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ELSEVIER Applied Mathematics and Computation 149 (2004) 147–164

APPLIED MATHEMATICS AND COMPUTATION

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Robust non-fragile control for uncertain discrete-delay large-scale systems with a class of controller gain variations

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Abstract

This paper considers the problems of robust non-fragile control for uncertain discrete-delay large-scale systems under state feedback gain variations. Two classes of controller gain variations are considered. Based on the Lyapunov method, the state feedback control design for robust stability is given in terms of solutions to a linear matrix inequality (LMI). The solutions of the LMI can be easily obtained using efficient convex optimization techniques. A numerical example is included to illustrate the design procedures.

Keywords: Large-scale system; Delay; Controller gain variation; Non-fragile control; Linear matrix inequality (LMI)

1. Introduction

With the enlargement of dimension of a control system, analysis and control for the system becomes very complicated. It is standard to divide such systems into a number of interconnected subsystems. In general, a large-scale interconnected dynamical system can be usually characterized by a large number of

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0096-3003/$ - see front matter
doi:10.1016/S0096-3003(02)00962-1

state variables, system parametric uncertainties, and a complex interaction between subsystems [14,17]. During the last decade, the problem of decentralized stabilization of large-scale systems has received considerable attention, because there are a large number of large scale interconnected dynamical systems in many practical control problems, e.g. transportation systems, power systems, communication systems, economic systems, social systems, and so on [2,3,8,20].

Time-delays, due to the information transmission between subsystems, naturally exist in large-scale systems and the existence of the delay is frequently a source of instability. Therefore, the stabilization problem of the large-scale system with time-delay in subsystem interconnections has been investigated by many researchers [10,13,15,16,18,19,21].

On the other hand, it is generally known that feedback systems designed for robustness with regard to plant parameters, may require very accurate controllers. Recently, it is shown that relatively small perturbations in controller parameters could even destabilize the closed-loop system [5,11]. Therefore, it is necessary that any controller should be able to tolerate some level of controller gain variations. This raises a new issue: how to design a controller for a given plant with uncertainty such that the controller is non-fragile with regard to its gain variations. More recently, there have been some studies to tackle the non-fragile controller design problem [4,6,7,9,12,22]. However, there are no papers considering non-fragile controller design methods of discrete-time large-scale systems with delays.

This paper is concerned with the design problem of robust non-fragile decentralized controller for discrete-delay large-scale systems with parametric uncertainties and controller gain variations. Multiplicative controller gain variations are assumed in existence in the state feedback gain. A stability criterion for robust stability of the system is derived in terms of linear matrix inequality (LMI) using the Lyapunov method. The result obtained can be extended to the systems with additive controller gain variations. In the approach, the controller parameters which satisfy the LMI can be easily found by various efficient convex optimization algorithms [1].

Notations: Throughout the paper, $\mathscr{R}^n$ denotes the n dimensional Euclidean space, $\mathscr{R}^{n\times m}$ is the set of all $n \times m$ real matrices, I is the identity matrix with appropriate dimensions, and *block diag*$(\cdot)$ denotes a block diagonal matrix. $*$ denotes the symmetric part. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \geqslant Y$) means that the matrix $X - Y$ is positive definite, (respectively, non-negative).

2. Problem formulation

Consider a class of uncertain discrete-delay large-scale system composed n interconnected subsystems described by

$$S_i : x_i(k+1) = (A_i + \Delta A_i(k))x_i(k) + \sum_{j\neq i}^{N}(A_{ij} + \Delta A_{ij}(k))x_j(k-h_j) + (B_i + \Delta B_i(k))u_i(k), \quad i = 1,2,\ldots,n, \tag{1}$$

where $x_i(k) \in \mathscr{R}^{n_i}$ is the state vector, $u_i(k) \in \mathscr{R}^{m_i}$ is the control vector, and the time-delays, h_j, are the positive constants. The system matrices A_i, B_i, and A_{ij} are of appropriate dimensions, and $\Delta A_i(k)$, $\Delta B_i(k)$, and $\Delta A_{ij}(k)$ are real-valued matrices representing time-varying parameter uncertainties in the system.

Assume that the pair $(A_i, B_i), i = 1,\ldots,n$, is stabilizable, and assume that the time-varying uncertainties are of the form

$$\begin{aligned} &\Delta A_i(k) = D_{ai}F_{ai}(k)E_{ai}, \quad \Delta B_i(k) = D_{bi}F_{bi}(k)E_{bi}, \\ &\Delta A_{ij}(k) = D_{aij}F_{aij}(k)E_{aij}, \end{aligned} \tag{2}$$

where D_{ai}, D_{bi}, D_{aij}, E_{ai}, E_{bi}, and E_{aij} are known constant real matrices with appropriate dimensions, and $F_{ai}(k)$, $F_{bi}(k)$, and $F_{aij}(k)$ are unknown matrix functions which are bounded as

$$\begin{aligned} &F_{ai}^{\mathrm{T}}(k)F_{ai}(k) \leqslant I, \quad F_{bi}^{\mathrm{T}}(k)F_{bi}(k) \leqslant I \\ &F_{aij}^{\mathrm{T}}(k)F_{aij}(k) \leqslant I, \quad \forall i,j \geqslant 0. \end{aligned} \tag{3}$$

Now, although one finds the controller $u_i(k) = -K_i x_i(k)$ for each subsystems, the actual controller implemented is

$$u_i(k) = -[K_i + \Delta K_i]x_i(k), \quad i = 1,2,\ldots,n, \tag{4}$$

where $K_i \in \mathscr{R}^{m_i \times n_i}$ is the nominal controller gain to be designed and ΔK_i represents the multiplicative gain perturbations of the form

$$\Delta K_i = H_i \Phi_i(k) G_i K_i \tag{5}$$

with H_i and G_i being known constant matrices, and $\Phi_i(k)$ the uncertain parameter matrix satisfying

$$\Phi_i^{\mathrm{T}}(k)\Phi_i(k) \leqslant I. \tag{6}$$

Remark 1. The controller gain perturbation can result from the actuator degradations, as well as from the requirement for re-adjustment of controller gains during the controller implementation state [5,11]. These perturbations in the controller gains are modelled here as uncertain gains that are dependent on uncertain parameters. In the literature [4,7,9] the models of additive uncertainties and multiplicative uncertainties are used to describe the controller gain variation. The uncertainty given in (5) is a class of multiplicative uncertainties.

With the control law (4), the resulting closed-loop subsystem becomes

$$x_i(k+1) = [A_i + \Delta A_i(k) - (B_i + \Delta B_i(k))(I + H_i \Phi_i(k) G_i) K_i] x_i(k) + \sum_{j \neq i}^{N} [A_{ij} + \Delta A_{ij}(k)] x_j(k - h_j). \tag{7}$$

From (7) we can write the overall system in the following way:

$$X(k+1) = [A + \Delta A - (B + \Delta B)K - (B + \Delta B) H \Phi(k) G K] X(k) + (A_D + \Delta A_D) X_d(k) \tag{8}$$

where

$$\begin{aligned}
&X(k) \triangleq [x_1^{\mathrm{T}}(k) x_2^{\mathrm{T}}(k) \cdots x_n^{\mathrm{T}}(k)]^{\mathrm{T}},\\
&X_d(k) \triangleq [x_1^{\mathrm{T}}(k-h_1) x_2^{\mathrm{T}}(k-h_2) \cdots x_n^{\mathrm{T}}(k-h_n)]^{\mathrm{T}},\\
&A \triangleq \text{block diag}(A_1, A_2, \ldots, A_n),\\
&\Delta A \triangleq D_a F_a(k) E_a,\\
&D_a \triangleq \text{block diag}(D_{a1}, D_{a2}, \ldots, D_{an}),\\
&F_a(k) \triangleq \text{block diag}(F_{a1}(k), F_{a2}(k), \ldots, F_{an}(k)),\\
&E_a \triangleq \text{block diag}(E_{a1}, E_{a2}, \ldots, E_{an}),\\
&B \triangleq \text{block diag}(B_1, B_2, \ldots, B_n),\\
&\Delta B \triangleq D_b F_b(k) E_b,\\
&D_b \triangleq \text{block diag}(D_{b1}, D_{b2}, \ldots, D_{bn}),\\
&F_b(k) \triangleq \text{block diag}(F_{b1}(k), F_{b2}(k), \ldots, F_{bn}(k)),\\
&E_b \triangleq \text{block diag}(E_{b1}, E_{b2}, \ldots, E_{bn}),\\
&H \triangleq \text{block diag}(H_1, H_2, \ldots, H_n),\\
&G \triangleq \text{block diag}(G_1, G_2, \ldots, G_n),\\
&\Phi(k) \triangleq \text{block diag}(\Phi_1(k), \Phi_2(k), \ldots, \Phi_n(k)),\\
&K \triangleq \text{block diag}(K_1, K_2, \ldots, K_n),\\
&A_D \triangleq \text{block matrix with elements } A_{ij} (i \neq j),\\
&\Delta A_D \triangleq \sum_{i=1}^{n} D_{di} F_{di}(k) E_{di},\\
&D_{di} \triangleq \text{block matrix with elements } D_{di}(r, j)\\
&\quad D_{di}(r,j) = \begin{cases} D_{aij}, & r = i,\ i \neq j,\\ 0, & \text{otherwise,} \end{cases}\\
&F_{di}(k) \triangleq \text{block diag with diagonal elements } F_{di}(j,j)\\
&\quad F_{di}(j,j) = \begin{cases} F_{aij}, & j \neq i,\ j = 1, \ldots, n,\\ 0, & \text{otherwise,} \end{cases}\\
&E_{di} \triangleq \text{block diag with diagonal elements } E_{di}(j,j)\\
&\quad E_{di}(j,j) = \begin{cases} E_{aij}, & j \neq i,\ j = 1, \ldots, n,\\ 0, & \text{otherwise.} \end{cases}
\end{aligned} \tag{9}$$

Then, the problem addressed in this paper is that of finding a robust stabilizing decentralized state feedback controllers of the form

$$U(k) = -(I + H\Phi(k)G)KX(k), \tag{10}$$

so that the closed-loop system (8) is asymptotically stabilized.

3. Design of robust non-fragile controller

In this section, we consider the problem of decentralized robust stabilization of the uncertain closed-loop system described by (8) using the Lyapunov method with LMI technique.

Before proceeding further, we will state well known lemma.

Lemma 1 [1]. *The LMI*

$$\begin{bmatrix} Y(x) & W(x) \\ W^{\mathrm{T}}(x) & R(x) \end{bmatrix} > 0,$$

is equivalent to

$$R(x) > 0, \quad Y(x) - W(x)R(x)^{-1}W^{\mathrm{T}}(x) > 0,$$

where $Y(x) = Y^{\mathrm{T}}(x), R(x) = R^{\mathrm{T}}(x)$ *and* $W(x)$ *depend affinely on* x.

Then, we have following theorem for robust stability of system (8).

Theorem 1. *The closed-loop system* (8) *is asymptotically stable, if there exist positive scalars* ε_1, ε_2, ε_3 *and* ε_4, *a block diagonal matrix* N, *and positive definite block diagonal matrices* Q *and* S *satisfying the following LMI* Ξ:

$$\Xi(N, Q, S, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4) = \begin{bmatrix} \Omega & BH & 0 & AQ - BN & 0 & 0 & 0 & 0 & A_D S E_D^{\mathrm{T}} \\ * & -I & H^{\mathrm{T}} E_b^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q & Q & N^{\mathrm{T}} E_b^{\mathrm{T}} & Q E_a^{\mathrm{T}} & N^{\mathrm{T}} G^{\mathrm{T}} & 0 \\ * & * & * & * & -S & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I + E_D S E_D^{\mathrm{T}} \end{bmatrix} < 0, \tag{11}$$

where $\Omega = -Q + \varepsilon_1 D_a D_a^{\mathrm{T}} + \varepsilon_2 D_D D_D^{\mathrm{T}} + \varepsilon_3 D_b D_b^{\mathrm{T}} + \varepsilon_4 D_b D_b^{\mathrm{T}} + A_D S A_D^{\mathrm{T}}$ *and*

$$D_D = \left(\sum_{i=1}^{N} D_{di} D_{di}^{\mathrm{T}} \right)^{1/2}, \qquad E_D = \left(\sum_{i=1}^{N} E_{di}^{\mathrm{T}} E_{di} \right)^{1/2}. \tag{12}$$

Then, the feedback gain K of the controller (10) *is*

$$K = NQ^{-1}. \tag{13}$$

Proof. Consider a Lyapunov function

$$V(X(k)) = X^{\mathrm{T}}(k)PX(k) + \sum_{i=k-h}^{k-1} X^{\mathrm{T}}(i)RX(i), \tag{14}$$

where P and R are the positive-definite matrices.

The difference of V is given by

$$\begin{aligned} \Delta V_k &= V(X(k+1)) - V(X(k)) \\ &= X^{\mathrm{T}}(k+1)PX(k+1) + \sum_{i=k+1-h}^{k} X^{\mathrm{T}}(i)RX(i) \\ &\quad - X^{\mathrm{T}}(k)PX(k) - \sum_{i=k-h}^{k-1} X^{\mathrm{T}}(i)RX(i) \\ &= X^{\mathrm{T}}(k)[(A + \Delta A - (B + \Delta B)K - (B + \Delta B)H\Phi(k)GK)^{\mathrm{T}} \\ &\quad \times P(A + \Delta A - (B + \Delta B)K - (B + \Delta B)H\Phi(k)GK) \\ &\quad - P + R]X(k) + 2X^{\mathrm{T}}(k)(A + \Delta A - (B + \Delta B)K \\ &\quad - (B + \Delta B)H\Phi(k)GK)^{\mathrm{T}}P(A_D + \Delta A_D)X_d(k) \\ &\quad + X_d^{\mathrm{T}}(k)[(A_D + \Delta A_D)^{\mathrm{T}}P(A_D + \Delta A_D) - R]X_d(k) \\ &\equiv \overline{X}^{\mathrm{T}}(k)M_0\overline{X}(k), \end{aligned} \tag{15}$$

where

$$\overline{X}(k) = [\, X^{\mathrm{T}}(k) \quad X_d^{\mathrm{T}}(k) \,]^{\mathrm{T}}$$

and

$$M_0 = \begin{bmatrix} \begin{pmatrix} [A + \Delta A - (B + \Delta B)K \\ -(B + \Delta B)H\Phi(k)GK]^{\mathrm{T}}P[A + \Delta A \\ -(B + \Delta B)K - (B + \Delta B)H\Phi(k)GK] \\ -P + R \end{pmatrix} & \begin{pmatrix} [A + \Delta A - (B + \Delta B)K \\ -(B + \Delta B)H\Phi(k)GK]^{\mathrm{T}} \\ \times P[A_D + \Delta A_D] \end{pmatrix} \\ * & (A_D + \Delta A_D)^{\mathrm{T}}P(A_D + \Delta A_D) - R \end{bmatrix}.$$

Hence, ΔV_k is negative if the matrix M_0 is negative definite. By Lemma 1 (Schur Complements), the fact that $M_0 < 0$ is equivalent to

$$\begin{aligned} M_1 &\equiv \begin{bmatrix} -P^{-1} & \begin{pmatrix} A+\Delta A-(B+\Delta B)K \\ -(B+\Delta B)H\Phi(k)GK \end{pmatrix} & A_D+\Delta A_D \\ * & -(P-R) & 0 \\ * & * & -R \end{bmatrix} \\ &= \begin{bmatrix} -P^{-1} & \begin{pmatrix} A+D_aF_a(k)E_a-(B+\Delta B)K \\ -(B+\Delta B)H\Phi(k)GK \end{pmatrix} & A_D+\sum_{i=1}^{N} D_{di}F_{di}(k)E_{di} \\ * & -(P-R) & 0 \\ * & * & -R \end{bmatrix} \\ &< 0. \end{aligned} \tag{16}$$

Using the well-known fact that

$$U\Delta^{\mathrm{T}}V^{\mathrm{T}} + V\Delta U^{\mathrm{T}} \leqslant \varepsilon UU^{\mathrm{T}} + \varepsilon^{-1}VV^{\mathrm{T}}, \quad \varepsilon > 0 \tag{17}$$

for any matrices U, V and Δ with $\Delta^{\mathrm{T}}\Delta \leqslant I$, we can eliminate the unknown factor, $F_a(k)$, $F_{di}(k)$ and $\Phi(k)$, in (16). Then we have

$$\begin{aligned} M_1 \leqslant M_2 &\equiv \begin{bmatrix} \begin{pmatrix} -P^{-1}+\varepsilon_1 D_aD_a^{\mathrm{T}} \\ +\varepsilon_2\sum_{i=1}^{N} D_{di}D_{di}^{\mathrm{T}} \\ +(B+\Delta B)H \\ \times H^{\mathrm{T}}(B+\Delta B)^{\mathrm{T}} \end{pmatrix} & A-(B+\Delta B)K & A_D \\ * & \begin{pmatrix} -(P-R) \\ +\varepsilon_1^{-1}E_a^{\mathrm{T}}E_a \\ +K^{\mathrm{T}}G^{\mathrm{T}}GK \end{pmatrix} & 0 \\ * & * & -R+\varepsilon_2^{-1}\sum_{i=1}^{N}E_{di}^{\mathrm{T}}E_{di} \end{bmatrix} \\ &= \begin{bmatrix} \begin{pmatrix} -P^{-1}+\varepsilon_1 D_aD_a^{\mathrm{T}} \\ +\varepsilon_2 D_DD_D^{\mathrm{T}} \\ +(B+\Delta B)H \\ \times H^{\mathrm{T}}(B+\Delta B)^{\mathrm{T}} \end{pmatrix} & A-(B+\Delta B)K & A_D \\ * & \begin{pmatrix} -(P-R) \\ +\varepsilon_1^{-1}E_a^{\mathrm{T}}E_a \\ +K^{\mathrm{T}}G^{\mathrm{T}}GK \end{pmatrix} & 0 \\ * & * & -R+\varepsilon_2^{-1}E_D^{\mathrm{T}}E_D \end{bmatrix}, \end{aligned} \tag{18}$$

where $\varepsilon_1 > 0$ and $\varepsilon_2 > 0$ and D_D and E_D are defined in (12).

Using Lemma 1, the condition $M_2 < 0$ is equivalent to

$$M_3 \equiv \begin{bmatrix} \begin{pmatrix} -P^{-1} \\ +\varepsilon_1 D_a D_a^{\mathrm{T}} \\ +\varepsilon_2 D_D D_D^{\mathrm{T}} \end{pmatrix} & (B+\Delta B)H & A-(B+\Delta B)K & 0 & 0 & A_D & 0 \\ * & -I & 0 & 0 & 0 & 0 & 0 \\ * & * & -P+R & E_a^{\mathrm{T}} & K^{\mathrm{T}}G^{\mathrm{T}} & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -R & E_D^{\mathrm{T}} \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0. \tag{19}$$

To eliminate the uncertain factor $F_b(k)$ in the term ΔB of the inequality (19), using the fact (17) we obtain

$$M_3 \leqslant M_4 \equiv \begin{bmatrix} (1,1) & BH & A-BK & 0 & 0 & A_D & 0 \\ * & (2,2) & 0 & 0 & 0 & 0 & 0 \\ * & * & (3,3) & E_a^{\mathrm{T}} & K^{\mathrm{T}}G^{\mathrm{T}} & 0 & 0 \\ * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -R & E_D^{\mathrm{T}} \\ * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix}, \tag{20}$$

where $\varepsilon_3 > 0, \varepsilon_4 > 0$, $(1,1) = -P^{-1} + \varepsilon_1 D_a D_a^{\mathrm{T}} + \varepsilon_2 D_D D_D^{\mathrm{T}} + \varepsilon_3 D_b D_b^{\mathrm{T}} + \varepsilon_4 D_b D_b^{\mathrm{T}}$, $(2,2) = -I + \varepsilon_3^{-1} H^{\mathrm{T}} E_b^{\mathrm{T}} E_b H$, and $(3,3) = -P + R + \varepsilon_4^{-1} K^{\mathrm{T}} E_b^{\mathrm{T}} E_b K$.

Again, using Lemma 1, the condition $M_4 < 0$ is also equivalent to

$$\begin{bmatrix} (1,1) & BH & 0 & A-BK & 0 & 0 & 0 & A_D & 0 \\ * & -I & H^{\mathrm{T}}E_b^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P+R & K^{\mathrm{T}}E_b^{\mathrm{T}} & E_a^{\mathrm{T}} & K^{\mathrm{T}}G^{\mathrm{T}} & 0 & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & * & * & -R & E_D^{\mathrm{T}} \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix} < 0$$

$$\Longleftrightarrow \begin{bmatrix} (1,1) & BH & 0 & A-BK & 0 & 0 & 0 & 0 \\ * & -I & H^{\mathrm{T}}E_b^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P+R & K^{\mathrm{T}}E_b^{\mathrm{T}} & E_a^{\mathrm{T}} & K^{\mathrm{T}}G^{\mathrm{T}} & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I \end{bmatrix}$$

$$+ \begin{bmatrix} A_D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ E_D \end{bmatrix} R^{-1} \begin{bmatrix} A_D^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 & E_D^{\mathrm{T}} \end{bmatrix}$$

$$= \begin{bmatrix} (1,1)+A_D R^{-1} A_D^{\mathrm{T}} & BH & 0 & A-BK & 0 & 0 & 0 & A_D R^{-1} E_D^{\mathrm{T}} \\ * & -I & H^{\mathrm{T}}E_b^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P+R & K^{\mathrm{T}}E_b^{\mathrm{T}} & E_a^{\mathrm{T}} & K^{\mathrm{T}}G^{\mathrm{T}} & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & \begin{pmatrix} -\varepsilon_2 I \\ +E_D R^{-1} E_D^{\mathrm{T}} \end{pmatrix} \end{bmatrix}$$

$$< 0. \tag{21}$$

Pre- and post-multiply inequality (21) by $\mathscr{T}^{\mathrm{T}}$ and $\mathscr{T}$, where $\mathscr{T} = \text{block diag}(I, I, I, P^{-1}, I, I, I, I)$, we have

$$\begin{bmatrix} (1,1)+A_D R^{-1} A_D^{\mathrm{T}} & BH & 0 & AP^{-1}-BKP^{-1} & 0 & 0 & 0 & A_D R^{-1} E_D^{\mathrm{T}} \\ * & -I & H^{\mathrm{T}}E_b^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -P^{-1}+P^{-1}RP^{-1} & P^{-1}K^{\mathrm{T}}E_b^{\mathrm{T}} & P^{-1}E_a^{\mathrm{T}} & P^{-1}K^{\mathrm{T}}G^{\mathrm{T}} & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I + E_D R^{-1} E_D^{\mathrm{T}} \end{bmatrix}$$

$$< 0. \tag{22}$$

Using some change of variables, $N = KP^{-1}, S = R^{-1}$ and $Q = P^{-1}$, the inequality (22) is changed to

$$\begin{bmatrix} \Omega & BH & 0 & AQ - BN & 0 & 0 & 0 & A_D S E_D^{\mathrm{T}} \\ * & -I & H^{\mathrm{T}} E_b^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q + QS^{-1}Q & N^{\mathrm{T}} E_b^{\mathrm{T}} & QE_a^{\mathrm{T}} & N^{\mathrm{T}} G^{\mathrm{T}} & 0 \\ * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & -\varepsilon_2 I + E_D S E_D^{\mathrm{T}} \end{bmatrix} < 0, \tag{23}$$

where Ω is defined in (12).

By Lemma 1, the inequality (23) is equivalent to the LMI (11). This completes the proof.

Remark 2. As a special case of the multiplicative uncertainty (5), consider the uncertainty of the form

$$\Delta K_i = \delta_i \Phi_i(k) K_i \tag{24}$$

with δ_i is an uncertain real parameter. The value of δ_i indicates the measure of non-fragility against controller gain variations for each subsystem S_i. In this case, the stability criterion for robust stability of the closed-loop system can be easily obtained as

$$\begin{aligned} &\Sigma(N, Q, S, \delta, \varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4) \\ &= \begin{bmatrix} \Omega & B\delta & 0 & AQ - BN & 0 & 0 & 0 & 0 & A_D S E_D^{\mathrm{T}} \\ * & -I & \delta^{\mathrm{T}} E_b^{\mathrm{T}} & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_3 I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -Q & Q & N^{\mathrm{T}} E_b^{\mathrm{T}} & QE_a^{\mathrm{T}} & N^{\mathrm{T}} & 0 \\ * & * & * & * & -S & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -\varepsilon_4 I & 0 & 0 & 0 \\ * & * & * & * & * & * & -\varepsilon_1 I & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_2 I + E_D S E_D^{\mathrm{T}} \end{bmatrix} \\ &< 0, \end{aligned} \tag{25}$$

where $\delta = \text{block diag}(\delta_1 I, \delta_2 I, \ldots, \delta_N I)$.

Then, the measure of non-fragility in each controller for subsystems, δ_i, can be obtained from the relation $\delta = \text{diag}\ (\delta_1 I, \delta_2 I, \ldots, \delta_n I)$ after finding the LMI solutions of (25). Since the proof of deriving the criterion (25) is similar to that of Theorem 1, it is omitted.

Remark 3. When the controller gain variations of the large-scale systems (1) are of the additive form [4,22]:

$$\Delta K_i = H_i \Phi_i(k) G_i, \quad \Phi_i^{\mathrm{T}}(k)\Phi_i(k) \leqslant I$$

with H_i and G_i being known constant matrices, and $\Phi(k)$ the uncertain parameter matrix, the stability criterion of the closed-loop system with the control law (10) is identical to the LMI (11) except that the (4, 8)th entry of $\Sigma(\cdot)$ given in (11) are changed as QG^{T}. The proof is trivial and omitted.

Remark 4. In order to solve the LMIs (11) and (25) given in Theorem 1 and Remark 2, we can utilize Matlab's LMI Control Toolbox, which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithms [1].

Numerical example: Consider a large-scale system which is composed of the following three interconnected subsystems

$$\begin{aligned}
x_1(k+1) &= \left(\begin{bmatrix} 0 & 0.5 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & 0.4\cos(k) \\ 0.2\sin(k) & 0 \end{bmatrix}\right) x_1(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_1(k) \\
&\quad + \left(\begin{bmatrix} 0 & 0.1 \\ 0.1 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.05\cos(k) \end{bmatrix}\right) x_2(k-h_2) \\
&\quad + \left(\begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} + \begin{bmatrix} 0.04\cos(k) & 0 \\ 0 & 0.04\sin(k) \end{bmatrix}\right) x_3(k-h_3), \\
x_2(k+1) &= \left(\begin{bmatrix} 0 & 1 \\ 0.5 & -0.5 \end{bmatrix} + \begin{bmatrix} 0 & 0.09\cos(k) \\ 0.09\sin(k) & 0 \end{bmatrix}\right) x_2(k) \\
&\quad + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u_2(k) + \left(\begin{bmatrix} 0.05 & 0 \\ 0 & 0.05 \end{bmatrix} + \begin{bmatrix} 0 & 0.04\cos(k) \\ 0 & 0 \end{bmatrix}\right) x_1(k-h_1) \\
&\quad + \left(\begin{bmatrix} 0 & 0.09 \\ 0.09 & 0 \end{bmatrix} + \begin{bmatrix} 0.05\cos(k) & 0 \\ 0 & 0.05\sin(k) \end{bmatrix}\right) x_3(k-h_3), \\
x_3(k+1) &= \left(\begin{bmatrix} -0.5 & 0 \\ 0 & 1.2 \end{bmatrix} + \begin{bmatrix} 0.1\cos(k) & 0 \\ 0 & 0.2\sin(k) \end{bmatrix}\right) x_3(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u_3(k) \\
&\quad + \left(\begin{bmatrix} 0 & 0.1 \\ 0.02 & 0.1 \end{bmatrix} + \begin{bmatrix} 0 & 0.04\cos(k) \\ 0.04\sin(k) & 0 \end{bmatrix}\right) x_1(k-h_1) \\
&\quad + \left(\begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix} + \begin{bmatrix} 0 & 0.04\cos(k) \\ 0.04\sin(k) & 0 \end{bmatrix}\right) x_2(k-h_2),
\end{aligned} \tag{26}$$

where

$$x_i(k) = [x_{i1}^{\mathrm{T}}(k)\ \ x_{i2}^{\mathrm{T}}(k)]^{\mathrm{T}}, \quad u_i(k) = [u_{i1}^{\mathrm{T}}(k)\ \ u_{i2}^{\mathrm{T}}(k)]^{\mathrm{T}}, \quad i = 1, 2, 3$$

and the time-delays and initial conditions are

$$\begin{aligned} &h_1 = 2, \quad h_2 = 5, \quad h_3 = 7, \\ &x_1(k) = [\,1 \quad -0.5\,]^{\mathrm{T}}, \quad x_2(k) = [\,2 \quad 1\,]^{\mathrm{T}}, \\ &x_3(k) = [\,-1 \quad 0.5\,]^{\mathrm{T}} \quad \text{for } -h_3 \leqslant k \leqslant 0. \end{aligned}$$

The above system is of the form of system (7) with

$$\begin{aligned} &D_{a1} = \begin{bmatrix} 0.6325 & 0 \\ 0 & 0.447 \end{bmatrix}, \quad D_{a2} = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.3 \end{bmatrix}, \quad D_{a3} = \begin{bmatrix} 0.3162 & 0 \\ 0 & 0.4472 \end{bmatrix}, \\ &E_{a1} = \begin{bmatrix} 0 & 0.6325 \\ 0.4472 & 0 \end{bmatrix}, \quad E_{a2} = \begin{bmatrix} 0 & 0.3 \\ 0.3 & 0 \end{bmatrix}, \quad E_{a3} = \begin{bmatrix} 0.3162 & 0 \\ 0 & 0.4472 \end{bmatrix}, \\ &D_{a12} = \begin{bmatrix} 0 & 0 \\ 0 & 0.2236 \end{bmatrix}, \quad D_{a13} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad D_{a21} = \begin{bmatrix} 0 & 0.2 \\ 0 & 0 \end{bmatrix}, \\ &D_{a23} = \begin{bmatrix} 0.2236 & 0 \\ 0 & 0.2236 \end{bmatrix}, \quad D_{a31} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad D_{a32} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ &E_{a12} = \begin{bmatrix} 0 & 0 \\ 0 & 0.2236 \end{bmatrix}, \quad E_{a13} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad E_{a21} = \begin{bmatrix} 0 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ &E_{a23} = \begin{bmatrix} 0.2236 & 0 \\ 0 & 0.2236 \end{bmatrix}, \quad E_{a31} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \quad E_{a32} = \begin{bmatrix} 0 & 0.2 \\ 0.2 & 0 \end{bmatrix}, \\ &D_{b1} = D_{b2} = D_{b3} = E_{b1} = E_{b2} = E_{b3} = 0. \end{aligned}$$

For the system (26), it is considered the multiplicative controller gain variations (5) with

$$\begin{aligned} &H_1 = [\,1 \quad 1\,], \quad G_1 = \begin{bmatrix} 0.35 \\ 0 \end{bmatrix}, \\ &H_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad G_2 = 0.3\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \\ &H_3 = 0.5, \quad G_3 = 0.5. \end{aligned} \tag{27}$$

Solving the LMI (11), we can obtain the solutions as

$$Q = \begin{bmatrix} 230.3605 & -5.0679 & 0 & 0 & 0 & 0 \\ -5.0679 & 52.6327 & 0 & 0 & 0 & 0 \\ 0 & 0 & 48.5473 & -28.5844 & 0 & 0 \\ 0 & 0 & -28.5844 & 74.0806 & 0 & 0 \\ 0 & 0 & 0 & 0 & 185.0797 & -14.4258 \\ 0 & 0 & 0 & 0 & -14.4258 & 31.0070 \end{bmatrix},$$

$$S = \begin{bmatrix} 874.4703 & -104.7458 & 0 & 0 & 0 & 0 \\ -104.7458 & 298.3944 & 0 & 0 & 0 & 0 \\ 0 & 0 & 55.0726 & -31.1762 & 0 & 0 \\ 0 & 0 & -31.1762 & 93.4527 & 0 & 0 \\ 0 & 0 & 0 & 0 & 373.7688 & -49.0746 \\ 0 & 0 & 0 & 0 & -49.0746 & 165.9314 \end{bmatrix},$$

$$N = \begin{bmatrix} -5.1095 & 53.0619 & 0 & 0 & 0 & 0 \\ 0 & 0 & -28.5767 & 74.0917 & 0 & 0 \\ 0 & 0 & 38.5488 & -51.3597 & 0 & 0 \\ 0 & 0 & 0 & 0 & -26.8875 & 33.3891 \end{bmatrix},$$

$$\varepsilon_1 = 63.3770, \quad \varepsilon_2 = 49.4389, \quad \varepsilon_3 = 15.9039.$$

Since $K = NQ^{-1} = \mathrm{diag}(K_1, K_2, K_3)$, the gain matrices, K_i, of the stabilizing controller, $u_i(k)$, for three subsystems are

$$K_1 = [-0.0000 \quad 1.0082],$$
$$K_2 = \begin{bmatrix} 0.0003 & 1.0003 \\ 0.4993 & -0.5006 \end{bmatrix},$$
$$K_3 = [-0.0637 \quad 1.0472].$$

This implies that the obtained robust decentralized controller guarantees the robust stability of the closed-loop system in spite of the given gain variations of the subsystems 1–3.

For numerical simulation, the following control laws are employed:

$$u_1(k) = -(1 + H_1\Phi_1(k)G_1)K_1x_1(k),$$
$$u_2(k) = -(1 + H_2\Phi_2(k)G_2)K_2x_2(k),$$
$$u_3(k) = -(1 + H_3\Phi_3(k)G_3)K_3x_3(k),$$

where $\Phi_i(k) = I$ for all i.

The simulation results are in Figs. 1–4. In the figures, one can see that the system is well stabilized irrespective of uncertainties and controller gain variations.

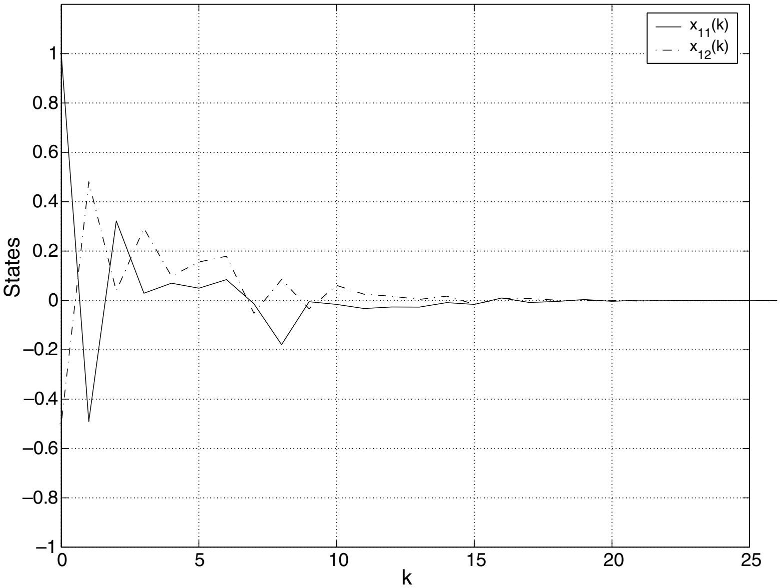

Fig. 1. State responses of subsystem 1.

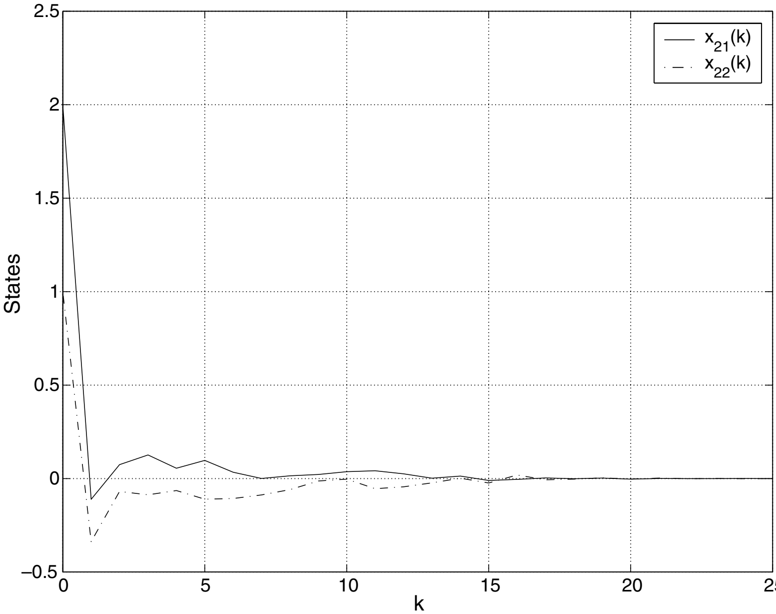

Fig. 2. State responses of subsystem 2.

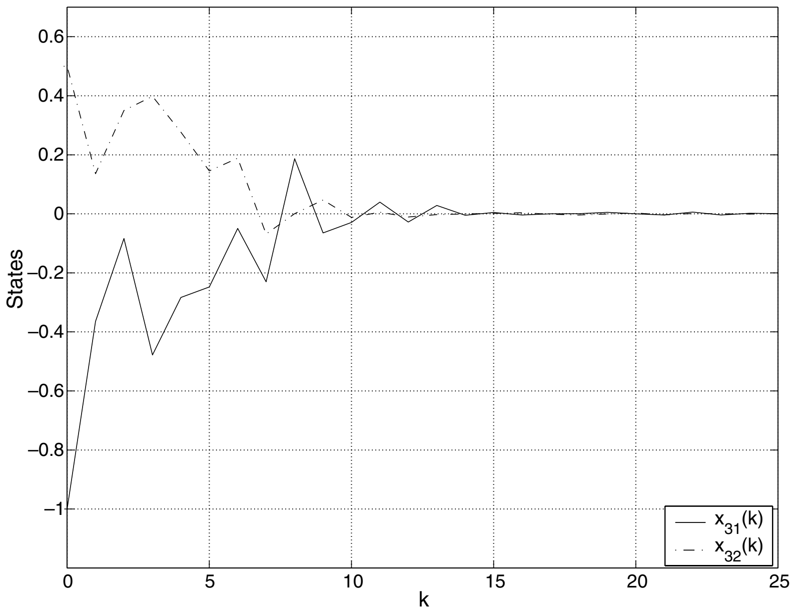

Fig. 3. State responses of subsystem 3.

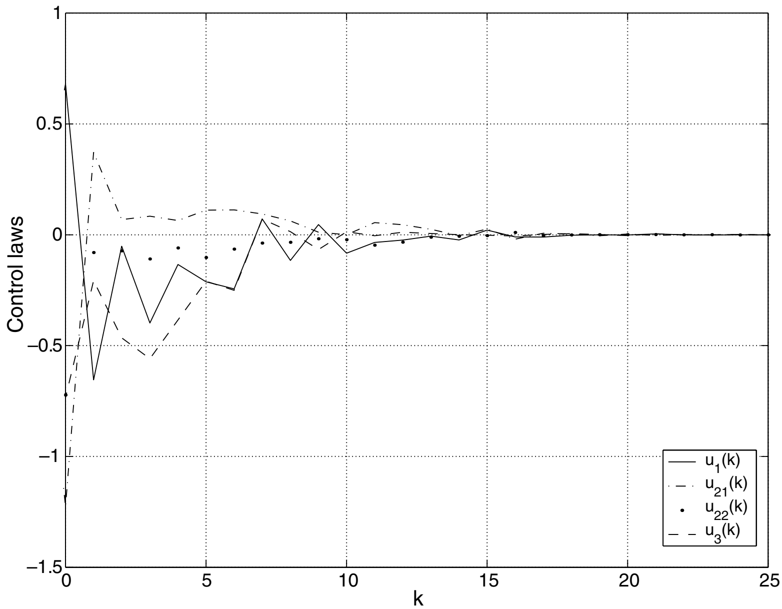

Fig. 4. Control inputs for subsystems 1–3.

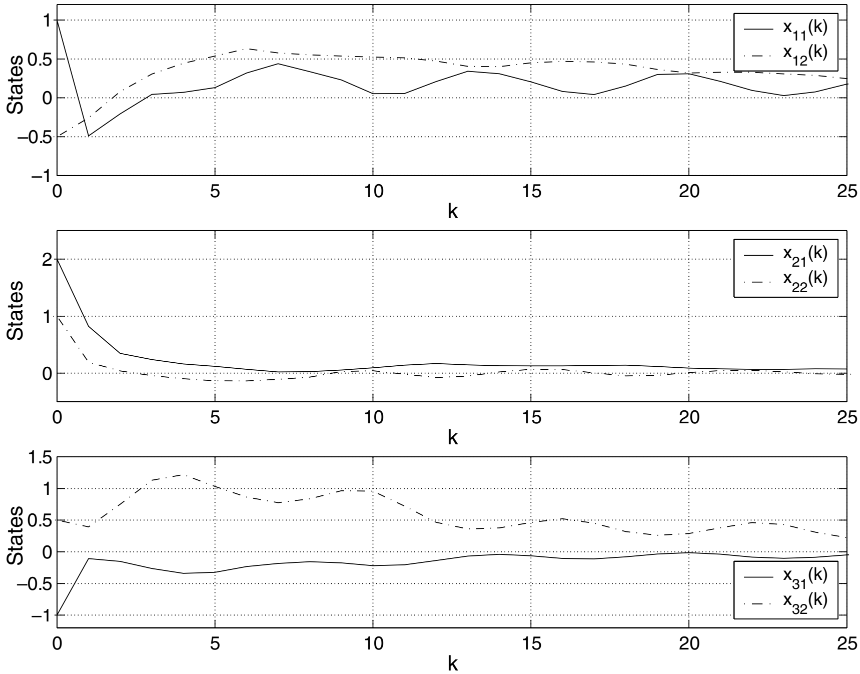

Fig. 5. Responses by conventional controller with gain variations.

Now, to briefly show the fragility of a controller designed without thinking over controller gain variations, consider a stabilizing controller obtained by classical pole-placement approach of the system (26):

$$K_1 = [\,0.24 \quad 0.3\,],$$
$$K_2 = \begin{bmatrix} -0.3 & 1 \\ 0.5 & -0.7 \end{bmatrix},$$
$$K_3 = [\,-0.4235 \quad 0.4235\,],$$

so that the closed-loop poles (eigenvalues of $A_i - B_iK_i$) of each subsystem are $\{0.3, 0.4\}, \{0.2, 0.3\}$, and $\{0.3, 0.4\}$, respectively. The system response with the controller gain variations (27) are given in Fig. 5. From this, one can see that the conventional stabilizing controller is fragile under the controller gain variations.

4. Conclusion

In this paper, we have investigated the problem of robust decentralized non-fragile control of uncertain discrete-delay large-scale systems under controller gain variations. Two classes of controller gain variations are considered. Using the Lyapunov method, the stability criteria for robust stability of the system are derived in terms of LMI. Finally, a numerical example is given for illustration of controller design, and simulation result shows that the system is well stabilized in spite of controller gain variations and uncertainties.

References

[1] S. Boyd, L.E. Ghaoui, E. Feron, V. Balakrishanan, Linear matrix inequalities in systems and control theory, SIAM Studies in Applied Mathematics, Philadelphia, PA, 1994.

[2] Y. Chen, G. Leitmann, X. Kai, Robust control design for interconnected systems with time-varying uncertainties, International Journal of Control 54 (1991) 1119–1142.

[3] Y. Chen, Decentralized robust control for large-scale uncertain systems: A design based on the bound uncertainty, Journal of Dynamic Systems Measurement and Control 114 (1992) 1–9.

[4] J. Corrado, W. Haddad, Static output controllers for systems with parametric uncertainty and control gain variation, in: Proceedings of the American Control Conference, San Diego, California, 1999, pp. 915–919.

[5] P. Dorato, Non-fragile controller design: An overview, in: Proceedings of American Control Conference, Philadelphia, Pennsylvania, 1998, pp. 2829–2831.

[6] P. Dorato, C. Abdallah, D. Famularo, On the design of non-fragile compensators via symbolic quantifier elimination, World Automation Congress, Anchorage, Alaska, 1998, pp. 9–14.

[7] D. Famularo, C. Abdallah, A. Jadbabaie, P. Dorato, W. Haddad, Robust non-fragile LQ controllers: The static state feedback case, in: Proceedings of American Control Conference, Philadelphia, Pennsylvania, 1998, pp. 1109–1113.

[8] J. Geromel, A. Yamakami, Stabilization of continuous and discrete linear systems subjected to control structure constraint, International Journal of Control 3 (1982) 429–444.

[9] W. Haddad, J. Corrado, Robust resilient dynamic controllers for systems with parametric uncertainty and controller gain variations, in: Proceedings of American Control Conference, Philadelphia, Pennsylvania, 1998, pp. 2837–2841.

[10] Z. Hu, Decentralized stabilization of large scale interconnected systems with delays, IEEE Transactions on Automatic Control 39 (1994) 180–182.

[11] L. Keel, S. Bhattacharyya, Robust, fragile, or optimal, IEEE Transactions on Automatic Control 42 (1997) 1098–1105.

[12] J. Kim, H. Park, Robust and non-fragile H_∞ control of time delay systems, Japan–Korea Joint Workshop on Robust and Predictive control of Time Delay Systems, Seoul National University, Korea, 1999, pp. 135–142.

[13] T. Lee, U. Radovic, Decentralized stabilization of linear continuous and discrete-time systems with delays in interconnections, IEEE Transactions on Automatic Control 33 (1988) 757–761.

[14] M. Mahmoud, M. Hassen, M. Darwish, Large-Scale Control Systems: Theories and Techniques, Marcel-Dekker, New York, 1985.

[15] M. Mahmoud, S. Bingulac, Robust design of stabilizing controllers for interconnected time-delay systems, Automatica 34 (1998) 795–800.

[16] S. Oucheriah, Decentralized stabilization of large scale systems with multiple delays in the interconnections, International Journal of Control 73 (2000) 1213–1223.

[17] D. Siljak, Large-Scale Dynamic Systems: Stability and Structure, North-Holland, Amsterdam, 1978.

[18] H. Trinh, M. Aldeen, A comment on decentralized stabilization of large scale interconnected systems with delays, IEEE Transactions on Automatic Control 40 (1995) 914–917.

[19] S. Won, J.H. Park, Observer-based controller design for uncertain large-scale systems with time-delays in subsystems interconnections, JSME International Journal: Series C 42 (1999) 123–128.

[20] H. Wu, K. Mizukami, S. Zhang, Robust stability conditions for large-scale interconnected systems with structured and unstructured uncertainties, Journal of Mathematical Analysis and Applications 230 (1999) 70–96.

[21] J. Yan, J. Tsai, F. Kung, Robust stabilization of large-scale systems with nonlinear uncertainties via decentralized state feedback, Journal of The Franklin Institute 335 (1998) 951–961.

[22] G. Yang, J. Wang, C. Lin, H_∞ control for linear systems with additive controller gain variations, International Journal of Control 73 (2000) 1500–1506.