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On guaranteed cost control of neutral systems by retarded integral state feedback

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Abstract

In this paper, the guaranteed cost control problem for a class of neutral delay-differential systems with a given quadratic cost functions is investigated. The problem is to design a memory state feedback controller such that the closed-loop system is asymptotically stable and the closed-loop cost function value is not more than a specified upper bound. Some criteria for the existence of such controllers is derived based on the linear matrix inequality (LMI) approach combined with the Lyapunov method. A parameterized characterization of the controllers is given in terms of the feasible solutions to the certain LMIs. A numerical example is given to illustrate the proposed method.

Keywords: Neutral systems; Guaranteed cost control; LMI; Lyapunov method

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Nomenclature

$\Re^n$	n-dimensional real space
$\Re^{m\times n}$	set of all real m by n matrices
x^{T} (or A^{T})	transpose of vector x (or matrix A)
$P > 0$	(respectively $P < 0$) matrix P is symmetric positive (respectively negative) definite
I	identity matrix of appropriate dimension
$\star$	the elements below the main diagonal of a symmetric block matrix
$\mathscr{C}_0$	a set of all continuous differentiable function on given interval
$\mathrm{diag}\{\cdots\}$	block diagonal matrix

1. Introduction

During the last three decades, the stability and stabilization problem of delay-differential systems has received considerable attention and many papers dealing with this problem have appeared because of the existence of delays in various practical control problems and also because of the fact that the delay is frequently a source of instability and performance degradation of systems. Especially, in recent years, the problem for various neutral delay-differential systems has also received some attention [1,2]. The theory of neutral delay-differential systems is of both theoretical and practical interest. In the literature, various stability analysis and stabilization techniques have been utilized to derive stability/stabilization criteria for asymptotic stability of the systems by many researchers [3–9]. On the other hand, in many practical system, it is desirable to design the control system which is not only stable but also guarantee an adequate level of performance. One way to address this problem is so-called guaranteed cost control [10]. The approach has the advantage of providing an upper bound on a given performance index and thus the system performance degradation incurred by time delays is guaranteed to be less than this bound. Based on this idea, some results have been proposed for discrete-delay systems [11] and for neutral delay-differential system [12] using memoryless feedback controller. However, if we design a memory state feedback controller with feedback provisions on current state and the past history of the state, we may expect to achieve an improved performance.

With this motivation, we consider a class of neutral delay-differential systems. Using the Lyapunov functional technique combined with LMI technique, we develop a guaranteed cost control for the system via retarded integral state feedback controller, which makes the closed-loop system asymptotically stable and guarantees an adequate level of performance. A stabilization criterion for the existence of the guaranteed cost controller is derived in terms of LMIs, and theirs solutions provide a parameterized representation of the control. The LMIs can be easily solved by various efficient convex optimization algorithms [13].

2. Problem statements

Consider a class of neutral delay-differential system of the form:

$$\begin{aligned} &\frac{d}{dt}[x(t) - A_2x(t-\tau)] = A_0x(t) + A_1x(t-h) + Bu(t), \\ &x(t_0+\theta) = \phi(\theta), \quad \forall\ \theta \in [-H, 0], \end{aligned} \tag{1}$$

where $x(t) \in \Re^n$ is the state vector, A_0, A_1, A_2, and B are known constant real matrices of appropriate dimensions, $u(t) \in \Re^m$ is the control input vector, h and τ are the positive constant time delays, $H = \max\{h, \tau\}$, $\phi(\cdot) \in \mathscr{C}_0 : [-H\ , 0] \to \Re^n$ is the initial vector. In this paper, it is assumed that the pair $(A_0 + A_1, B)$ is completely controllable. This is a basic requirement for controller design.

Now, we are interested in designing a memory retarded integral state feedback controller for the system (1) as

$$u(t) = -K\left(x(t) + \int_{t-h}^{t} A_1x(s)\,ds - A_2x(t-\tau)\right), \tag{2}$$

where K is a control gain to be designed.

Associated with the system (1) is the following quadratic cost function

$$J = \int_0^{inf} \left(x^T(t)Qx(t) + u^T(t)Su(t)\right)dt, \tag{3}$$

where $Q \in \Re^{n\times n}$ and $S \in \Re^{m\times m}$ are given positive-definite matrices.

Here, the objective of this paper is to develop a procedure to design a memory state feedback controller $u(t)$ for the system (1) and cost function (3) such that the resulting closed-loop system is asymptotically stable and the closed-loop value of the cost function (3) satisfies $J \leqslant J^*$, where J^* is some specified constant.

Definition 1. For the neutral system (1) and cost function (3), if there exist a control law $u^*(t)$ and a positive J^* such that for all admissible delays, the system (1) is asymptotically stable and the closed-loop value of the cost function (3) satisfies $J \leqslant J^*$, then J^* is said to be a guaranteed cost and $u^*(t)$ is said to be a guaranteed cost control law of the system (1) and cost function (3).

Before proceeding further, we will state a well known fact and two lemmas.

Fact 1. The linear matrix inequality

$$\begin{bmatrix} Z(x) & Y(x) \\ Y^{\mathrm{T}}(x) & W(x) \end{bmatrix} > 0$$

is equivalent to

$$W(x) > 0, \quad Z(x) - Y(x)W^{-1}(x)Y^{\mathrm{T}}(x) > 0,$$

where $Z(x) = Z^{\mathrm{T}}(x)$, $W(x) = W^{\mathrm{T}}(x)$ and $Y(x)$ depend affinely on x.

Lemma 1 [14]. *For any constant matrix $M \in \Re^{n\times n}$, $M = M^T > 0$, scalar $\gamma > 0$, vector function $\omega : [0,\gamma] \to \Re^n$ such that the integrations concerned are well defined, then*

$$\left(\int_0^{\gamma} \omega(s)\,\mathrm{d}s\right)^{\mathrm{T}} M \left(\int_0^{\gamma} \omega(s)\,\mathrm{d}s\right) \leqslant \gamma \int_0^{\gamma} \omega^{\mathrm{T}}(s) M \omega(s)\,\mathrm{d}s.$$

Lemma 2 [9]. *For given positive scalars h and τ and any A_1, $A_2 \in \Re^{n\times n}$, the operator $\mathscr{D}(x_t) : \mathscr{C}_0 \to \Re^n$ defined by*

$$\mathscr{D}(x_t) = x(t) + \int_{t-h}^{t} A_1 x(s)\,\mathrm{d}s - A_2 x(t-\tau) \tag{4}$$

is stable if there exist a positive definite matrix Γ_0 and positive scalars α_1 and α_2 such that

$$\alpha_1 + \alpha_2 < 1, \quad \begin{bmatrix} A_2^{\mathrm{T}}\Gamma_0 A_2 - \alpha_1\Gamma_0 & hA_2^{\mathrm{T}}\Gamma_0 A_1 \\ \star & h^2 A_1^{\mathrm{T}}\Gamma_0 A_1 - \alpha_2\Gamma_0 \end{bmatrix} < 0. \tag{5}$$

Differentiating $\mathscr{D}(x_t)$ and combining Eqs. (1) and (2) leads to

$$\begin{aligned} \dot{\mathscr{D}}(x_t) &= \dot{x}(t) + A_1 x(t) - A_1 x(t-h) - A_2\dot{x}(t-\tau) \\ &= (A_0 + A_1 - BK)x(t) - BK\int_{t-h}^{t} A_1 x(s)\,\mathrm{d}s + BKA_2 x(t-\tau) \\ &= (A - BK)\mathscr{D}(x_t) - A\int_{t-h}^{t} A_1 x(s)\,\mathrm{d}s + AA_2 x(t-\tau), \end{aligned} \tag{6}$$

where $A = A_0 + A_1$.

Now, we establish a criterion in terms of LMIs, for asymptotic stabilization of (1) using the Lyapunov method.

Theorem 1. *Suppose that there exist* $\Gamma_0 > 0$, $\alpha_1 > 0$, *and* $\alpha_2 > 0$ *satisfying* (5). *Then, for given* $Q > 0$ *and* $S > 0$, *the controller* $u(t)$ *given in* (2) *is a guaranteed cost controller for the system* (1) *if there exist the positive-definite matrices* X, Z_1, Z_2 *and a matrix* Y *satisfying the following LMI*:

$$\begin{bmatrix} \begin{pmatrix} AX+XA^{\mathrm T} \\ -BY-Y^{\mathrm T}B^{\mathrm T} \end{pmatrix} & Y^{\mathrm T}S & -AA_1Z_1 & AA_2Z_2 & hX & X & XQ \\ \star & -S & 0 & 0 & 0 & 0 & 0 \\ \star & \star & -Z_1 & 0 & -hZ_1A_1 & -Z_1A_1 & -Z_1A_1Q \\ \star & \star & \star & -Z_2 & hZ_2A_2 & Z_2A_2 & Z_2A_2Q \\ \star & \star & \star & \star & -Z_1 & 0 & 0 \\ \star & \star & \star & \star & \star & -Z_2 & 0 \\ \star & \star & \star & \star & \star & \star & -Q \end{bmatrix} < 0. \tag{7}$$

Also, the gain matrix of the controller (2) *is* $K = YX^{-1}$, *and the upper bound of the quadratic cost function* J *is*

$$J^* = \mathscr{D}^{\mathrm T}(0)X^{-1}\mathscr{D}(0) + h\int_{-h}^{0}(s+h)x^{\mathrm T}(s)Z_1^{-1}x(s)\,\mathrm{d}s + \int_{-\tau}^{0}x^{\mathrm T}(s)Z_2^{-1}x(s)\,\mathrm{d}s, \tag{8}$$

where $\mathscr{D}(0)$ *denotes* $\mathscr{D}(x_t)|_{t=0}$.

Proof. For $P > 0$, $R_1 > 0$, and $R_2 > 0$, the functional given by

$$V = V_1 + V_2 + V_3, \tag{9}$$

where

$$V_1 = \mathscr{D}(x_t)^{\mathrm T}P\mathscr{D}(x_t), \tag{10}$$

$$V_2 = \int_{t-h}^{t}(s-t+h)x^{\mathrm T}(s)R_1x(s)\,\mathrm{d}s, \tag{11}$$

$$V_3 = \int_{t-\tau}^{t}x^{\mathrm T}(s)R_2x(s)\,\mathrm{d}s \tag{12}$$

is a legitimate Lyapunov functional candidate [1].

Taking the time derivative of V along the solution of (6) gives that

$$\frac{\mathrm{d}V_1}{\mathrm{d}t} = 2\mathscr{D}(x_t)^{\mathrm T}P\left[(A-BK)\mathscr{D}(x_t) - A\int_{t-h}^{t}A_1x(s)\,\mathrm{d}s + AA_2x(t-\tau)\right], \tag{13}$$

$$\frac{\mathrm{d}V_2}{\mathrm{d}t} = hx^{\mathrm{T}}(t)R_1x(t) - \int_{t-h}^{t} x^{\mathrm{T}}(s)R_1x(s)\,\mathrm{d}s, \leqslant hx^{\mathrm{T}}(t)R_1x(t) - \left(\int_{t-h}^{t} x(s)\,\mathrm{d}s\right)^{\mathrm{T}}(h^{-1}R_1)\left(\int_{t-h}^{t} x(s)\,\mathrm{d}s\right), \tag{14}$$

$$\frac{\mathrm{d}V_3}{\mathrm{d}t} = x^{\mathrm{T}}(t)R_2x(t) - x^{\mathrm{T}}(t-\tau)R_2x(t-\tau), \tag{15}$$

where Lemma 1 is utilized in (14).

Here, let $M = hR_1 + R_2$ and note that

$$\begin{aligned} x^{\mathrm{T}}(t)Mx(t) &= \left(\mathscr{D}(x_t) - \int_{t-h}^{t} A_1x(s)\,\mathrm{d}s + A_2x(t-\tau)\right)^{\mathrm{T}} \\ &\quad \times M\left(\mathscr{D}(x_t) - \int_{t-h}^{t} A_1x(s)\,\mathrm{d}s + A_2x(t-\tau)\right) \\ &= \mathscr{D}^{\mathrm{T}}(x_t)M\mathscr{D}(x_t) - 2\mathscr{D}^{\mathrm{T}}(x_t)M\int_{t-h}^{t} A_1x(s)\,\mathrm{d}s + 2\mathscr{D}^{\mathrm{T}}(x_t)MA_2x(t-\tau) \\ &\quad + \left(\int_{t-h}^{t} A_1x(s)\,\mathrm{d}s\right)^{\mathrm{T}} M\left(\int_{t-h}^{t} A_1x(s)\,\mathrm{d}s\right) \\ &\quad - 2\left(\int_{t-h}^{t} A_1x(s)\,\mathrm{d}s\right)^{\mathrm{T}} MA_2x(t-\tau) + x(t-\tau)^{\mathrm{T}}A_2^{\mathrm{T}}MA_2x(t-\tau). \end{aligned} \tag{16}$$

Then, a new bound of the time-derivative of V is as follows:

$$\frac{\mathrm{d}V}{\mathrm{d}t} = \sum_{i=1}^{3}\frac{\mathrm{d}V_i}{\mathrm{d}t} \leqslant \chi^{\mathrm{T}}(t)\Omega\chi(t), \tag{17}$$

where

$$\chi(t) = \begin{bmatrix} \mathscr{D}(x_t) \\ \int_{t-h}^{t} x(s)\,\mathrm{d}s \\ x(t-\tau) \end{bmatrix} \tag{18}$$

and

$$\Omega = \begin{bmatrix} \begin{pmatrix} P(A-BK)+ \\ (A-BK)^{\mathrm{T}}P + M \end{pmatrix} & -PAA_1 - MA_1 & MA_2 + PAA_2 \\ \star & -h^{-1}R_1 + A_1^{\mathrm{T}}MA_1 & -A_1^{\mathrm{T}}MA_2 \\ \star & \star & -R_2 + A_2^{\mathrm{T}}MA_2 \end{bmatrix}. \tag{19}$$

Again, applying the relation (16) to the terms $x^{\mathrm{T}}(t)Qx(t)$ and using

$$u^{\mathrm{T}}(t)Su(t) = \mathscr{D}^{\mathrm{T}}(x_t)K^{\mathrm{T}}SK\mathscr{D}(x_t), \tag{20}$$

gives that

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leqslant \chi^{\mathrm{T}}(t)\Omega_1\chi(t) - \left(x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Su(t)\right), \tag{21}$$

where

$$\begin{aligned}
\Omega_1 &= \begin{bmatrix} \begin{pmatrix} P(A-BK)+ \\ (A-BK)^{\mathrm{T}}P + M \\ +Q + K^{\mathrm{T}}SK \end{pmatrix} & -PAA_1 - MA_1 - QA_1 & MA_2 + PAA_2 + QA_2 \\ \star & -h^{-1}R_1 + A_1^{\mathrm{T}}MA_1 + A_1^{\mathrm{T}}QA_1 & -A_1^{\mathrm{T}}MA_2 - A_1^{\mathrm{T}}QA_2 \\ \star & \star & -R_2 + A_2^{\mathrm{T}}MA_2 + A_2^{\mathrm{T}}QA_2 \end{bmatrix} \\
&= \begin{bmatrix} P(A-BK) + (A-BK)^{\mathrm{T}}P + K^{\mathrm{T}}SK & -PAA_1 & PAA_2 \\ \star & -h^{-1}R_1 & 0 \\ \star & \star & -R_2 \end{bmatrix} \\
&\quad + \begin{bmatrix} I \\ -A_1^{\mathrm{T}} \\ A_2^{\mathrm{T}} \end{bmatrix} (M+Q)[I \quad -A_1 \quad A_2].
\end{aligned} \tag{22}$$

Therefore, if $\Omega_1 < 0$, there exists the positive scalar γ such that

$$\frac{\mathrm{d}V}{\mathrm{d}t} \leqslant -\gamma\|x(t)\|^2. \tag{23}$$

By Fact 1, the inequality $\Omega_1 < 0$ is equivalent to

$$\Omega_2 = \begin{bmatrix} \begin{pmatrix} P(A-BK) + (A-BK)^{\mathrm{T}}P \\ +K^{\mathrm{T}}SK \end{pmatrix} & -PAA_1 & PAA_2 & hI & I & I \\ \star & -h^{-1}R_1 & 0 & -hA_1 & -A_1 & -A_1 \\ \star & \star & -R_2 & hA_2 & A_2 & A_2 \\ \star & \star & \star & -hR_1^{-1} & 0 & 0 \\ \star & \star & \star & \star & -R_2^{-1} & 0 \\ \star & \star & \star & \star & \star & -Q^{-1} \end{bmatrix} < 0. \tag{24}$$

Letting $X = P^{-1}$, $Y = KX$, $Z_1 = hR_1^{-1}$, $Z_2 = R_2^{-1}$, and pre- and post-multiplying the matrix Ω_2 by $\mathrm{diag}\{X, Z_1, Z_2, I, I, Q\}$, give that $\Omega_2 < 0$ is equivalent to the following inequality:

$$\begin{bmatrix} \begin{pmatrix} AX + XA^{\mathrm{T}} - BY \\ -Y^{\mathrm{T}}B^{\mathrm{T}} + Y^{\mathrm{T}}SY \end{pmatrix} & -AA_1Z_1 & AA_2Z_2 & hX & X & XQ \\ \star & -Z_1 & 0 & -hZ_1A_1 & -Z_1A_1 & -Z_1A_1Q \\ \star & \star & -Z_2 & hZ_2A_2 & Z_2A_2 & Z_2A_2Q \\ \star & \star & \star & -Z_1 & 0 & 0 \\ \star & \star & \star & \star & -Z_2 & 0 \\ \star & \star & \star & \star & \star & -Q \end{bmatrix} < 0. \tag{25}$$

Again, by Fact 1, the inequality (25) is equivalent to the LMI (7). This implies that both the system (1) and (6) with stable operator $\mathscr{D}(x_t)$ are asymptotically stable by Theorem 9.8.1 in [1]. Furthermore, we have

$$x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Su(t) < -\frac{\mathrm{d}V}{\mathrm{d}t}.$$

Integrating both sides of the above inequality from 0 to T_f leads to

$$\begin{aligned} \int_0^{T_f} (x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Su(t))\,\mathrm{d}t &< V(0) - V(T_f) \\ &= \left(\mathscr{D}^{\mathrm{T}}(0)P\mathscr{D}(0) - \mathscr{D}^{\mathrm{T}}(T_f)P\mathscr{D}(T_f)\right) \\ &\quad + \left(\int_{-h}^{0}(s+h)x^{\mathrm{T}}(s)R_1x(s)\,\mathrm{d}s - \int_{T_f-h}^{T_f}(s+h)x^{\mathrm{T}}(s)R_1x(s)\,\mathrm{d}s\right) \\ &\quad + \left(\int_{-\tau}^{0}x^{\mathrm{T}}(s)R_2x(s)\,\mathrm{d}s - \int_{T_f-\tau}^{T_f}x^{\mathrm{T}}(s)R_2x(s)\,\mathrm{d}s\right). \end{aligned}$$

As both the operator $\mathscr{D}(x_t)$ and the system (1) are stable, when $T_f \to \infty$,

$$\mathscr{D}^{\mathrm{T}}(T_f)P\mathscr{D}(T_f) \to 0, \qquad \int_{T_f-h}^{T_f} x^{\mathrm{T}}(s)R_1x(s)\,\mathrm{d}s \to 0,$$
$$\int_{T_f-\tau}^{T_f} x^{\mathrm{T}}(s)R_2x(s)\,\mathrm{d}s \to 0.$$

Hence we get

$$\begin{aligned} \int_0^{\infty} (x^{\mathrm{T}}(t)Qx(t) + u^{\mathrm{T}}(t)Su(t))\,\mathrm{d}t &< V(0) \\ &= \mathscr{D}^{\mathrm{T}}(0)P\mathscr{D}(0) + \int_{-h}^{0}(s+h)x^{\mathrm{T}}(s)R_1x(s)\,\mathrm{d}s + \int_{-\tau}^{0}x^{\mathrm{T}}(s)R_2x(s)\,\mathrm{d}s \triangleq J^*. \end{aligned} \tag{26}$$

This completes our proof. □

Theorem 1 presents a method of designing a state feedback guaranteed cost controller. The following theorem presents a method of selecting a controller minimizing the upper bound of the guaranteed cost (8).

Theorem 2. *Consider the system* (1) *with cost function* (3). *If the following optimization problem*

$$\begin{aligned}
&\min_{X>0,\Gamma_1>0,\Gamma_2>0,Z_1>0,Z_2>0,Y,\alpha>0} \quad \{\alpha + \mathrm{tr}(\Gamma_1) + \mathrm{tr}(\Gamma_2)\} \\
&\text{subject to} \quad \text{(i) LMI (7)} \\
&\qquad \text{(ii)} \begin{bmatrix} -\alpha & \mathscr{D}^{\mathrm{T}}(0) \\ \mathscr{D}(0) & -X \end{bmatrix} < 0, \\
&\qquad \text{(iii)} \begin{bmatrix} -\Gamma_1 & hN_1^{\mathrm{T}} \\ hN_1 & -hZ_1 \end{bmatrix} < 0, \\
&\qquad \text{(iv)} \begin{bmatrix} -\Gamma_2 & N_2^{\mathrm{T}} \\ N_2 & -Z_2 \end{bmatrix} < 0,
\end{aligned} \tag{27}$$

has a positive solution set $(X, \Gamma_1, \Gamma_2, Z_1, Z_2, Y, \alpha)$, *then the control law* (2) *is an optimal robust guaranteed cost control law which ensures the minimization of the guaranteed cost* (8) *for neutral system* (1), *where* $\int_{-h}^{0}(s+h)x(s)x^{\mathrm{T}}(s)\,\mathrm{d}s = N_1N_1^{\mathrm{T}}$ *and* $\int_{-\tau}^{0}x(s)x^{\mathrm{T}}(s)\,\mathrm{d}s = N_2N_2^{\mathrm{T}}$.

Proof. By Theorem 1, (i) in (27) is clear. Also, it follows from the Lemma 1 that (ii), (iii), and (iv) in (27) are equivalent to $\mathscr{D}^{\mathrm{T}}(0)X^{-1}\mathscr{D}(0) < \alpha$, $hN_1^{\mathrm{T}}Z_1^{-1}N_1 < \Gamma_1$, and $N_2^{\mathrm{T}}Z_2^{-1}N_2 < \Gamma_2$, respectively. On the other hand,

$$\begin{aligned}
\int_{-h}^{0}(s+h)x^{\mathrm{T}}(s)R_1x(s)\,\mathrm{d}s &= \int_{-h}^{0}\mathrm{tr}((s+h)x^{\mathrm{T}}(s)R_1x(s))\,\mathrm{d}s = \mathrm{tr}(N_1N_1^{\mathrm{T}}R_1) \\
&= \mathrm{tr}(N_1^{\mathrm{T}}hZ^{-1}N_1) < \mathrm{tr}(\Gamma_1), \\
\int_{-\tau}^{0}x^{\mathrm{T}}(s)R_2x(s)\,\mathrm{d}s &= \int_{-\tau}^{0}\mathrm{tr}(x^{\mathrm{T}}(s)R_2x(s))\,\mathrm{d}s \\
&= \mathrm{tr}(N_2N_2^{\mathrm{T}}R_2) = \mathrm{tr}(N_2^{\mathrm{T}}Z_2^{-1}N_2) < \mathrm{tr}(\Gamma_2).
\end{aligned}$$

Hence, it follows from (8) that

$$J^* < \alpha + \mathrm{tr}(\Gamma_1) + \mathrm{tr}(\Gamma_2).$$

Thus, the minimization of α + tr(Γ_1) + tr (Γ_2) implies the minimization of the guaranteed cost for the system (1). Note that this convex optimization problem guarantees that a global optimum, when it exists, is reachable (see Remark 1). □

Remark 1. The problem (27) is to determine whether the problem is feasible or not. It is called the feasibility problem. Also, the solutions of the problem can be found by solving eigenvalue problem in X, Z_1, Z_2, Y, Γ_1, and Γ_2, which is a convex optimization problem. For details, see Boyd et al. [13]. Various efficient convex optimization algorithms can be used to check whether the matrix inequality (7) is feasible. In this paper, in order to solve the matrix inequality, we utilize Matlab's LMI Control Toolbox [15], which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithms [13].

Numerical Example 1. Consider the following linear differential system of neutral type:

$$\frac{\mathrm{d}}{\mathrm{d}t}[x(t) - A_2x(t-0.3)] = A_0x(t) + A_1x(t-0.3) + Bu(t), \tag{28}$$

where

$$A_0 = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix}, \qquad A_1 = \begin{bmatrix} 0 & 0.5 \\ -0.2 & -0.2 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.2 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0.5 \end{bmatrix}$$

and the initial condition of the system is as follows:

$$x(t) = [\,0.5\mathrm{e}^{t} \quad -0.5\mathrm{e}^{-t}\,]^{\mathrm{T}}, \quad \text{for} \quad -0.3 \leqslant t \leqslant 0.$$

Actually, when the control input is not forced to the system (28), i.e., $u(t) = 0$, the system is unstable since the states of the system go to infinity as $t \to \infty$.

Here, associated with this system is the cost function of (3) with $Q = I$ and $S = 0.1I$.

From the relations $\mathscr{D}(0) = x(0) + A_1\int_{-0.3}^{0} x(s)\,\mathrm{d}s - A_2x(-0.3)$, $N_1N_1^{\mathrm{T}} = \int_{-0.3}^{0}(s+0.3)x(s)x^{\mathrm{T}}(s)\,\mathrm{d}s$, $N_2N_2^{\mathrm{T}} = \int_{-0.3}^{0} x(s)x^{\mathrm{T}}(s)\,\mathrm{d}s$, we have

$$\mathscr{D}(0) = \begin{bmatrix} 0.3450 \\ -0.3559 \end{bmatrix}, \qquad N_1 = \begin{bmatrix} 0.0876 & -0.0402 \\ -0.0402 & 0.1919 \end{bmatrix},$$

$$N_2 = \begin{bmatrix} 0.1614 & -0.1742 \\ -0.1742 & 0.2691 \end{bmatrix}.$$

First, checking the stability condition (5) for operator $\mathscr{D}(x_t)$ gives the solutions:

$$\Gamma_0 = \begin{bmatrix} 0.6284 & -0.0002 \\ -0.0002 & 0.6280 \end{bmatrix}, \qquad \alpha_1 = 0.3333, \qquad \alpha_2 = 0.3333.$$

Next, by solving the optimization problem of Theorem 2, we find the solutions of the LMIs (27) for the system as

$$X = \begin{bmatrix} 0.4150 & -0.2580 \\ -0.2580 & 0.3868 \end{bmatrix}, \quad Z_1 = \begin{bmatrix} 0.4564 & 0.1156 \\ 0.1156 & 0.5830 \end{bmatrix},$$
$$Z_2 = \begin{bmatrix} 3.4351 & -0.5909 \\ -0.5909 & 3.6150 \end{bmatrix}, \quad Y = [-0.0000 \quad 5.0000],$$
$$\Gamma_1 = \begin{bmatrix} 0.0072 & -0.0092 \\ -0.0092 & 0.0232 \end{bmatrix}, \quad \Gamma_2 = \begin{bmatrix} 0.0137 & -0.0182 \\ -0.0182 & 0.0251 \end{bmatrix}, \quad \alpha = 0.3752.$$

Therefore, the gain matrix of stabilizing optimal guaranteed cost controller $u(t)$ for the system (28) is

$$K = YX^{-1} = [\,13.7189 \quad 22.0741\,],$$

and the optimal guaranteed cost of the closed-loop system is as follows:

$$J^* = \alpha + \mathrm{tr}(\Gamma_1) + \mathrm{tr}(\Gamma_2) = 0.4444.$$

However, when the memoryless guaranteed cost state-feedback controller presented in [12] is applied to the system (28), the optimal guaranteed cost is 5.8617. This shows that the memory guaranteed cost feedback controller improves the performance of the system.

3. Concluding remarks

In this paper, the optimal guaranteed cost control problem via a retarded integral state feedback controller for neutral delay-differential systems has been investigated using the Lyapunov method and the LMI framework. The controller can be obtained through a convex optimization problem which can be solved by various efficient convex optimization algorithms.

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