the controlled output error $y_e(t)$ is depicted in Fig. 4, which shows the dynamic $\mathcal{H}_\infty$ controller (7) and secondary (12) reduces the effect of the disturbance input $w(t)$ on the controlled output error $y_e(t)$ to within a prescribed level $\gamma = 0.2$.

5. Conclusions

The problem of $\mathcal{H}_\infty$ synchronization for a general class of chaotic systems with disturbances has been presented. Based on Lyapunov theory and LMI formulation, a new control scheme, dynamic feedback control plus nonlinear static control, has proposed to guarantee synchronization for the master and slave systems and reduce the $\mathcal{H}_\infty$-norm from the disturbance to the output error within a prescribed level. Furthermore, a model of Hopfield neural network is given to illustrate the effectiveness of the proposed control scheme.

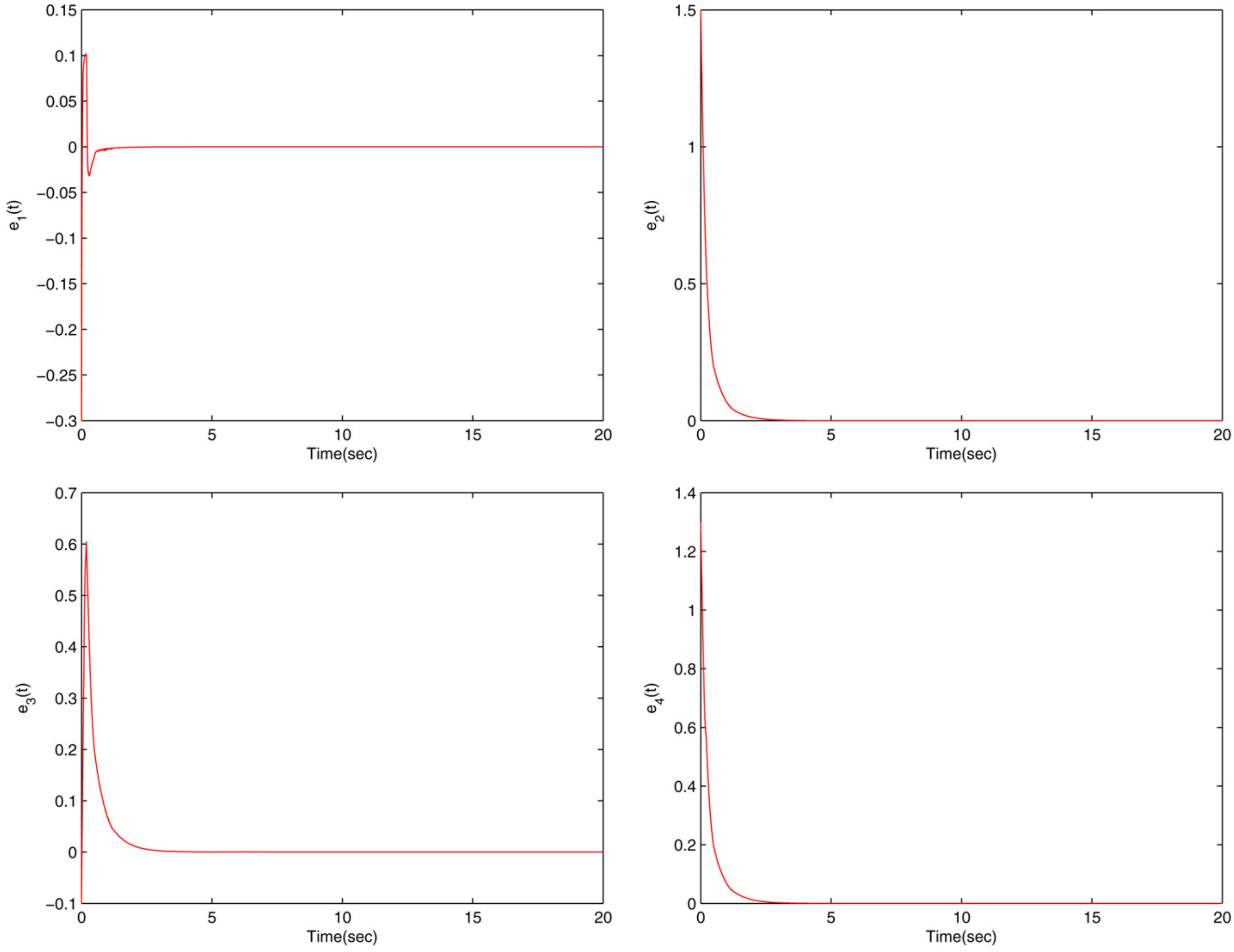

Fig. 1. The time responses of synchronization of Hopfield neural networks without disturbance signal $w(t)$.

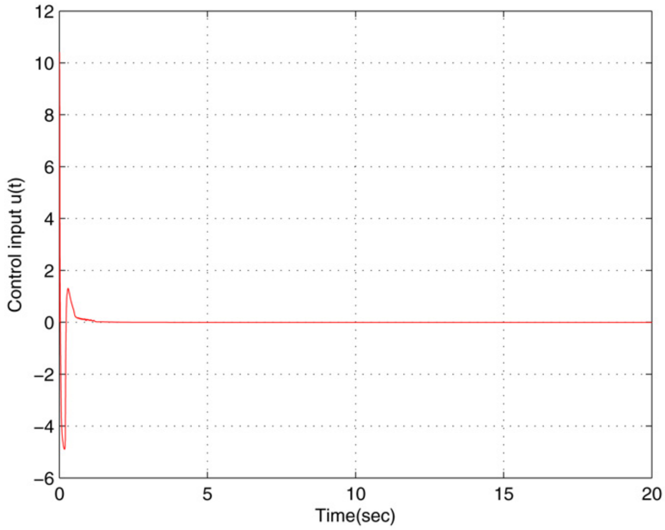

Fig. 2. The dynamic control input $u(t)$ for Hopfield neural networks without disturbance signal $w(t)$.

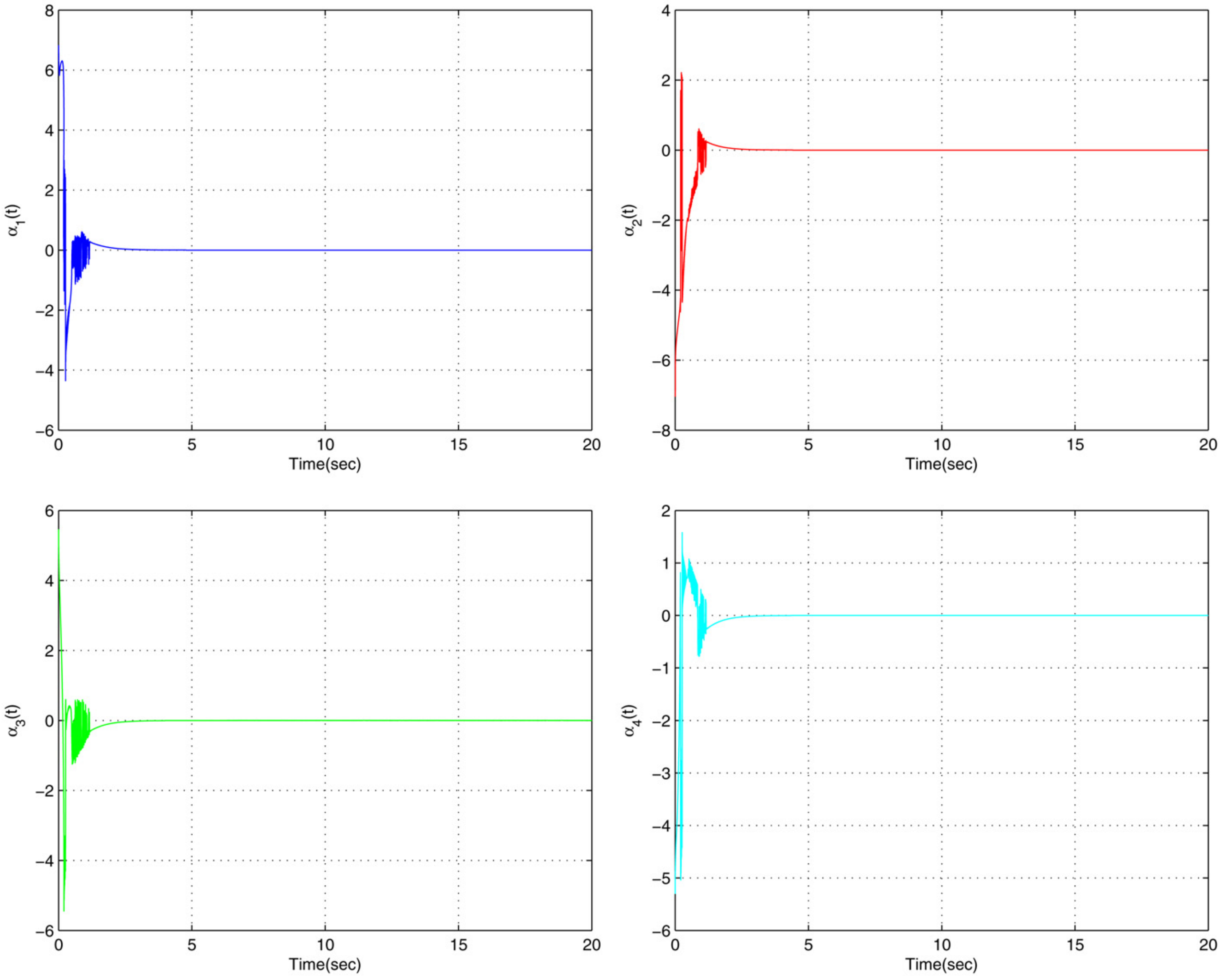

Fig. 3. The secondary control input $\alpha(t)$ for Hopfield neural networks without disturbance signal $w(t)$.

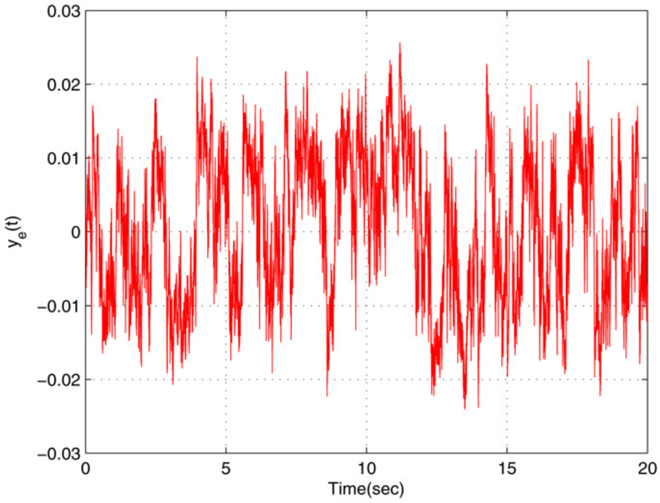

Fig. 4. The time response of the output error $y_e(t)$ of Hopfield neural networks with disturbance signal $w(t)$ and zero initial conditions.

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