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On design of dynamic output feedback controller for GCS of large-scale systems with delays in interconnections: LMI optimization approach

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Abstract

This paper considers the dynamic output feedback controller design problem for decentralized guaranteed cost stabilization (GCS) of large-scale systems with time delays in subsystem interconnections. Based on the Lyapunov method, a linear matrix inequality (LMI) convex optimization problem is formulated to find the controller which guarantees the asymptotic stability and minimizes the upper bound of a given quadratic cost function.

Keywords: Large-scale system; Dynamic output feedback controller; GCS; LMI; Time delay

1. Introduction

A large-scale dynamical system can be usually characterized by a large number of variables representing the system, a strong interaction between subsystem variables, and a complex interaction between subsystems [6,11]. Also, time delays are often encountered in large-scale systems because of

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computation data, measurement of system variables, and signal transmission between subsystems. The existence of the delay is frequently a source of instability and poor performance. Therefore, the stabilization problem of large-scale system with time-delay has been one of the most popular research topics in control systems during the last decades (see [5,7,8,12,13] and reference therein). However, no further design method is investigated to select a particular controller amongst all the admissible stabilizing controller. One way to address this performance problem is to consider a linear quadratic cost function. This approach is the so-called GCS [2,3,9,14]. Up to date, unfortunately, the topic of GCS for large-scale systems has been received very little attention.

This paper considers a class of large-scale systems with delays in subsystem interconnections. Using the Lyapunov method and LMI approach, the design method of a dynamic output feedback controller for GCS of the system, which makes the closed-loop system asymptotically stable and guarantees an adequate level of performance, is presented. Existence criteria of the controller for GCS are derived in terms of LMIs. The LMIs can be easily solved by various efficient convex optimization algorithms [1].

Notations. Through the paper, R^n denotes the n dimensional Euclidean space, $\mathsf{R}^{n\times m}$ is the set of all $n \times m$ real matrices, and I is the identity matrix with appropriate dimensions. diag$\{\cdots\}$ denotes the block diagonal matrix. $\bigstar$ denotes the symmetric part. For $X \in \mathsf{R}^{n\times n}$, the notation $X > 0$ ($X < 0$) means that matrix X is symmetric and positive-definite (negative-definite).

2. Problem formulation

Consider a class of large-scale system composed of n interconnected subsystems described by

$$\begin{aligned} \dot{x}_i(t) &= A_i x_i(t) + \sum_{j\neq i}^{n} A_{ij} x_j(t - h_{ij}) + B_i u_i(t), \\ y_i(t) &= C_i x_i(t), \quad i = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where $x_i(t) \in \mathsf{R}^{n_i}$ is the state vector, $u_i(t) \in \mathsf{R}^{m_i}$ is the control vector, $y_i(t) \in \mathsf{R}^{q_i}$ is the output vector, the time-delays h_{ij} are the positive constants, and the system matrices A_i, B_i, C_i, and A_{ij} are of appropriate dimensions. It is assumed that the triple (A_i, B_i, C_i), $i = 1, \ldots, N$, is stabilizable and detectable.

In order to stabilize system (1), let us consider the following dynamic output feedback controller for subsystem i:

$$\begin{aligned} \dot{\xi}_i(t) &= A_{ci}\xi_i(t) + B_{ci} y_i(t), \\ u_i(t) &= C_{ci}\xi_i(t), \quad \xi_i(0) = 0, \end{aligned} \tag{2}$$

where $\xi_i(t) \in \mathrm{R}^{n_i}$, and A_{ci}, B_{ci}, and C_{ci} are constant matrices of appropriate dimensions to be determined later.

The performance index associated with subsystem i is the following quadratic function:

$$J_i = \int_0^\infty (x_i^{\mathrm{T}}(t)Q_i x_i(t) + u_i^{\mathrm{T}}(t)R_i u_i(t))\,\mathrm{d}t, \tag{3}$$

where $Q_i \in \mathrm{R}^{n_i \times n_i}$ and $R_i \in \mathrm{R}^{m_i \times m_i}$ are given positive-definite matrices.

Applying the controller (2) to the system (1) results in the closed-loop system

$$\dot{z}_i(t) = \overline{A}_i z_i(t) + \sum_{j=1, j\neq i}^{n} \overline{A}_{ij} z_j(t - h_{ij}), \tag{4}$$

where for $i = 1, 2, \ldots, n$,

$$\overline{A}_i = \begin{bmatrix} A_i & B_i C_{ci} \\ B_{ci} C_i & A_{ci} \end{bmatrix}, \quad \overline{A}_{ij} = \begin{bmatrix} A_{ij} & 0 \\ 0 & 0 \end{bmatrix}, \quad z_i(t) = \begin{bmatrix} x_i(t) \\ \xi_i(t) \end{bmatrix}. \tag{5}$$

The corresponding closed-loop cost function is

$$J_i = \int_0^\infty z_i^{\mathrm{T}}(t) \begin{bmatrix} Q_i & 0 \\ 0 & C_{ci}^{\mathrm{T}} R_i C_{ci} \end{bmatrix} z_i(t)\,\mathrm{d}t \equiv \int_0^\infty z_i^{\mathrm{T}}(t)\overline{Q}_i z_i(t)\,\mathrm{d}t. \tag{6}$$

Here, the objective of this paper is to develop a procedure to design a dynamic output feedback controller (2) for system (1) and performance index (3), such that the resulting closed-loop system is asymptotically stable and the closed-loop value of the cost function (3) satisfies $J_i \leqslant J_i^\star$, where $J_i^\star$ is some specified constant.

Definition 1. For the dynamic system (1) and cost function (3), if there exist a control law $u_i^\star(t)$ and a positive constant $J_i^\star$ such that for all admissible delays, the closed-loop system (4) is asymptotically stable and the closed-loop value of the cost function (3) satisfies $J_i \leqslant J_i^\star$, then $J_i^\star$ is said to be a guaranteed cost and $u_i^\star(t)$ is said to be a guaranteed cost control law of subsystem i and its corresponding cost function (3).

Before proceeding further, we will give a well-known fact.

Fact 2 (Schur complement). Given constant symmetric matrices Σ_1, Σ_2, Σ_3 where $\Sigma_1 = \Sigma_1^{\mathrm{T}}$ and $0 < \Sigma_2 = \Sigma_2^{\mathrm{T}}$, then $\Sigma_1 + \Sigma_3^{\mathrm{T}}\Sigma_2^{-1}\Sigma_3 < 0$ if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^{\mathrm{T}} \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^{\mathrm{T}} & \Sigma_1 \end{bmatrix} < 0.$$

3. Controller design

In this section, two criteria for the existence of a dynamic output feedback controller (2) for GCS of system (1), will be derived using the Lyapunov theory and LMI convex optimization technique.

The following is a main result of the paper.

Theorem 3. *For given $Q_i > 0, R_i > 0$ and $h_{ij} > 0$, there exists a dynamic output feedback controller* (2) *for system* (1) *if there exist positive-definite matrices S_i, Y_i, $\widehat{X}_i$, and matrices $\widehat{A}_i$, $\widehat{B}_i$, $\widehat{C}_i$ satisfying the following LMIs*:

$$\begin{bmatrix} \Omega_{i1} & \alpha Y_i & Y_i Q_i & \widehat{C}_i^{\mathrm{T}} R_i & \Omega_{i2} & A_{di} \\ \star & -I & 0 & 0 & 0 & 0 \\ \star & \star & -Q_i & 0 & 0 & 0 \\ \star & \star & \star & -R_i & 0 & 0 \\ \star & \star & \star & \star & \Omega_{i3} & S_i A_{di} \\ \star & \star & \star & \star & \star & -I \end{bmatrix} < 0, \tag{7}$$

$$\begin{bmatrix} Y_i & I \\ \star & S_i \end{bmatrix} > 0 \quad \text{for} \quad i = 1, 2, \ldots, n, \tag{8}$$

where

$$\begin{aligned} &\alpha = \sqrt{n-1}, \\ &A_{di} = \left(\sum_{j \neq i}^{N} A_{ij} A_{ij}^{\mathrm{T}} \right)^{1/2}, \\ &\Omega_{i1} = A_i Y_i + Y_i A_i^{\mathrm{T}} + B_i \widehat{C}_i + \widehat{C}_i^{\mathrm{T}} B_i^{\mathrm{T}} + (n-1)\widehat{X}_i, \\ &\Omega_{i2} = A_i + \widehat{A}_i^{\mathrm{T}} + \alpha^2 Y_i + Y_i Q_i, \\ &\Omega_{i3} = S_i A_i + A_i^{\mathrm{T}} S_i + C_i^{\mathrm{T}} \widehat{B}_i^{\mathrm{T}} + \widehat{B}_i C_i + \alpha^2 I + Q_i. \end{aligned} \tag{9}$$

Then, the upper bound of cost function for subsystem i is

$$J_i \leqslant x_i^{\mathrm{T}}(0) S_i x(0) + \sum_{j=1, j \neq i}^{n} \int_{-h_{ij}}^{0} x_j^{\mathrm{T}}(s) x_j(s)\, \mathrm{d}s \triangleq J_i^{\star}. \tag{10}$$

Proof. Consider a Lyapunov function for system (4)

$$V = \sum_{i=1}^{n} V_i \equiv \sum_{i=1}^{n} \left(z_i^{\mathrm{T}}(t) P_i z_i(t) + \sum_{j=1, j \neq i}^{n} \int_{t-h_{ij}}^{t} z_j^{\mathrm{T}}(s) z_j(s)\, \mathrm{d}s \right),$$

where $P_i > 0$, $i = 1, 2, \ldots, n$.

The time derivative of V along with the solution of (4) is

$$\dot{V} = \sum_{i=1}^{n} \dot{V}_i = \sum_{i=1}^{n} \left\{ z_i^{\mathrm{T}}(t)(\overline{A}_i^{\mathrm{T}} P_i + P_i \overline{A}_i) z_i(t) + 2 z_i^{\mathrm{T}}(t) P_i \sum_{j=1, j\neq i}^{n} \overline{A}_{ij} z_j(t - h_{ij}) \right.$$
$$\left. + \sum_{j=1, i\neq j}^{n} (z_j^{\mathrm{T}}(t) z_j(t) - z_j^{\mathrm{T}}(t - h_{ij}) z_j(t - h_{ij})) \right\}. \tag{11}$$

Note that

$$\sum_{i=1}^{n} \sum_{j=1, j\neq i}^{n} z_j^{\mathrm{T}}(t) z_j(t) \equiv (n-1) \sum_{i=1}^{n} z_i^{\mathrm{T}}(t) z_i(t). \tag{12}$$

Using the known fact that

$$ab^{\mathrm{T}} + ba^{\mathrm{T}} \leqslant \varepsilon aa^{\mathrm{T}} + \varepsilon^{-1} bb^{\mathrm{T}}, \quad \varepsilon > 0$$

for any vectors a, b, we obtain

$$2 \sum_{i=1}^{N} z_i^{\mathrm{T}}(t) P_i \sum_{j\neq i}^{N} \overline{A}_{ij} z_j(t - h_{ij}) \leqslant \sum_{i=1}^{N} \left(z_i^{\mathrm{T}}(t) P_i \overline{A}_{di} \overline{A}_{di}^{\mathrm{T}} P_i z_i(t) \right.$$
$$\left. + \sum_{j\neq i}^{N} z_j^{\mathrm{T}}(t - h_{ij}) z_j(t - h_{ij}) \right), \tag{13}$$

where

$$\overline{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}.$$

Thus, we have

$$\dot{V} \leqslant \sum_{i=1}^{N} z_i^{\mathrm{T}}(t) M_i z_i(t) - \sum_{i=1}^{n} z_i^{\mathrm{T}}(t) \overline{Q}_i z_i(t), \tag{14}$$

where $M_i = \overline{A}_i^{\mathrm{T}} P_i + P_i \overline{A}_i + P_i \overline{A}_{di} \overline{A}_{di}^{\mathrm{T}} P_i + (n-1)I + \overline{Q}_i$.

Therefore, if $M_i < 0$ for all i, there exists the positive scalars γ_i such that

$$\dot{V} \leqslant -\sum_{i=1}^{n} z_i^{\mathrm{T}}(t) \overline{Q}_i z_i(t) \leqslant -\sum_{i=1}^{n} \gamma_i \|x_i(t)\|^2, \tag{15}$$

which guarantees the asymptotic stability of the system by Lyapunov stability theory.

By Fact 2, the inequality, $M_i < 0$, is equivalent to the following inequality:

$$\overline{M}_i \equiv \begin{bmatrix} \overline{A}_i^{\mathrm{T}} P_i + P_i \overline{A}_i + (n-1)I + \overline{Q}_i & P_i \overline{A}_{di} \\ \star & -I \end{bmatrix} < 0 \quad \text{for} \quad i = 1, 2, \ldots, n. \tag{16}$$

Note that in matrix $\overline{M}_i$, the matrices P_i and the controller parameters A_{ci}, B_{ci} and C_{ci}, which included in the matrix $\overline{A}_i$, are unknown. In the following, we will use a method of changing variables such that the inequality can be solved as convex optimization algorithms [10].

First, partition the matrix P_i and its inverse as

$$P_i = \begin{bmatrix} S_i & N_i \\ N_i^{\mathrm{T}} & U_i \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} Y_i & M_i \\ M_i^{\mathrm{T}} & W_i \end{bmatrix}, \tag{17}$$

where $S_i, Y_i \in \mathrm{R}^{n_i \times n_i}$ are positive definite matrices, and M_i and N_i are invertible matrices. Note that the equality $P_i^{-1}P_i = I$ gives that

$$M_i N_i^{\mathrm{T}} = I - Y_i S_i. \tag{18}$$

Define

$$F_{i1} = \begin{bmatrix} Y_i & I \\ M_i^{\mathrm{T}} & 0 \end{bmatrix}, \quad F_{i2} = \begin{bmatrix} I & S_i \\ 0 & N_i^{\mathrm{T}} \end{bmatrix}. \tag{19}$$

Then, it follows that

$$P_i F_{i1} = F_{i2}, \quad F_{i1}^{\mathrm{T}} P_i F_{i1} = F_{i1}^{\mathrm{T}} F_{i2} = \begin{bmatrix} Y_i & I \\ I & S_i \end{bmatrix} > 0. \tag{20}$$

Next, postmultiplying and premultiplying the matrix inequality, $\overline{M}_i < 0$, by the matrix $\mathrm{diag}\{F_{i1}^{\mathrm{T}}, I\}$ and by its transpose, respectively, gives

$$\begin{bmatrix} F_{i2}^{\mathrm{T}}\overline{A}_i F_{i1} + F_{i1}^{\mathrm{T}}\overline{A}_i^{\mathrm{T}} F_{i2} + F_{i1}^{\mathrm{T}}((n-1)I + \overline{Q}_i)F_{i1} & F_{i2}^{\mathrm{T}}\overline{A}_{di} \\ \star & -I \end{bmatrix} < 0. \tag{21}$$

By utilizing relations (17)–(20), it can be easily obtained that the inequality (21) is modified to

$$\begin{bmatrix} (1,1) & (1,2) & (1,3) & 0 \\ \star & (2,2) & (2,3) & 0 \\ \star & \star & -I & 0 \\ \star & \star & \star & -I \end{bmatrix} < 0, \tag{22}$$

where

$$\begin{aligned}(1,1) = {} & A_i Y_i + Y_i A_i^{\mathrm{T}} + B_i C_{ci} M_i^{\mathrm{T}} + M_i C_{ci}^{\mathrm{T}} B_i^{\mathrm{T}} + (n-1) Y_i Y_i + Y_i Q_i Y_i \\ & + (n-1) M_i M_i^{\mathrm{T}} + M_i C_{ci}^{\mathrm{T}} R_i C_{ci} M_i^{\mathrm{T}},\end{aligned}$$

$$(1,2) = A_i + Y_i A_i^{\mathrm{T}} S_i + M_i C_{ci}^{\mathrm{T}} B_i^{\mathrm{T}} S_i + Y_i C_i^{\mathrm{T}} B_{ci}^{\mathrm{T}} N_i^{\mathrm{T}} + M_i A_{ci}^{\mathrm{T}} N_i^{\mathrm{T}} + (n-1) Y_i + Y_i Q_i,$$

$$(1,3) = A_{di},$$

$$(2,2) = S_i A_i + N_i B_{ci} C_i + A_i^{\mathrm{T}} S_i + C_i^{\mathrm{T}} B_{ci}^{\mathrm{T}} N_i^{\mathrm{T}} + (n-1)I + Q_i,$$

$$(2,3) = S_i A_{di}.$$

By defining a new set of variables as follows:

$$\begin{aligned} \widehat{A}_i &= S_i A_i Y_i + S_i B_i \widehat{C}_i + \widehat{B}_i C_i Y_i + N_i A_{ci} M_i^{\mathrm{T}}, \\ \widehat{B}_i &= N_i B_{ci}, \\ \widehat{C}_i &= C_{ci} M_i^{\mathrm{T}}, \\ \widehat{X}_i &= M_i M_i^{\mathrm{T}}. \end{aligned} \tag{23}$$

The inequality (22) is simplified to the following inequality:

$$\begin{bmatrix} \Omega_{i1} + (n-1)Y_i Y_i + Y_i Q_i Y_i + \widehat{C}_i^{\mathrm{T}} R_i \widehat{C}_i & \Omega_{i2} & A_{di} & 0 \\ \star & \Omega_{i3} & S_i A_{di} & 0 \\ \star & \star & -I & 0 \\ \star & \star & \star & -I \end{bmatrix} < 0, \tag{24}$$

where Ω_{i1}, Ω_{i2}, and Ω_{i3} are defined in (9).

By Fact 2 (Schur complement), the inequality (24) is equivalent to the LMI (7).

On the other hand, from (15) we have

$$\dot{V}_i \leqslant -z_i^{\mathrm{T}}(t)\overline{Q}_i z_i(t). \tag{25}$$

Integrating both sides of the above inequality from 0 to T_f leads to

$$\int_0^{\mathrm{T}_f} z_i^{\mathrm{T}}(t)\overline{Q}_i z_i(t) < V(0) - V(T_f). \tag{26}$$

Since the asymptotic stability of the system has already been established, we conclude that $V(T_f) \to 0$ as $t \to \infty$. Hence we have

$$\begin{aligned} J_i &\leqslant z_i^{\mathrm{T}}(0) P_i z_i(0) + \sum_{j=1, j\neq i}^{n} \int_{-h_{ij}}^{0} z_j^{\mathrm{T}}(s) z^j(s)\,\mathrm{d}s \\ &= x_i^{\mathrm{T}}(0) S_i x(0) + \sum_{j=1, j\neq i}^{n} \int_{-h_{ij}}^{0} x_j^{\mathrm{T}}(s) x_j(s)\,\mathrm{d}s = J_i^{\star}. \end{aligned} \tag{27}$$

This completes the proof. $\square$

Theorem 3 presents a method of designing a dynamic output feedback controller for GCS of system (1). In the following, we will present a method of selecting the optimal controller minimizing the upper bound of the guaranteed cost (10).

Theorem 4. *Consider the system* (1) *with cost function* (3). *For all i, if the following LMI optimization problem,*

$$\min_{\beta_i,\widehat{X}_i,\widehat{A}_i,\widehat{B}_i,\widehat{C}_i,Y_i,S_i} \beta_i \tag{28}$$

subject to

$$\text{(i) LMIs}(7) \text{ and } (8), \tag{29}$$

$$\text{(ii)} \begin{bmatrix} -\beta_i & x_i^{\mathrm{T}}(0)S_i \\ \star & -S_i \end{bmatrix} < 0, \tag{30}$$

has the solution set (β_i, $\widehat{X}_i$, $\widehat{A}_i$, $\widehat{B}_i$, $\widehat{C}_i$, Y_i, S_i), *the controller* (2) *is the optimal dynamic output feedback controller which ensures the minimization of the guaranteed cost* (10) *of the system. The optimal cost of each subsystem is* $J_i^\star = \beta_i + \Phi_i$, *where* $\Phi_i = \sum_{j=1,j\neq i}^{n} \int_{-h_{ij}}^{0} x_j^{\mathrm{T}}(s)x_j(s)\,\mathrm{d}s$.

Proof. By Theorem 3, (i) in the optimization problem (28) is clear, and from Fact 2 (ii) is equivalent to $x_i^{\mathrm{T}}(0)S_ix_i(0) < \beta_i$. So, it follows from (10) that $J_i^\star = \beta_i + \Phi_i$. Thus, the minimization of β_i implies the minimization of the guaranteed cost (10). It is well-known that the convexity of the LMI optimization problem ensures that a global optimum, when it exists, is reachable. This completes the proof. □.

Remark 5. The problem of Theorems 3 and 4 is to determine whether the problem is feasible or not. It is called the feasibility problem. The solutions of the problem can be found by solving eigenvalue problem for variables, which is a convex optimization problem. Various efficient convex optimization algorithms can be used to check whether the LMIs is feasible. A well-known LMI solver is the Matlab's LMI Control Toolbox [4], which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithms [1].

Remark 6. Given any solution of the LMIs in Theorem 4, a corresponding controller of the form (2) will be constructed as follows:

- Using the solution $\widehat{X}_i$, compute the invertible matrices M_i satisfying the relation $\widehat{X}_i = M_iM_i^{\mathrm{T}}$.
- Using the matrix M_i, compute the invertible matrix N_i satisfying (18).

- Utilizing the matrices M_i and N_i obtained above, solve the system of equations (23) for B_{ci}, C_{ci} and A_{ci} (in this order).

Remark 7. Uncertainties in system (1) are not considered for simplicity. However, the results obtained can be easily generalized to system with uncertainties.

4. Conclusions

In this paper, the design problem of output feedback controller for GCS of a class of large-scale systems with delays in subsystem interconnections has been investigated by using the Lyapunov method. Two criteria for GCS have been presented in terms of LMIs. The LMIs can be easily solved by various efficient convex optimization algorithms.

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