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A novel criterion for delayed feedback control of time-delay chaotic systems

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Abstract

This paper investigated stability criterion of time-delay chaotic systems via delayed feedback control (DFC) using the Lyapunov stability theory and linear matrix inequality (LMI) technique. A stabilization criterion is derived in terms of LMIs which can be easily solved by efficient convex optimization algorithms. A numerical example is given to illuminate the design procedure and advantage of the result derived.

1. Introduction

Since Mackay and Glass [1] first found chaos in time-delay systems, there has been increasing interest in chaotic systems with delays (see e.g. Lu and He [2], Tian and Gao [3], Chen and Yu [8], and the references therein). Time-delay occurs in many physical systems such as population dynamics, neural networks, automatic control systems, biology, economy, and so on. Time-delay is frequently a source of instability and poor performance. Therefore, stability analysis and controller synthesis for time-delay systems have been one of the most challenging problems (see e.g. [4–6]). In the literature [7–11], one of the most frequent objectives consists in the stabilization of chaotic behaviors to one of unstable fixed points or unstable periodic orbits embedded within a chaotic attractor. That is, to design a stabilizing controller that guarantees the closed-loop system dynamics converges to the fixed point or periodic orbit. With this motivation, Guan et al. [9] and Sun [10] have investigated the controller design problem of a class of time-delay chaotic systems using the famous OGY-method [12]. Using the Lyapunov method, they proposed two kind of controller, i.e., standard feedback control (SFC) and DFC, and derived the stabilization criteria which are expressed in terms of norms of certain matrices. However, when the order of system matrix increases, it may lead conservatism to apply the criteria. Furthermore, their methods need pre-selection of some variables to find controller gains satisfying their stability conditions. To overcome this disadvantage, more recently, Park and Kwon [11] proposed a new SFC controller design method using LMI framework, and derived a new stabilization criterion which gives less conservative results than result in [9,10].

In this paper, we consider a class of time-delay chaotic system studied in [9–11]. Using the Lyapunov method and LMI technique, we propose a novel DFC for time-delay chaotic systems by extending the method [11], and derive a new stability criterion, which can be easily solved by various convex optimization algorithms. Note that DFC method does not require a reference signal corresponding to the desired unstable periodic orbit [7].

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Through the paper, ★ denotes the symmetric part. $X > 0$ ($X \geqslant 0$) means that X is a real symmetric positive definitive matrix (positive semi-definite). I denotes the identity matrix with appropriate dimensions. $\|\cdot\|$ refers to the induced matrix 2-norm. diag$\{\cdots\}$ denotes the block diagonal matrix. $\mathscr{C}_{n,h} = \mathscr{C}([-h,0],\mathscr{R}^n)$ denotes the Banach space of continuous functions mapping the interval $[-h,0]$ into $\mathscr{R}^n$, with the topology of uniform convergence.

2. Statement of the problem

Consider the following chaotic system with an additional feedback force:

$$\dot{x}(t) = Ax(t) + Bx(t-h) + f_1(t,x(t)) + f_2(t,x(t-h)) + u(t), \tag{1}$$

where $x(t) \in \mathscr{R}^n$ is the state vector, $u(t) \in \mathscr{R}^n$ is the control input vector, $A, B \in \mathscr{R}^{n\times n}$ are constant system matrices representing the linear parts of the system, $f_1(\cdot), f_2(\cdot) \in \mathscr{R}^n$ are the nonlinear parts of the system, and $h > 0$ is the constant time-delay.

Suppose that the chaotic system (1) has an unstable fixed point or an unstable periodic orbit $\bar{x}(t)$, and is currently in a chaotic state. Then the purpose of this paper is to control the system asymptotically converges towards $\bar{x}(t)$ via the following DFC feedback controller:

$$u(t) = K(x(t-T) - x(t)), \tag{2}$$

where K is a gain matrix of the controller and T is the feedback time-delay. In the case of $T \ll 1$, the time-delayed feedback control technique is equivalent to the derivative control technique.

Since the chaotic system (1) has an unstable fixed point $\bar{x} =$ constant, then

$$\dot{\bar{x}}(t) = A\bar{x}(t) + B\bar{x}(t-h) + f_1(t,\bar{x}(t)) + f_2(t,\bar{x}(t-h)). \tag{3}$$

By applying DFC (2) into system (1), we have

$$\dot{x}(t) = Ax(t) + Bx(t-h) + f_1(t,x(t)) + f_2(t,x(t-h)) + K(x(t-T) - x(t)). \tag{4}$$

Since $\bar{x}(t)$ is the fixed point of chaotic system, we have $\bar{x}(t-T) = \bar{x}(t)$, where T is the constant time-delay of delayed feedback controller. Then, the following equation holds:

$$\dot{\bar{x}}(t) = A\bar{x}(t) + B\bar{x}(t-h) + f_1(t,\bar{x}(t)) + f_2(t,\bar{x}(t-h)) + K(\bar{x}(t-T) - \bar{x}(t)). \tag{5}$$

Then the error dynamics based on DFC is as follows:

$$\begin{aligned} \dot{e}(t) &= Ae(t) + Be(t-h) + F_1(t,e(t)) + F_2(t,e(t-h)) + K[e(t-T) - e(t)] \\ &= (A-K)e(t) + Be(t-h) + Ke(t-T) + F_1(t,e(t)) + F_2(t,e(t-h)), \end{aligned} \tag{6}$$

where $e(t) = x(t) - \bar{x}(t), e(t-h) = x(t-h) - \bar{x}(t-h) = x(t-h) - \bar{x}(t)$, and

$$\begin{aligned} F_1(t,e(t)) &= f_1(t,(e(t)+\bar{x}(t))) - f_1(t,\bar{x}(t)), \\ F_2(t,e(t-h)) &= f_2(t,(e(t-h)+\bar{x}(t-h))) - f_1(t,\bar{x}(t-h)). \end{aligned}$$

Therefore, the control goal is force $\|e(t)\| \to 0$ as $t \to \infty$.

3. Stability analysis

In this section, based on the Lyapunov stability theory and LMI framework, we design a novel DFC for system (1).

For error system (6), since zero is a fixed point of $F_1(t,e(t)) + F_2(t,e(t-h))$, we have a Taylor expansion

$$F_1(t,e(t)) + F_2(t,e(t-h)) = \beta_0 e(t) + [\mathrm{HOT}]_1 + \beta_1 e(t-h) + [\mathrm{HOT}]_2, \tag{7}$$

where $\beta_0 = F_1'(t,e(t)), \beta_1 = F_2'(t,e(t-h))$, $[\mathrm{HOT}]_1$ and $[\mathrm{HOT}]_2$ are higher order term in $e(t)$ and $e(t-h)$, respectively, and F_i' denotes the time derivative of F_i, $(i = 1,2)$.

From the OGY-method [12], the control goal is the zero fixed point, so we can only consider the linearized part near zero point. Rewrite the local error system (6) as follows:

$$\dot{e}(t) = (A - K + \beta_0 I)e(t) + (B + \beta_1 I)e(t-h) + Ke(t-T). \tag{8}$$

Here, we introduce a well-known fact and two lemmas which are essential for the proof of main result in this paper.

Fact 1 (Schur Complement). The linear matrix inequality

$$\begin{bmatrix} Z(x) & Y(x) \\ Y^{\mathrm{T}}(x) & W(x) \end{bmatrix} > 0 \tag{9}$$

is equivalent to $W(x) > 0$ and $Z(x) - Y(x)W^{-1}(x)Y^{\mathrm{T}}(x)$ where $Z(x) = Z^{\mathrm{T}}(x)$, $W(x) = W^{\mathrm{T}}(x)$ and $Y(x)$ depend affinely on x.

Lemma 2 [15]. *For any constant matrix* $M \in \mathscr{R}^{n\times n}$, $M = M^{\mathrm{T}} > 0$, *scalar* $\gamma > 0$, *vector function* $\omega : [0,\gamma] \to \mathscr{R}^n$ *such that the integrations concerned are well defined, then*

$$\left(\int_0^{\gamma} \omega(s)\mathrm{d}s\right)^{\mathrm{T}} M \left(\int_0^{\gamma} \omega(s)\mathrm{d}s\right) \leqslant \gamma \int_0^{\gamma} \omega^{\mathrm{T}}(s)M\omega(s)\mathrm{d}s. \tag{10}$$

Lemma 3 [13]. *Consider an operator* $\mathscr{D}(\cdot) : \mathscr{C}_{n,h} \to \mathscr{R}^n$ *with* $\mathscr{D}(x_t) = x(t) + \hat{B}\int_{t-h}^{t} x(s)\mathrm{d}s$, *where* $x(t) \in \mathscr{R}^n$ *and* $\hat{B} \in \mathscr{R}^{n\times n}$. *For a given scalar* δ, *where* $0 < \delta < 1$, *if a positive definite symmetric matrix* M *exists, such that*

$$\begin{bmatrix} -\delta M & h\hat{B}^{\mathrm{T}}M \\ hM\hat{B} & -M \end{bmatrix} < 0 \tag{11}$$

holds, then the operator $\mathscr{D}(x_t)$ *is stable.*

As a tool for stability analysis, let us define an operator $\mathscr{D}(e_t) : \mathscr{C}_{n,h} \to \mathscr{R}^n$ as

$$\mathscr{D}(e_t) = e(t) + \int_{t-h}^{t} Ge(s)\mathrm{d}s, \tag{12}$$

where $e_t = e(t+s), s \in [-h, 0]$ and $G \in \mathscr{R}^{n\times n}$ is a constant matrix which will be chosen.

With the above operator, the transformed system is

$$\dot{\mathscr{D}}(e_t) = \dot{e}(t) + Ge(t) - Ge(t-h) = (A - K + \beta_0 I + G)e(t) + (B + \beta_1 I - G)e(t-h) + Ke(t-T). \tag{13}$$

This transformation is called parameterized neutral model transformation. Then, we have the following theorem.

Theorem 4. *For given* $h > 0$ *and* $\alpha > 1$, *the system* (1) *under the control* (2) *is asymptotically stable if there exist positive definite matrices* X, W_1, W_2, F_{11}, F_{22}, F_{33}, F_{44}, *and any matrices* Y, Z, F_{12}, F_{13}, F_{14}, F_{23}, F_{24}, F_{34} *which satisfy the following LMIs*:

$$\begin{bmatrix} \Pi & X\bar{A}^{\mathrm{T}} + Y^{\mathrm{T}} - Z^{\mathrm{T}} + F_{12} & \bar{B}X - Y + hF_{13} & Z + hF_{14} & h\alpha Y^{\mathrm{T}} \\ \star & -h^{-1}(\alpha-1)X & \bar{B}X - Y + F_{23} & Z + F_{24} & 0 \\ \star & \star & -W_1 + hF_{33} & hF_{34} & 0 \\ \star & \star & \star & -W_2 + hF_{44} & 0 \\ \star & \star & \star & \star & -h\alpha X \end{bmatrix} < 0, \tag{14}$$

$$-X + F_{22} < 0, \tag{15}$$

$$\begin{bmatrix} -X & hY^{\mathrm{T}} \\ \star & -X \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14} \\ \star & F_{22} & F_{23} & F_{24} \\ \star & \star & F_{33} & F_{34} \\ \star & \star & \star & F_{44} \end{bmatrix} > 0, \tag{17}$$

where $\Pi = \bar{A}X + X\bar{A}^{\mathrm{T}} + Y + Y^{\mathrm{T}} - Z - Z^{\mathrm{T}} + hF_{11} + W_1 + W_2$, $\bar{A} = A + \beta_0 I$, *and* $\bar{B} = B + \beta_1 I$.

Such that when $\|e(t)\|$ *is small enough, the control goal by DFC* $u(t) = ZX^{-1}(x(t-T) - x(t))$ *is guaranteed, i.e.,* $\|e(t)\| \to 0$ *as* $t \to \infty$.

Proof. Consider a legitimate Lyapunov function candidate [4] as

$$V = V_1 + V_2 + V_3 + V_4 + V_5, \tag{18}$$

where

$$V_1 = \mathscr{D}^{\mathrm{T}}(e_t)P\mathscr{D}(e_t), \tag{19}$$

$$V_2 = \alpha\int_{t-h}^{t}\int_{s}^{t} e^{\mathrm{T}}(u)G^{\mathrm{T}}PGe(u)\mathrm{d}u\,\mathrm{d}s, \tag{20}$$

$$V_3 = \int_{t-h}^{t} e^{\mathrm{T}}(s)Q_1e(s)\mathrm{d}s, \tag{21}$$

$$V_4 = \int_{t-T}^{t} e^{\mathrm{T}}(s)Q_2e(s)\mathrm{d}s, \tag{22}$$

$$V_5 = \int_0^t\int_{s-h}^{s}\begin{bmatrix} e(s)\\ Ge(u)\\ e(s-h)\\ e(s-T)\end{bmatrix}^{\mathrm{T}}\bar{P}\begin{bmatrix} F_{11} & F_{12} & F_{13} & F_{14}\\ \star & F_{22} & F_{23} & F_{24}\\ \star & \star & F_{33} & F_{34}\\ \star & \star & \star & F_{44}\end{bmatrix}\bar{P}\begin{bmatrix} e(s)\\ Ge(u)\\ e(s-h)\\ e(s-T)\end{bmatrix}\mathrm{d}u\,\mathrm{d}s, \tag{23}$$

where $Q_1 > 0, Q_2 > 0$, $P > 0$, and $\bar{P} = \mathrm{diag}\{P,P,P,P\}$.

The differential of the Lyapunov functional along the trajectory of system (13) is

$$\begin{aligned}\dot{V}_1 &= 2\mathscr{D}^{\mathrm{T}}(e_t)P\dot{\mathscr{D}}(e_t)\\ &= 2\left\{e(t) + \int_{t-h}^{t} Ge(s)\mathrm{d}s\right\}^{\mathrm{T}} P\{(A-K+\beta_0I+G)e(t) + (B+\beta_1I-G)e(t-h) + Ke(t-T)\}\\ &= e^{\mathrm{T}}(t)\left\{P(A-K+\beta_0I+G) + (A-K+\beta_0I+G)^{\mathrm{T}}P\right\}e(t) + 2e^{\mathrm{T}}(t)P(B+\beta_1I-G)e(t-h)\\ &\quad + 2\left(\int_{t-h}^{t} Ge(s)\mathrm{d}s\right)^{\mathrm{T}}P(A-K+\beta_0I+G)e(t) + 2\left(\int_{t-h}^{t} Ge(s)\mathrm{d}s\right)^{\mathrm{T}}P(B+\beta_1I-G)e(t-h)\\ &\quad + 2e^{\mathrm{T}}(t)PKe(t-T) + 2\left(\int_{t-h}^{t} Ge(s)\mathrm{d}s\right)^{\mathrm{T}}PKe(t-T),\end{aligned} \tag{24}$$

$$\begin{aligned}\dot{V}_2 &= \alpha he^{\mathrm{T}}(t)G^{\mathrm{T}}PGe(t) - \alpha\int_{t-h}^{t} e^{\mathrm{T}}(s)G^{\mathrm{T}}PGe(s)\mathrm{d}s \leqslant \alpha he^{\mathrm{T}}(t)G^{\mathrm{T}}PGe(t)\\ &\quad - \int_{t-h}^{t} e^{\mathrm{T}}(s)G^{\mathrm{T}}PGe(s)\mathrm{d}s - h^{-1}(\alpha-1)\left(\int_{t-h}^{t} Ge(s)\mathrm{d}s\right)^{\mathrm{T}}P\left(\int_{t-h}^{t} Ge(s)\mathrm{d}s\right)^{\mathrm{T}},\end{aligned} \tag{25}$$

$$\dot{V}_3 = e^{\mathrm{T}}(t)Q_1e(t) - e^{\mathrm{T}}(t-h)Q_1e(t-h), \tag{26}$$

$$\dot{V}_4 = e^{\mathrm{T}}(t)Q_2e(t) - e^{\mathrm{T}}(t-T)Q_2e(t-T), \tag{27}$$

$$\begin{aligned}\dot{V}_5 &= he^{\mathrm{T}}(t)PF_{11}Pe(t) + 2e^{\mathrm{T}}(t)PF_{12}P\int_{t-h}^{t} Ge(s)\mathrm{d}s + 2he^{\mathrm{T}}(t)PF_{13}Pe(t-h) + 2he^{\mathrm{T}}(t)PF_{14}Pe(t-T)\\ &\quad + \int_{t-h}^{t} e^{\mathrm{T}}(s)G^{\mathrm{T}}PF_{22}GPe(s)\mathrm{d}s + 2\left(\int_{t-h}^{t} Ge(s)\mathrm{d}s\right)^{\mathrm{T}}PF_{23}Pe(t-h) + 2\left(\int_{t-h}^{t} Ge(s)\mathrm{d}s\right)^{\mathrm{T}}PF_{24}Pe(t-T)\\ &\quad + he^{\mathrm{T}}(t-h)PF_{33}Pe(t-h) + 2he^{\mathrm{T}}(t-h)PF_{34}Pe(t-T) + he^{\mathrm{T}}(t-T)PF_{44}Pe(t-T),\end{aligned} \tag{28}$$

where Lemma 2 was utilized in (25).

From (24)–(28), the time-derivative of V has new upper bound as follows:

$$\dot{V} \leqslant \begin{bmatrix} e(t)\\ \int_{t-h}^{t} Ge(s)\mathrm{d}s\\ e(t-h)\\ e(t-T)\end{bmatrix}^{\mathrm{T}}\Omega\begin{bmatrix} e(t)\\ \int_{t-h}^{t} Ge(s)\mathrm{d}s\\ e(t-h)\\ e(t-T)\end{bmatrix} + \int_{t-h}^{t} e^{\mathrm{T}}(s)G^{\mathrm{T}}(-P+PF_{22}P)Ge(s)\mathrm{d}s, \tag{29}$$

where

$$\Omega = \begin{bmatrix} \Sigma & (A-K+\beta_0 I+G)^{\mathrm{T}}P+PF_{12}P & P(B+\beta_1 I-G)+hPF_{13}P & PK+hPF_{14}P \\ \star & -h^{-1}(\alpha-1)P & P(B+\beta_1 I-G)+PF_{23}P & PK+PF_{24}P \\ \star & \star & -Q_1+hPF_{33}P & hPF_{34}P \\ \star & \star & \star & -Q_2+hPF_{44}P \end{bmatrix}, \tag{30}$$

$$\Sigma = P(A-K+\beta_0 I+G)+(A-K+\beta_0 I+G)^{\mathrm{T}}P+\alpha hG^{\mathrm{T}}PG+Q_1+Q_2+hPF_{11}P. \tag{31}$$

Therefore, if the two inequalities $\Omega < 0$ and $-P+PF_{22}P<0$ hold, then a positive scalar λ exists which satisfies

$$\dot{V} < -\lambda\|e(t)\|^2. \tag{32}$$

Let

$$X = P^{-1}, \quad W_1 = XQ_1X, \quad W_2 = XQ_2X, \quad Y = GX, \quad Z = KX. \tag{33}$$

By pre- and post-multiplying inequalities $\Omega<0$ and $-P+PF_{22}P<0$ by $\mathrm{diag}\{X,X,X,X\}$ and X, respectively, the resulting inequalities are equivalent to (14) and (15) by Fact 1 (Schur Complement).

The inequality (16) is equivalent to

$$\begin{bmatrix} -P & hG^{\mathrm{T}}P \\ \star & -P \end{bmatrix} < 0 \tag{34}$$

by pre- and post-multiplying the inequality (16) by $\mathrm{diag}\{X^{-1},X^{-1}\}$. If the above inequality (34) holds, then we can prove that a positive scalar δ which is less than one exists such that

$$\begin{bmatrix} -\delta P & hG^{\mathrm{T}}P \\ \star & -P \end{bmatrix} < 0 \tag{35}$$

according to matrix theory. Therefore, from Lemma 3, if the inequality (16) holds, then operator $\mathscr{D}(e_t)$ is stable. The inequality (17) means that V_4 is positive-definite. According the Theorem 9.8.1 in [4], we conclude that if matrix inequalities (14)–(17) holds, then, system (8) is asymptotically stable. This completes our proof. □

Remark 5. Note that the stability criterion of Theorem 4 is independent of feedback time-delay T and dependent of system time-delay h.

Remark 6. In this paper, we use the operator $\mathscr{D}(e_t) = e(t) + \int_{t-h}^{t} Ge(s)\mathrm{d}s$ to transform the original system. Note that if G is A_1, then the transformation is the neutral model transformation one [4]. Since the operator $\mathscr{D}(e_t)$ has free weighting matrix, it is less conservative than the results obtained by using the neutral model transformation.

Remark 7. The solutions of Theorem 4 can be obtained by solving a eigenvalue problem, which is a convex optimization problem. In this paper, we utilize Matlab's LMI Control Toolbox [16] which implements interior-point algorithms. These algorithms are significantly faster than classical convex optimization algorithms [14].

Example 8. Consider a chaotic system of the form [17]:

$$\zeta\frac{\mathrm{d}x(t)}{\mathrm{d}t} = -x(t) + \frac{G}{1+\mu}(x(t-h)+U_B)[1+\mu\cos(\pi x(t-h)+U_0+U_M)],$$

where $x(t)$ is the normalized output voltage variation, $G \geqslant 0$ is the feedback gain and $h \geqslant 0$ is the feedback delay, $\mu \geqslant 0$ is the fringe constant, $\zeta \geqslant 0$ is the response time, U_0 and U_B are the constant phase shifts, and U_M is the input phase shift induced by the fiber strain. Let us keep $U_0 = 2.0, U_B = 1.0, U_M = -2.5$ and put $\mu = 1.0$ and $G = 2.0$. When $\zeta = 0.1$ and $h = 0.01$, the system demonstrates chaotic behavior (for simulation results, see [9]). In this case, we know $\bar{x} = 0.85$ is one of the fixed points. Based on this fixed point, the standard SFC can be chosen as $u = K(0.85 - x(t))$. As we consider DFC method, the controller is of the form $u(t) = K(x(t-T) - x(t))$, and its corresponding error system is obtained that [9]

$$\dot{e}(t) = (-10-K)e(t) + 10e(t-h) + Ke(t-T) + F_1(t,e(t)) + F_2(t,e(t-h)), \tag{36}$$

where $F_1(\cdot) = 0$, $\beta_0 = 0$, $F_2(\cdot) = 10(e(t-h) + \bar{x}(t-h))\cos(\pi(e(t-h) + \bar{x}(t-h)) - 0.5) + 10\cos(\pi(e(t-h) + \bar{x}(t-h)) - 0.5) - 10\bar{x}(t-h)\cos(\pi\bar{x}(t-h) - 0.5) - 10\cos(\pi\bar{x}(t-h) - 0.5)$. Then, we have

$$\beta_1 = 10[\cos(\pi\bar{x}(t-h) - 0.5) - \pi(\bar{x}(t-h) + 1)\sin(\pi\bar{x}(t-h) - 0.5)]\Big|_{\bar{x}(t-h)=0.85} = -53.6254.$$

According to Guan et al. [9] and Sun [10], the controller gains designed are $K > 45.4381$ for DFC [9] and $K = 1.4$ for DFC [10], respectively.

Now, let us take $\alpha = 2.3$, and by applying Theorem 4 to the above system, we found that the LMI solutions of Theorem 1 are as follows:

$$\begin{aligned} &X = 0.1296, \quad W_1 = 2.7796, \quad W_2 = 2.541, \quad Y = -4.0819, \quad Z = 0.0974, \quad F_{11} = 3.1139, \\ &F_{12} = 0.0282, \quad F_{13} = 0.0161, \quad F_{14} = -0.0016, \quad F_{22} = 0.0649, \quad F_{23} = -0.0020, \\ &F_{24} = -0.0103, \quad F_{33} = 3.1051, \quad F_{34} = 10^4 \times 1.9541, \quad F_{44} = 3.1086. \end{aligned} \tag{37}$$

With the above solution, the our controller gain matrix is

$$K = ZX^{-1} = 0.7514,$$

which is lower gain than those in literature [9,10]. For one-dimensional system as this example, from the relation $K = ZX^{-1}$, we can get lower controller gain by solving the LMIs of Theorem 4 so that the parameter Z is minimized or X is maximized. This can be done by using a function *mincx* in Matlab's LMI Control Toolbox [16]. By applying the function to this example, we obtain a new lower gain

$$K = 0.1805,$$

where $\alpha = 5.8$ is used.

Finally, note that our method can give various stabilizing control gains as α varies.

4. Conclusions

In this paper, we present a novel DFC method for stabilization of time-delay chaotic systems. Utilizing the operator which has free weighting matrix, we transform the original system to the equivalent time-delay system. Then, the delay-dependent stability criteria is derived in terms of LMIs by establishing the Lyapunov functional which have free weighting matrices. An example is discussed to illustrate the advantage of the our result.

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