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Robust non-fragile guaranteed cost control of uncertain large-scale systems with time-delays in subsystem interconnections

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In this paper, the robust non-fragile guaranteed cost-control problem is studied for a class of uncertain linear large-scale systems with time-delays in subsystem interconnections and given quadratic cost functions. The uncertainty in the system is assumed to be norm-bounded and time-varying. Also, the state-feedback gains for subsystems of the large-scale system are assumed to have norm-bounded controller gain variations. The problem is to design a state feedback control law such that the closed-loop system is asymptotically stable, and the closed-loop cost function value is not more than a specified upper bound for all admissible uncertainties. Sufficient conditions for the existence of such controllers are derived based on the linear matrix inequality (LMI) approach combined with the Lyapunov method. A parameterized characterization of the robust non-fragile guaranteed cost controllers is given in terms of the feasible solutions to a certain LMI. A numerical example is given to illustrate the proposed method.

1. Introduction

There exist many real-world systems that can be modelled as large-scale systems: examples are power systems, communication systems, economic systems, social systems, transportation systems, and so on. It is also well known that the control of such large-scale systems can become very complicated owing to the high dimensionality of the system equation, uncertainties, and time-delays. So, it is standard practice to divide such systems into a number of interconnected subsystems (Siljak 1978, Mahmoud *et al*. 1985). In view of reliability and practical implementation, the decentralized stabilization of large-scale interconnected systems becomes a very important problem and has been studied extensively for more than two decades (see, for example, Geromel and Yamakami 1982, Shi and Gao 1987, Chen *et al*. 1991, Chen 1992, Ho *et al*. 1992, Yan *et al*. 1998). Also, since time-delays are frequently introduced because of computation of data, measurement of system variables, or signal transmission between subsystems, many researchers have considered the problem of stability analysis and control of various large-scale systems with time-delays (see, for example, Lee and Radovic 1988, Hu 1994, Jiang and Wang 2000, Oucheriah 2000, Cheng *et al*. 2001, and references therein). However, when controlling a real plant, it is also desirable to design a control system which is not only stable but also guarantees an adequate level of performance. One way to address the robust performance problem is to consider a linear quadratic cost function. This approach is the so-called guaranteed cost control (Chang and Peng 1972). The approach has the advantage of providing an upper bound on a given performance index, and thus the system performance degradation incurred by the uncertainties is guaranteed to be less than this bound. Recently, there have been considerable efforts to tackle the guaranteed-cost controller design problem (Petersen 1995, Petersen *et al*. 1998, Guan *et al*. 1999, Yu and Chu 1999, Aliyu 2000, Arzelier and Peaucelle 2000).

While the above methods yield controllers that are robust to uncertainties in the plant under control, their robustness with respect to uncertainties in the controllers themselves has not been considered. In a recent study by Keel and Bhattacharyya (1997), it is shown that the controllers may be very sensitive, or fragile with respect to errors in the controller

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coefficients, although they are robust with respect to plant uncertainty. This raises a new issue: how to design a controller for a given plant with uncertainty such that the controller is insensitive to some amount of error with respect to its gain, i.e. the controller is non-fragile. More recently, there have been some efforts to tackle the non-fragile controller design problem (Dorato 1998, Dorato *et al.* 1998, Famularo *et al.* 1998, Corrado and Haddad 1999). Unfortunately, untill now, the topic of robust non-fragile control for large-scale systems has received little attention. In this paper, we consider a class of linear large-scale systems with parametric uncertainties in the system matrices, controller gain variations, and time-delays in subsystem interconnections. The uncertainty is time-varying and is assumed to be norm-bounded. Using the Lyapunov functional technique combined with a linear matrix inequality (LMI) technique, we develop a robust non-fragile guaranteed cost control for this system via memoryless state feedback, which makes the closed-loop system robustly stable for all admissible uncertainties and guarantees an adequate level of performance. A stability criterion for the existence of the guaranteed cost controller is derived in terms of LMIs, and their solutions provide a parameterized representation of the control. The LMIs can be easily solved by various efficient convex optimization algorithms (Boyd *et al.* 1994). Finally, a numerical example is given to illustrate the proposed design method.

Notations: Throughout the paper, $\mathcal{R}^n$ denotes the n-dimensional Euclidean space, and $\mathcal{R}^{n\times m}$ is the set of all $n \times m$ real matrices. I denotes the identity matrix with appropriate dimensions. $\lambda_M(\cdot)$ denotes the largest eigenvalue of the given matrix. $diag\{\cdots\}$ denotes the diagonal matrix. For symmetric matrices X and Y, the notation $X > Y$ (respectively, $X \geq Y$) means that the matrix $X - Y$ is positive definite, (respectively, nonnegative).

2. Problem formulation

Consider a class of uncertain large-scale system composed of N interconnected subsystems described by

$$S_i : \dot{x}_i(t) = \Big[A_i + \Delta A_i(t)\Big]x_i(t) + \sum_{j\neq i}^{N}\Big[A_{ij} + \Delta A_{ij}(t)\Big]x_j(t-\tau_{ij}) + B_i u_i(t), \quad i = 1, 2, \ldots, N, \tag{1}$$

where $x_i(t) \in \mathcal{R}^{n_i}$ is the state vector, $u_i(t) \in \mathcal{R}^{m_i}$ is the control vector, and τ_{ij} is the time-delay between subsystem i and j. The system matrices A_i, B_i, and A_{ij} are of appropriate dimensions, and $\Delta A_i(t)$ and $\Delta A_{ij}(t)$ are real-valued matrices representing time-varying parameter uncertainties in the system.

Assume that the pair $(A_i, B_i), i = 1, \ldots, N,$ is stabilizable, and the time-varying uncertainties are of the form

$$\Delta A_i(t) = D_{ai}F_{ai}(t)E_{ai}, \quad \Delta A_{ij}(t) = D_{aij}F_{aij}(t)E_{aij}, \tag{2}$$

where D_{ai}, D_{aij}, E_{ai}, and E_{aij} are known constant real matrices with appropriate dimensions, and $F_{ai}(t)$ and $F_{aij}(t)$ are unknown matrix functions which are bounded as

$$F_{ai}^T(t)F_{ai}(t) \leq I, \quad F_{aij}^T(t)F_{aij}(t) \leq I, \quad \forall i, j \geq 0. \tag{3}$$

Associated with each subsystem S_i is the following quadratic cost function

$$J_i = \int_0^{\infty}\Big[x_i^T(t)Q_i x_i(t) + u_i^T(t)R_i u_i(t)\Big]\mathrm{d}t \tag{4}$$

where $Q_i \in \mathcal{R}^{n_i\times n_i}$ and $R_i \in \mathcal{R}^{m_i\times m_i}$ are given positive-definite matrices.

Now, although one finds the controller $u_i(t) = K_i x_i(t)$ for each subsystem, the actual controller implemented is

$$u_i(t) = -[K_i + \Delta K_i]x_i(t), \quad i = 1, 2, \ldots, N, \tag{5}$$

where $K_i \in \mathcal{R}^{m_i\times n_i}$ is the nominal controller gain to be designed, and ΔK_i represents the multiplicative gain perturbations of the form

$$\Delta K_i = H_i\Phi_i(t)E_i K_i, \tag{6}$$

with H_i and E_i being known constant matrices, and uncertain parameter matrix $\Phi_i(t)$ satisfying $\Phi_i^T(t)\Phi_i(t) \leq I$.

Here, the objective of this paper is to develop a procedure to design a state feedback controller $u_i(t)$ for uncertain system (1) and cost function (4), such that the resulting closed-loop subsystem given by

$$\dot{x}_i(t) = [A_i + \Delta A_i(t) - B_i(I + H_i\Phi_i(t)E_i)K_i]\,x_i(t) + \sum_{j\neq i}^{N}[A_{ij} + \Delta A_{ij}(t)]x_j(t-\tau_{ij}) \tag{7}$$

is asymptotically stable, and the closed-loop value of the cost function (4) satisfies $J_i \leq J_i^*$, where J_i^* is some specified constant.

Definition 1: For an uncertain large-scale system (1) and cost function (4), if there exist a control law $u_i^*(t)$ and a positive J_i^* such that for all admissible uncertainties the closed-loop system (7) is asymptotically stable and the closed-loop value of the cost function (4) satisfies $J_i \le J_i^*$, then J_i^* is said to be a guaranteed cost, and $u_i^*(t)$ is said to be a guaranteed cost-control law of system (1) and cost function (4).

Remark 1: The controller gain perturbation can result from the actuator degradations, as well as from the requirement for readjustment of controller gains during the controller implementation state (Keel and Bhatacharyya 1997, Dorato 1998). These perturbations in the controller gains are modelled here as uncertain gains that are dependent on uncertain parameters. In the literature (Famularo *et al.* 1998, Haddad and Corrado 1998, Corrado and Haddad 1999), the models of additive uncertainties and multiplicative uncertainties are used to describe the controller gain variation. The uncertainty given in (6) is a class of multiplicative uncertainties.

Before proceeding further, we will state a well-known lemma (Boyd *et al.* 1994).

Lemma 1: *The linear matrix inequality*

$$\begin{bmatrix} Q(x) & S(x) \\ S^T(x) & R(x) \end{bmatrix} > 0,$$

is equivalent to

$$R(x) > 0, \quad Q(x) - S(x)R^{-1}(x)S^T(x) > 0,$$

where $Q(x) = Q^T(x), R(x) = R^T(x)$ *and* $S(x)$ *depend affinely on* x.

3. Design of robust decentralized guaranteed cost controller

In this section, we consider the problem of decentralized robust non-fragile guaranteed cost control for the uncertain closed-loop system described by (7) using the Lyapunov method combined with the LMI technique.

For simplicity, we define

$$A_{di} = \left(\sum_{j\neq i}^{N} A_{ij}A_{ij}^T\right)^{1/2}, \qquad D_{di} = \left(\sum_{j\neq i}^{N} D_{aij}D_{aij}^T\right)^{1/2},$$
$$R_{di} = \left(\sum_{j\neq i}^{N}\left(I + E_{aji}^T E_{aji}\right)\right)^{1/2}, \qquad \mu_i = \lambda_M^{1/2}(H_i^T R_i H_i). \tag{8}$$

Theorem 1: $u_i(t) = -K_i x_i(t)$ *is a guaranteed cost controller for each subsystems if there exist positive-definite matrix* P_i *and positive scalars* $\beta_i, \varepsilon_{0i}$, *and* ε_i *such that for any admissible uncertain matrices* $F_i(t), F_{aij}(t)$, *and* $\Phi_i(t)$, *the following matrix inequality holds:*

$$\begin{aligned} &A_i^T P_i + P_i A_i + \varepsilon_{0i}P_i D_{ai}D_{ai}^T P_i + \varepsilon_{0i}^{-1}E_{ai}^T E_{ai} - P_i B_i K_i \\ &\quad - K_i^T B_i^T P_i + \varepsilon_i^{-1}K_i^T E_i^T E_i K_i + \varepsilon_i P_i B_i H_i H_i^T B_i^T P_i \\ &\quad + P_i A_{di}A_{di}^T P_i + P_i D_{di}D_{di}^T P_i + R_{di}^T R_{di} + Q_i + K_i^T R_i K_i \\ &\quad + \beta_i^{-1}K_i^T E_i^T E_i K_i + \beta_i K_i^T R_i H_i H_i^T R_i K_i \\ &\quad + \mu_i^2 K_i^T E_i^T E_i K_i < 0 \quad \text{for } i = 1, 2, \ldots, N. \end{aligned} \tag{9}$$

Proof: Consider a Lyapunov function candidate

$$\begin{aligned} V &= \sum_{i=1}^{N} V_i \\ &= \sum_{i=1}^{N}\left(x_i^T(t)P_i x_i(t) + \sum_{j=1, j\neq i}^{N}\int_{t-\tau_{ij}}^{t} x_j^T(s)R_{ij}x_j(s)\mathrm{d}s\right) \end{aligned} \tag{10}$$

where R_{ij} is the positive definite matrix to be chosen later.

The time derivative of V is given by

$$\begin{aligned} \dot{V} = \sum_{i=1}^{N}\Bigg\{ &\dot{x}_i^T(t)P_i x_i(t) + x_i^T(t)P_i\dot{x}(t) \\ &+ \sum_{j=1, j\neq i}^{N}\left[x_j^T(t)R_{ij}x_j(t) - x_j^T(t-\tau_{ij})R_{ij}x_j(t-\tau_{ij})\right]\Bigg\}. \end{aligned} \tag{11}$$

Substituting (7) into (11), we have

$$\begin{aligned} \dot{V} = \sum_{i=1}^{N}\Bigg\{ &x_i^T(t)\Big[A_i^T P_i + P_i A_i + 2P_i D_{ai}F_{ai}(t)E_{ai} \\ &- 2P_i B_i K_i - 2P_i B_i H_i \Phi_i(t) E_i K_i\Big]x_i(t) \\ &+ 2x_i^T(t)P_i \sum_{j=1, i\neq j}^{N}(A_{ij} + D_{aij}F_{aij}(t)E_{aij})x_j(t-\tau_{ij}) \\ &+ \sum_{j=1, j\neq i}^{N}\left[x_j^T(t)R_{ij}x_j(t) - x_j^T(t-\tau_{ij})R_{ij}x_j(t-\tau_{ij})\right]\Bigg\}. \end{aligned} \tag{12}$$

Using the well-known inequality that

$$U\Delta V^T + V\Delta U^T \le \varepsilon UU^T + \varepsilon^{-1}VV^T, \quad \varepsilon > 0 \tag{13}$$

for any matrices U, V and Δ with $\Delta^T\Delta \le I$, we can eliminate the unknown factor, $F_{ai}(t), F_{aij}(t)$ and $\Phi_i(t)$, of parameter uncertainties. Then, the terms on the right-hand side of (12) are bounded as

$$\begin{aligned}
&\sum_{i=1}^{N} 2x_i^T(t)P_iD_{ai}F_{ai}(t)E_{ai}x_i(t)\\
&\quad\le \sum_{i=1}^{N}\big(\varepsilon_{0i}x_i^T(t)P_iD_{ai}F_{ai}(t)F_{ai}^T(t)D_{ai}^TP_ix_i(t)\\
&\qquad+\varepsilon_{0i}^{-1}x_i^T(t)E_{ai}^TE_{ai}x_i(t)\big)\\
&\quad\le \sum_{i=1}^{N}\big(\varepsilon_{0i}x_i^T(t)P_iD_{ai}D_{ai}^TP_ix_i(t)+\varepsilon_{0i}^{-1}x_i^T(t)E_{ai}^TE_{ai}x_i(t)\big)\\
&\qquad-\sum_{i=1}^{N}2x_i^T(t)P_iB_iH_i\Phi_i(t)E_iK_ix_i(t)\\
&\quad\le \sum_{i=1}^{N}\Big(\varepsilon_i^{-1}x_i^T(t)K_i^TE_i^TE_iK_ix_i(t)\\
&\qquad\qquad+\varepsilon_ix_i^T(t)P_iB_iH_i\Phi_i(t)\Phi_i^T(t)H_i^TB_i^TP_ix_i(t)\Big)\\
&\quad\le \sum_{i=1}^{N}\Big(\varepsilon_i^{-1}x_i^T(t)K_i^TE_i^TE_iK_ix_i(t)\\
&\qquad\qquad+\varepsilon_ix_i^T(t)P_iB_iH_iH_i^TB_i^TP_ix_i(t)\Big)\\
&\sum_{i=1}^{N}2x_i^T(t)P_i\sum_{j\ne i}^{N}A_{ij}x_j(t-\tau_{ij})\\
&\quad\le \sum_{i=1}^{N}\left(x_i^T(t)P_i\sum_{j\ne i}^{N}A_{ij}A_{ij}^TP_ix_i(t)+\sum_{j\ne i}^{N}x_j^T(t-\tau_{ij})x_j(t-\tau_{ij})\right)\\
&\quad=\sum_{i=1}^{N}\left(x_i^T(t)P_iA_{di}A_{di}^TP_ix_i(t)+\sum_{j\ne i}^{N}x_j^T(t-\tau_{ij})x_j(t-\tau_{ij})\right)
\end{aligned}$$

$$\begin{aligned}
&\sum_{i=1}^{N}2x_i^T(t)P_i\sum_{j\ne i}^{N}D_{aij}F_{aij}(t)E_{aij}x_j(t-\tau_{ij})\\
&\quad\le\sum_{i=1}^{N}\left(x_i^T(t)P_i\sum_{j\ne i}^{N}D_{aij}F_{aij}(t)F_{aij}^T(t)D_{aij}^TP_ix_i(t)\right.\\
&\qquad\qquad\left.+\sum_{j\ne i}^{N}x_j^T(t-\tau_{ij})E_{aij}^TE_{aij}x_j(t-\tau_{ij})\right)\\
&\quad\le\sum_{i=1}^{N}\left(x_i^T(t)P_i\sum_{j\ne i}^{N}D_{aij}D_{aij}^TP_ix_i(t)\right.\\
&\qquad\qquad\left.+\sum_{j\ne i}^{N}x_j^T(t-\tau_{ij})E_{aij}^TE_{aij}x_j(t-\tau_{ij})\right)\\
&\quad=\sum_{i=1}^{N}\left(x_i^T(t)P_iD_{di}D_{di}^TP_ix_i(t)\right.\\
&\qquad\qquad\left.+\sum_{j\ne i}^{N}x_j^T(t-\tau_{ij})E_{aij}^TE_{aij}x_j(t-\tau_{ij})\right),
\end{aligned}\tag{14}$$

where A_{di} and D_{di} are defined in (8), and ε_{0i} and ε_i are positive scalars to be chosen.

Substituting (14) into (12) gives

$$\begin{aligned}
\dot V \le \sum_{i=1}^{N}\Bigg\{&x_i^T(t)\Big[A_i^TP_i+P_iA_i+\varepsilon_{0i}P_iD_{ai}D_{ai}^TP_i+\varepsilon_{0i}^{-1}E_{ai}^TE_{ai}\\
&+\varepsilon_i^{-1}K_i^TE_i^TE_iK_i-2P_iB_iK_i+\varepsilon_iP_iB_iH_iH_i^TB_i^TP_i\\
&+P_iA_{di}A_{di}^TP_i+P_iD_{di}D_{di}^TP_i\Big]x_i(t)\\
&+\sum_{j=1,j\ne i}^{N}\Big[x_j^T(t)R_{ij}x_j(t)-x_j^T(t-\tau_{ij})\\
&\times\Big(R_{ij}-I-E_{aij}^TE_{aij}\Big)x_j(t-\tau_{ij})\Big]\Bigg\}.
\end{aligned}\tag{15}$$

Now, let us choose R_{ij} as $I+E_{aij}^TE_{aij}$ and note that

$$\sum_{i=1}^{N}\sum_{j=1,\,i\ne j}^{N}x_j^T(t)R_{ij}x_j(t)=\sum_{i=1}^{N}x_i^T(t)\left(\sum_{j\ne i}^{N}R_{ji}\right)x_i(t).\tag{16}$$

Then (15) is simplified as

$$\begin{aligned}
\dot V&=\sum_{i=1}^{N}V_i\\
&\le\sum_{i=1}^{N}\Bigg\{x_i^T(t)\Big[A_i^TP_i+P_iA_i+\varepsilon_{0i}P_iD_{ai}D_{ai}^TP_i+\varepsilon_{0i}^{-1}E_{ai}^TE_{ai}\\
&\qquad-2P_iB_iK_i+R_{di}^TR_{di}+\varepsilon_i^{-1}K_i^TE_i^TE_iK_i\\
&\qquad+\varepsilon_iP_iB_iH_iH_i^TB_i^TP_i+P_iA_{di}A_{di}^TP_i\\
&\qquad+P_iD_{di}D_{di}^TP_i\Big]x_i(t)\Bigg\}\\
&\equiv\sum_{i=1}^{N}x_i^T(t)\Omega x_i(t)\\
&=\sum_{i=1}^{N}\Bigg\{x_i^T(t)\Big[\Omega+Q+K_i^TR_iK_i+2K_i^TR_i\Delta K_i\\
&\qquad+\Delta K_i^TR_i\Delta K_i\Big]x_i(t)\\
&\qquad-x_i^T(t)\Big[Q_i+u_i^T(t)R_iu_i(t)\Big]x_i(t)\Bigg\}.
\end{aligned}\tag{17}$$

Again, using the inequality (13), for $\beta_i>0$, we have

$$\begin{aligned}
&2x_i^T(t)K_i^TR_i\Delta K_ix(t)\\
&\le\beta_i^{-1}x_i^T(t)K_i^TE_i^TE_iK_ix_i(t)+\beta_ix_i^T(t)K_i^TR_iH_iH_i^TR_iK_ix(t)\\
&\qquad x_i^T(t)\Delta^TK_iR_i\Delta K_ix(t)\le\mu_i^2x_i^T(t)K_i^TE_i^TE_iK_ix_i(t).
\end{aligned}\tag{18}$$

By substituting (18) into (17), the matrix inequality (9) implies that

$$\dot{V} = \sum_{i=1}^{N} \dot{V}_i < -\sum_{i=1}^{N} x_i^T(t)\big[Q_i + u_i^T(t)R_i u_i(t)\big]x_i(t) < 0. \tag{19}$$

Noting $Q_i > 0$ and $R_i > 0$, this implies that the system (7) is asymptotically stable by Lyapunov stability theory. Furthermore, from (19), we have

$$x_i^T(t)(Q_i + K_i^T R_i K_i)x_i(t) < \dot{V}_i.$$

Integrating both sides of the above inequality from 0 to T leads to

$$\begin{aligned}\int_0^T & x_i^T(t)(Q_i + K_i^T R_i K_i)x_i(t)\,\mathrm{d}t \\ &< x_i^T(0)P_i x_i(0) - x_i^T(T)P_i x_i(T) \\ &+ \sum_{j\neq i}^{N} \int_{-\tau_{ij}}^{0} x_j(s)^T R_{ij} x_j(s)\,\mathrm{d}s \\ &- \sum_{j\neq i}^{N} \int_{T-\tau_{ij}}^{T} x_j(s)^T R_{ij} x_j(s)\,\mathrm{d}s.\end{aligned}$$

As the closed-loop system (7) is asymptotically stable, when $T \to \infty$,

$$x_i^T(T)P_i x_i(T) \to 0 \quad \text{and} \quad \int_{T-\tau_{ij}}^{T} x_j(s)^T R_{ij} x_j(s)\,\mathrm{d}s \to 0.$$

Hence we get

$$\begin{aligned}\int_0^\infty & x_i^T(t)(Q_i + K_i^T R_i K_i)x_i(t)\,\mathrm{d}t < x_i^T(0)P_i x_i(0) \\ &+ \sum_{j\neq i}^{N} \int_{-\tau_{ij}}^{0} x_j(s)^T R_{ij} x_j(s)\,\mathrm{d}s \overset{\Delta}{=} J_i^*,\end{aligned} \tag{20}$$

which completes the proof. □

In the following, we will show that the above sufficient condition for the existence of guaranteed cost controllers is equivalent to the feasibility of LMI.

Theorem 2: *For given $R_i > 0$ and $Q_i > 0$, if there exist a matrix M_i, positive-definite matrices X_i, and positive scalars, $\beta_i, c\varepsilon_{0i}$, and ε_i, such that for $i = 1, 2, \ldots, N$, the following LMI is feasible:*

$$\begin{bmatrix} \Sigma_1 & X_i E_{ai}^T & M_i^T E_i^T & X_i R_{di}^T & X_i & \Sigma_2 \\ * & -\varepsilon_{0i} I & 0 & 0 & 0 & 0 \\ * & * & -\varepsilon_i I & 0 & 0 & 0 \\ * & * & * & -I & 0 & 0 \\ * & * & * & * & -Q_i^{-1} & 0 \\ * & * & * & * & * & -\Sigma_3 \end{bmatrix} < 0 \tag{21}$$

where $X_i = P_i^{-1}$ and

$$\begin{aligned}\Sigma_1 &= X_i A_i^T + A_i X_i + \varepsilon_{0i} D_{ai} D_{ai}^T - B_i M_i - M_i^T B_i^T \\ &\quad + \varepsilon_i B_i H_i H_i^T B_i^T + A_{di} A_{di}^T + D_{di} D_{di}^T, \\ \Sigma_2 &= \big[M_i^T \;\; M_i^T E_i^T \;\; \beta_i M_i^T R_i H_i \;\; \mu_i M_i^T E_i^T\big], \\ \Sigma_3 &= diag\{R_i^{-1}, \beta_i I, \beta_i I, I\},\end{aligned}$$

then the state feedback control law

$$u_i(t) = -K_i x_i(t) = -M_i X_i^{-1} x_i(t) \tag{22}$$

is a non-fragile guaranteed cost control law for robust decentralized stabilization of the uncertain large-scale systems (7), *and the corresponding closed-loop value of the cost function satisfies $J_i \le J_i^*$, in which J_i^* is given in* (20).

Proof: By premultiplying and postmultiplying X_i onto (9), we get

$$\begin{aligned}& X_i A_i^T + A_i X_i + \varepsilon_{0i} D_{ai} D_{ai}^T + \varepsilon_{0i}^{-1} X_i E_{ai}^T E_{ai} X_i - B_i K_i X_i \\ &- X_i K_i^T B_i^T + \varepsilon_i^{-1} X_i K_i^T E_i^T E_i K_i X_i + \varepsilon_i B_i H_i H_i^T B_i^T \\ &+ A_{di} A_{di}^T + D_{di} D_{di}^T + X_i^T R_{di}^T R_{di} X_i + X_i Q_i X_i \\ &+ X_i K_i^T R_i K_i X_i + \beta_i^{-1} X_i K_i^T E_i^T E_i K_i X_i \\ &+ \beta_i X_i K_i^T R_i H_i H_i^T R_i K_i X_i + \mu_i^2 X_i K_i^T E_i^T E_i K_i X_i < 0.\end{aligned} \tag{23}$$

Using a change of variable, $M_i = K_i X_i$, and Lemma 1, the inequality (23) is equivalent to (21). This completes the proof. □

Remark 2: Since the inequality (21) is a linear matrix inequality in $X_i, M_i, \varepsilon_{0i}, \varepsilon_i, \beta_i$, the inequality (21) defines a convex solution set of $(X_i, M_i, \beta_i, \varepsilon_{0i}, \varepsilon_i)$, and therefore various efficient convex optimization algorithms can be used to check whether the LMI is feasible. Moreover, the decentralized gain matrix K_i can be calculated from the relation $M_i = K_i P_i^{-1}$ after finding the LMI solutions, $X_i\ (= P_i^{-1})$ and M_i from (21). In this paper, in order to solve the LMI, we utilize Matlab's LMI

Control Toolbox (Gahinet *et al*. 1995), which implements state-of-the-art interior-point algorithms, which is significantly faster than classical convex optimization algorithms (Boyd *et al*. 1994).

Theorem 2 presents a method of designing a state feedback guaranteed cost controller. The following theorem presents a method of selecting a controller minimizing the upper bound of the guaranteed cost (20).

Theorem 3: *Consider system* (7) *with cost function* (4). *If the following optimization problem*

$$\min_{X_i, M_i, \varepsilon_{0i}, \varepsilon_i, \beta_i, \alpha_i} \quad \alpha_i$$

$$(i) \ \text{LMI (21)} \qquad (24)$$

$$(ii) \begin{bmatrix} -(\alpha_i - \Gamma_i) & x_i^T(0) \\ x_i(0) & -X_i \end{bmatrix} < 0, \quad \text{for } i = 1, 2, \ldots, N \qquad (25)$$

has a solution set $(\alpha_i, \beta_i, X_i, M_i, \varepsilon_{0i}, \varepsilon_i)$, *then the control law* (22) *is an optimal non-fragile guaranteed cost control law which ensures the minimization of the guaranteed cost* (20) *for the uncertain large-scale system* (7), *where* $\Gamma_i = \sum_{j \neq i}^{N} \int_{-\tau_{ij}}^{0} x_j(s)^T R_{ij} x_j(s) \, \mathrm{d}s$.

Proof: By Theorem 2, (i) in (24) is clear. Also, it follows from the Lemma 1 that (ii) in (24) is equivalent to $x_i^T(0) X_i^{-1} x_i(0) + \Gamma_i < \alpha_i$. So, it follows from (20) that

$$J_i^* < \alpha_i, \quad \text{for } i = 1, 2, \ldots, N.$$

Thus, the minimization of α_i implies the minimization of the guaranteed cost for the subsystem (7). The convexity of this optimization problem ensures that a global optimum, when it exists, is reachable. □

To illustrate the application of the proposed method, we present the following example.

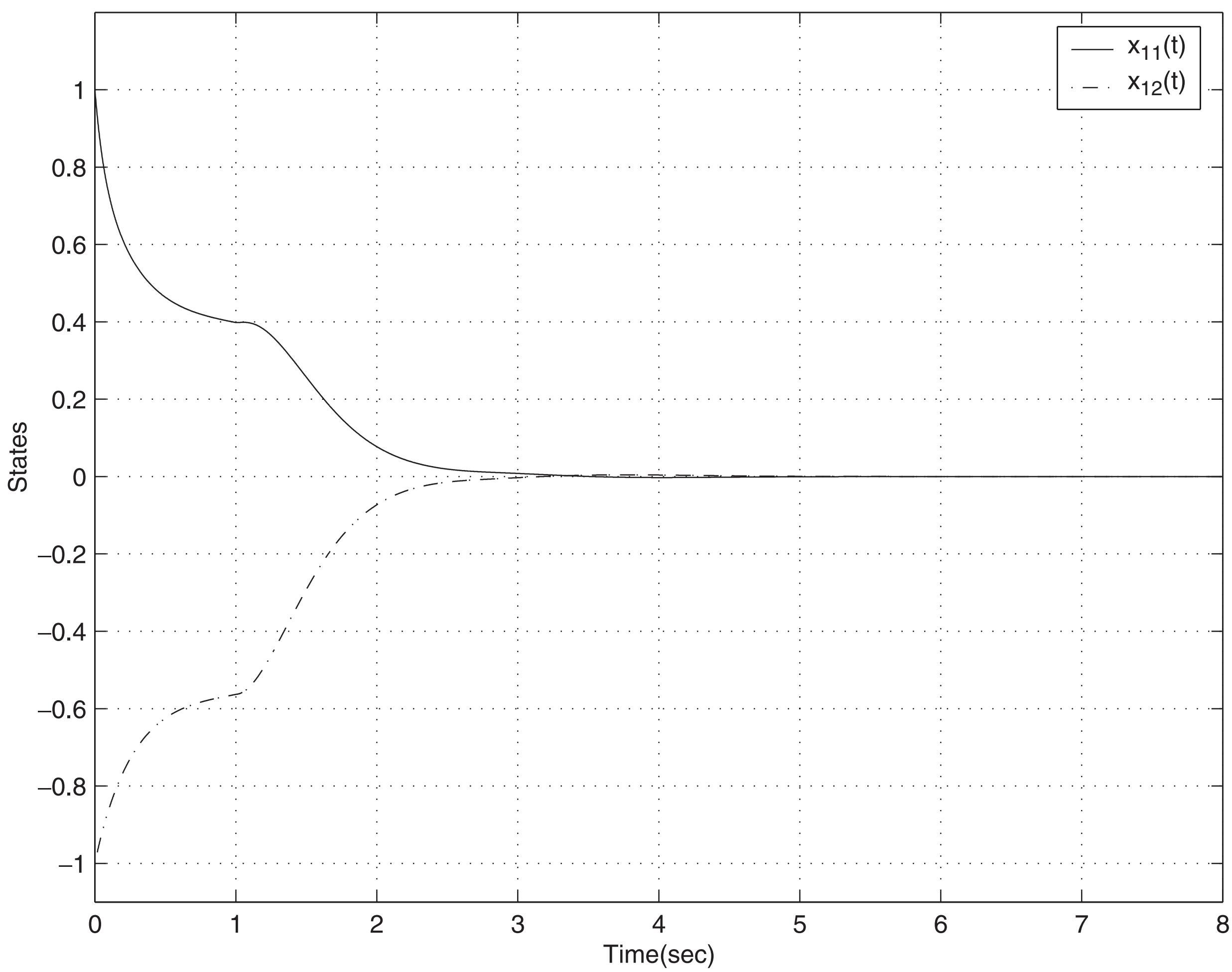

Figure 1. State responses of subsystem 1.

Example 1: Consider a large-scale system which is composed of the following two interconnected subsystems

$$\dot{x}_1(t) = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} x_1(t) + \begin{bmatrix} 1 & -0.1 & 0.5 \\ -0.5 & 0.5 & 1 \end{bmatrix} x_2(t-1) + \Delta A_1(t) x_1(t) + \Delta A_{12}(t) x_2(t-1) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u_1(t),$$

$$\dot{x}_2(t) = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 1 & -3 \end{bmatrix} x_2(t) + \begin{bmatrix} 1 & 0 \\ 0 & 0.5 \\ 0.5 & 1 \end{bmatrix} x_1(t-0.5) + \Delta A_2(t) x_2(t) + \Delta A_{21}(t) x_1(t-0.5) + \begin{bmatrix} 1 & 0 \\ 1 & 0.5 \\ 0 & 1 \end{bmatrix} u_2(t)$$

where

$$\Delta A_1(t) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.4 \end{bmatrix} \begin{bmatrix} \sin(t) & 0 \\ 0 & \sin(2t) \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_{12}(t) = \begin{bmatrix} 0.5 \\ 0 \end{bmatrix} \sin(2t) \begin{bmatrix} 0 & 1 & 1 \end{bmatrix},$$

$$\Delta A_2(t) = \begin{bmatrix} 0 & 0 & 0.3 \\ 0.3 & 0 & 0.2 \\ 0.1 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} \sin(2t) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & \sin(t) \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

$$\Delta A_{21}(t) = \begin{bmatrix} 0.5 \\ 0.1 \\ 0.5 \end{bmatrix} \sin(t) \begin{bmatrix} 1 & 1 \end{bmatrix},$$

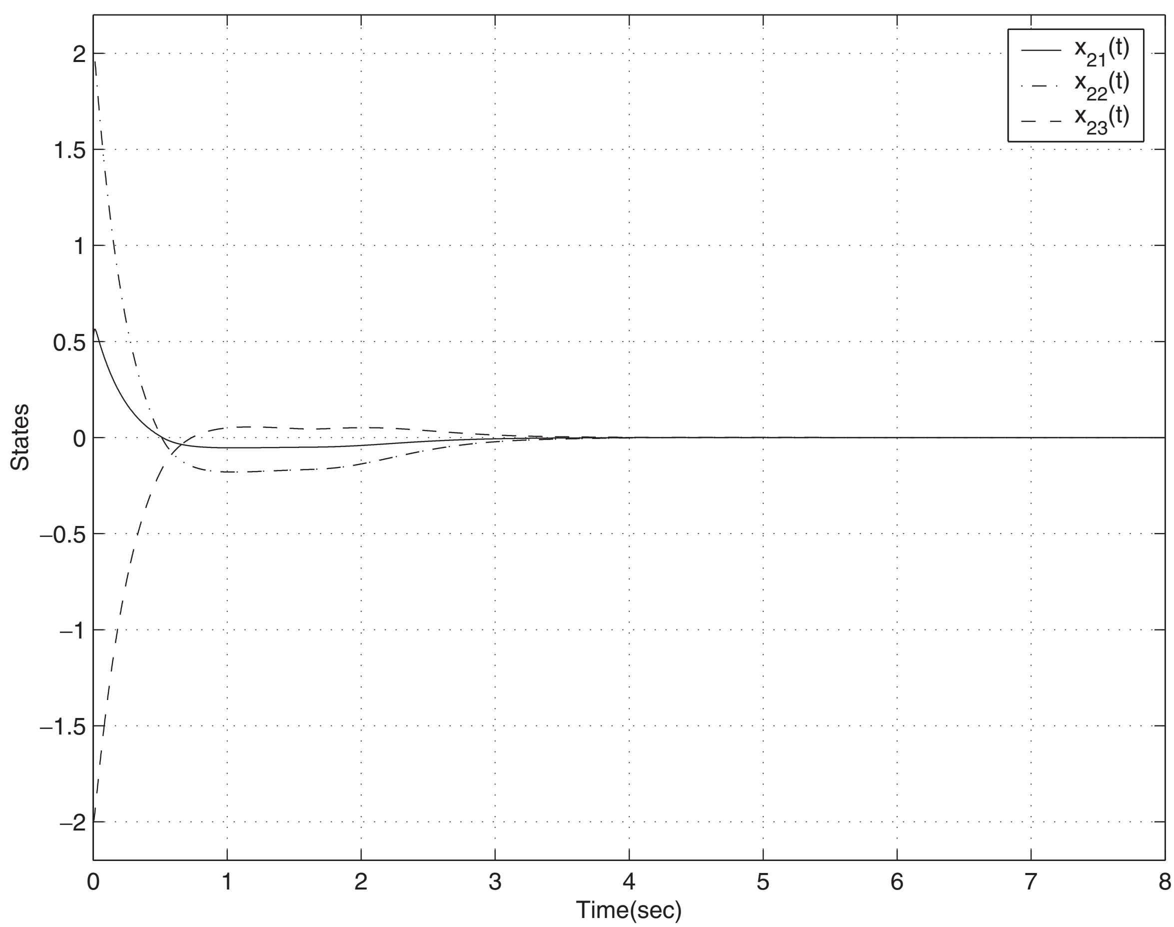

Figure 2. State responses of subsystem 2.

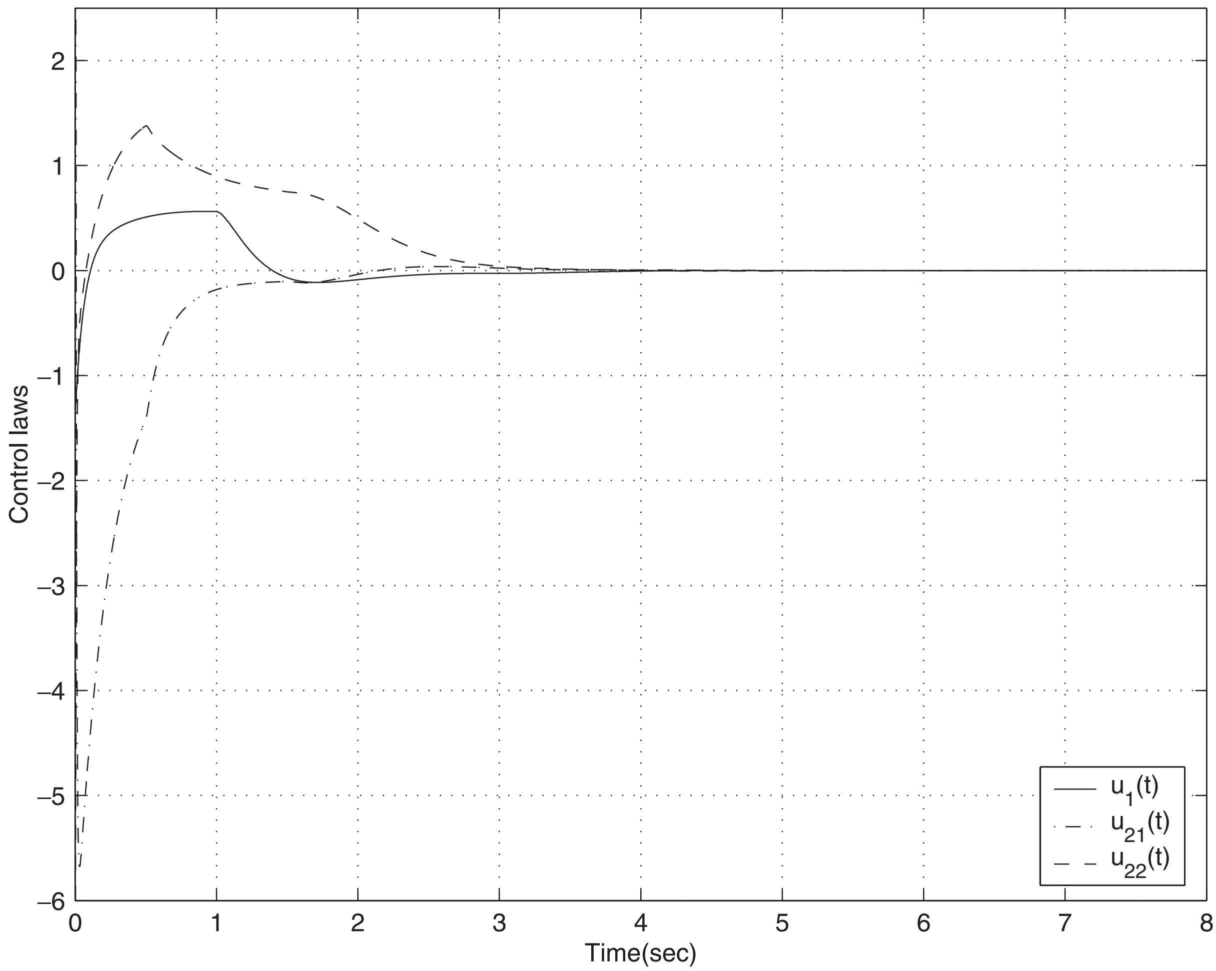

Figure 3. Control inputs for subsystems 1 and 2.

and the initial conditions of each subsystem are as follows:

$$\begin{cases} x_1(t) = [0\ \ 0]^T, \quad x_2(t) = [0\ \ 0\ \ 0]^T, & \text{for } -1 \le t < 0 \\ x_1(t) = [1\ \ -1]^T, \quad x_2(t) = [0.5\ \ 2\ \ -2]^T, & \text{for } t = 0. \end{cases}$$

Also, the following multiplicative controller uncertainties of the form (6) are considered:

$$H_1 = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0.2 \\ 0.2 \end{bmatrix}$$

$$H_2 = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}, \quad E_e = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}.$$

Associated with this system is the cost function of (4) with $Q_1 = I, Q_2 = I, R_1 = 0.2I$ and $R_2 = 0.2I$.

Here, solving the optimization problem of Theorem 3, we find the positive solutions of the LMIs for the subsystem 1 as

$$X_1 = \begin{bmatrix} 1.3380 & -0.6686 \\ -0.6686 & 0.7932 \end{bmatrix}, \quad M_1 = \begin{bmatrix} 5.1725 & 0.0000 \end{bmatrix},$$

$$\varepsilon_{01} = 2.1854, \quad \varepsilon_1 = 0.5173,$$

$$\beta_1 = 10^8 \times 7.3832, \quad \alpha_1 = 9.5426.$$

Similarly, the solutions for the subsystem 2 are as follows:

$$X_2 = \begin{bmatrix} 0.1193 & 0.3397 & -0.1337 \\ 0.3397 & 1.1078 & -0.6813 \\ -0.1337 & -0.6813 & 2.0792 \end{bmatrix},$$

$$M_2 = \begin{bmatrix} 3.5294 & 3.5294 & 0.0000 \\ -0.0000 & 1.7647 \times 3.5295 & \end{bmatrix},$$

$$\varepsilon_{02} = 1.2958, \quad \varepsilon_2 = 3.5294,$$

$$\beta_2 = 10^8 \times 3.5267, \quad \alpha_2 = 6.4915.$$

Therefore, the gain matrices, K_i, of the stabilizing controller, u_i, for two subsystems are

$$K_1 = M_1 X_1^{-1} = \begin{bmatrix} 6.6788 & 5.6296 \end{bmatrix}$$
$$K_2 = M_2 X_2^{-1} = \begin{bmatrix} 200.0072 & -62.9207 & -7.7596 \\ -63.8996 & 24.6793 & 5.6763 \end{bmatrix},$$

and the optimal guaranteed costs of the uncertain closed-loop system are as follows:

$$J_1^* = \alpha_1 = 9.5426$$
$$J_2^* = \alpha_2 = 6.4915.$$

For computer simulation, the following control laws are employed:

$$u_1(t) = -(I + H_1\Phi_1(t)E_1)K_1x_1(t)$$
$$u_2(t) = -(I + H_2\Phi_2(t)E_2)K_2x_2(t),$$

where

$$\Phi_1(t) = \begin{bmatrix} \sin(t) & 0 \\ 0 & \cos(t) \end{bmatrix}, \qquad \Phi_2(t) = \begin{bmatrix} \cos(2t) & 0 \\ 0 & \sin(t) \end{bmatrix}.$$

The simulation results are shown in figures 1–3. In the figures, one can see that the system is indeed well stabilized irrespective of uncertainties and controller gain variations.

4. Conclusion

In this paper, a robust non-fragile guaranteed cost-controller design method for uncertain large-scale interconnected systems with time-delays in subsystem interconnections is presented by the LMI framework. Using the Lyapunov method, the controller is obtained through a convex optimization problem. Finally, a numerical example is given for illustration of controller design, and simulation results show that the system is well stabilized in spite of controller gain variations and uncertainties.

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