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Convex Optimization Approach to Dynamic Output Feedback Control for Delay Differential Systems of Neutral Type[1,2]

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Abstract. In this paper, the design problem of the dynamic output feedback controller for the asymptotic stabilization of a class of linear delay differential systems of the neutral type is considered. A criterion for the existence of such controller is derived based on the matrix inequality approach combined with the Lyapunov method. A parametrized characterization of the controller is given in terms of the feasible solutions to certain matrix inequalities, which can be solved by various convex optimization algorithms. A numerical example is given to illustrate the proposed design method.

Key Words. Neutral systems, dynamic controllers, Lyapunov methods, matrix inequalities.

1. Introduction

Delay differential equations/systems are assuming an increasingly important role in many disciplines like mathematics, science, and engineering. Especially, the stability and stabilization problem for neutral delay differential dynamic systems has received considerable attention during the last decades. Many papers dealing with this problem have appeared because of the existence of delays in various practical control problems and also because of the fact that the delay is frequently a source of system instability and performance degradation. The theory of neutral

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delay differential systems is of both theoretical and practical interest. For various practical examples of neutral systems, see the Refs. 1–3. Recently, various techniques for stability analysis have been proposed to derive less conservative stability criteria for the asymptotic stability of several classes of neutral systems; see e.g. Refs. 4–10 and the references therein. Also, the design problem of controllers for the stabilization of the systems has been explored by some researchers (Refs. 11–13). However, these works are restricted to the static state feedback control schemes, although output measurement based control is a necessary prerequisite for practical control problems. Furthermore, in some situation, there is a strong need to construct a dynamic controller instead of a static controller in order to obtain better performance and better dynamical behavior of the state response. To the knowledge of this author, the topic of dynamic output feedback control for neutral differential systems has received little attention.

This paper is concerned with the design problem of the output dynamic feedback controller for linear delay differential systems of the neutral type. Using the Lyapunov functional stability theory combined with a matrix inequality technique, a stabilization criterion for the existence of the controller is derived in terms of matrix inequalities; their solutions provide a parametrized representation of the controller. The matrix inequality can solved easily by various efficient convex optimization algorithms (Ref. 15). Finally, a numerical example is given to illustrate the proposed design method.

Notations. $\Re^n$ denotes the n-dimensional Euclidean space, $\Re^{n\times m}$ is the set of all $n\times m$ real matrices, I denotes the identity matrix of appropriate order, and an asterisk represents the elements below the main diagonal of a symmetric block matrix; $\|\cdot\|$ denotes the Euclidean norm of a given vector and the induced norm of a matrix; $\lambda_m(\cdot)$ denotes the minimum eigenvalue of a matrix $(\cdot)$; diag$\{\cdot\}$ denotes a block diagonal matrix. The notation $W>0$ $(\geq, <, \leq 0)$ denotes a symmetric positive-definite (positive-semidefinite, negative, negative-semidefinite) matrix W.

2. Problem Statement and Main Result

Consider the class of neutral differential system of the form

$$\dot{x}(t)=A_0x(t)+A_1x(t-h)+A_2\dot{x}(t-h)+Bu(t), \tag{1a}$$

$$y(t)=Cx(t), \tag{1b}$$

with the initial condition function

$$x(t_0+\theta)=\phi(\theta), \quad \forall\theta\in[-h,0], \tag{2}$$

where $x(t)\in\Re^n$ is the state vector, $A_0, A_1, A_2\in\Re^{n\times n}$, $B\in\Re^{n\times m}$, $C\in\Re^{l\times n}$ are constant system matrices, $u(t)\in\Re^m$ is the control input, $y(t)\in\Re^l$ is the measured output, h is a positive constant time delay, and $\phi(\cdot)\in\mathcal{C}_0$ is the initial vector, where $\mathcal{C}_0$ is a set of all continuous differentiable functions on $[-h,0]$ to $\Re^n$.

Now, in order to stabilize the system (1), let us consider the following dynamic output feedback controller:

$$\dot{\xi}(t)=A_c\xi(t)+B_c y(t), \tag{3a}$$

$$u(t)=C_c\xi(t), \tag{3b}$$

where $\xi(t)\in\Re^n$ is the controller state and A_c, B_c, C_c are gain matrices with appropriate dimensions to be determined later. Applying the controller (3) to the system (1) results in the closed-loop system

$$\dot{z}(t)=\bar{A}_0 z(t)+\bar{A}_1 z(t-h)+\bar{A}_2\dot{z}(t-h), \tag{4}$$

where

$$z(t)=\begin{bmatrix} x(t) \\ \xi(t) \end{bmatrix}, \quad \bar{A}_0=\begin{bmatrix} A_0 & BC_c \\ B_cC & A_c \end{bmatrix},$$
$$\bar{A}=\begin{bmatrix} A_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{A}_2=\begin{bmatrix} A_2 & 0 \\ 0 & 0 \end{bmatrix}.$$

Before proceeding further, we give a well-known fact and two lemmas.

Fact 2.1. Schur Complement. Given the constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$, where

$$\Sigma_1=\Sigma_1^T \quad \text{and} \quad 0<\Sigma_2=\Sigma_2^T,$$

then

$$\Sigma_1+\Sigma_3^T\Sigma_2^{-1}\Sigma_3<0$$

if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix}<0 \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & -\Sigma_1 \end{bmatrix}<0.$$

Lemma 2.1. See Ref. 16. For any constant symmetric positive-definite matrix Θ, positive scalar σ, and vector function $\omega:[0,\sigma]\to\Re^m$ such that the following integrations are well defined, then

$$\sigma\int_0^{\sigma}\omega^T(s)\Theta\omega(s)ds\ge\left(\int_0^{\sigma}\omega(s)ds\right)^T\Theta\left(\int_0^{\sigma}\omega(s)ds\right).$$

Lemma 2.2. See Ref. 14. For given positive scalar h and any $E_1, E_2, \in \Re^{n\times n}$, the operator $\mathcal{D}(x_t):\mathcal{C}_0\to\Re^n$ defined by

$$\mathcal{D}(x_t)=x(t)+E_1\int_{t-h}^{t}x(s)ds-E_2x(t-h) \tag{5}$$

is stable if there exist a positive-definite matrix Γ and positive scalars α_1 and α_2 such that

$$\alpha_1+\alpha_2<1, \tag{6a}$$

$$\begin{bmatrix} E_2^T\Gamma E_2-\alpha_1\Gamma & hE_2^T\Gamma E_1 \\ \star & h^2E_1^T\Gamma E_1-\alpha_2\Gamma \end{bmatrix}<0. \tag{6b}$$

Remark 2.1. The well-known criterion for the stability of the operator $\mathcal{D}(x_t)$ given in (5) is

$$h\,\|E_1\|+\|\,E_2\,\|<1,$$

which is more conservative than the criterion (6) (Ref. 14).

To obtain the main result of the paper, let us rewrite the system (4) in the following form:

$$(d/dt)\left[z(t)+\bar{A}_1\int_{t-h}^{t}z(s)ds-\bar{A}_2z(t-h)\right]=\bar{A}z(t),\quad t\ge 0, \tag{7}$$

where

$$\bar{A}=\bar{A}_0+\bar{A}_1.$$

Define a new operator $\mathcal{D}(z_t):\mathcal{C}_0\to\Re^n$ as

$$\mathcal{D}(z_t)=z(t)+\bar{A}_1\int_{t-h}^{t}z(s)ds-\bar{A}_2z(t-h). \tag{8}$$

From Lemma 2.2, we can obtain easily the following lemma, which will be used in main theorem (Theorem 2.1).

Lemma 2.3. By Lemma 2.2, the operator $\mathcal{D}(z_t)$ is stable if there exist a positive-definite matrix Γ and positive scalars α_1, α_2 such that

$$\begin{bmatrix} \bar{A}_2^T \Gamma \bar{A}_2 - \alpha_1 \Gamma & h\bar{A}_2^T \Gamma \bar{A}_1 \\ \star & h^2 \bar{A}_1^T \Gamma \bar{A}_1 - \alpha_2 \Gamma \end{bmatrix} < 0 \quad \text{and} \quad \alpha_1 + \alpha_2 < 1. \tag{9}$$

Defining

$$\Gamma = \text{diag}\{\Gamma_1, \Gamma_2\}$$

and using matrix operations, we see that the operator $\mathcal{D}(z_t)$ is stable if there exist positive-definite matrices Γ_1, Γ_2 and positive scalars α_1, α_2 such that

$$\alpha_1 + \alpha_2 < 1, \tag{10a}$$

$$\begin{bmatrix} m_{11} & 0 & m_{13} & 0 \\ \star & -\alpha_1 \Gamma_2 & 0 & 0 \\ \star & \star & m_{33} & 0 \\ \star & \star & \star & -\alpha_2 \Gamma_2 \end{bmatrix} < 0, \tag{10b}$$

where

$$m_{11} = A_2^T \Gamma_1 A_2 - \alpha_1 \Gamma_1, \quad m_{13} = hA_2^T \Gamma_1 A_1, \quad m_{33} = h^2 A_1^T \Gamma_1 A_1 - \alpha_2 \Gamma_1.$$

Now, using the Lyapunov stability theory, we estabilish a criterion in terms of matrix inequalities for the dynamic output feedback controller of the neutral delay differential system (1).

Theorem 2.1. For given scalar $h > 0$, suppose that there exist matrices $\Gamma_1 > 0, \Gamma_2 > 0$ and scalars $\alpha_1 > 0, \alpha_2 > 0$ satisfying (10). Then there exist a dynamic output feedback controller (3) for the system (1) if there exist a positive scalar ε, positive-definite matrices $S, Y, X, \mathcal{R}$ and matrices $\hat{A}, \hat{B}, \hat{C}$ satisfying the following matrix inequalities:

$$\begin{bmatrix} \Omega_1 & \varepsilon Y & \Omega_2 & -\hat{A}^T A_2 & 0 & h\hat{A}^T A_1 & 0 \\ \star & -\varepsilon I & 0 & 0 & 0 & 0 & 0 \\ \star & \star & \Omega_3 & \Omega_4 & 0 & \Omega_5 & 0 \\ \star & \star & \star & -\varepsilon I/2 & \star & 0 & 0 \\ \star & \star & \star & \star & -\mathcal{R}/2 & 0 & 0 \\ \star & \star & \star & \star & \star & -\varepsilon I/2 & 0 \\ \star & \star & \star & \star & \star & \star & -\mathcal{R}/2 \end{bmatrix} < 0, \tag{11}$$

$$\begin{bmatrix} Y & I \\ I & S \end{bmatrix} > 0, \tag{12}$$

where

$$A = A_0 + A_1, \tag{13a}$$

$$\Omega_1 = AY + YA^T + B\hat{C} + \hat{C}^T B^T + X, \tag{13b}$$

$$\Omega_2 = \hat{A}^T + A + \varepsilon Y, \tag{13c}$$

$$\Omega_3 = SA + A^T S + C^T \hat{B}^T + \hat{B}C + \varepsilon I, \tag{13d}$$

$$\Omega_4 = -(A^T S + C^T \hat{B}^T)A_2, \tag{13e}$$

$$\Omega_5 = h(A^T S + C^T \hat{B}^T)A_1. \tag{13f}$$

Proof. Let us consider the following legitimate Lyapunov functional candidate (Ref. 1):

$$V = \mathcal{D}^T(z_t)P\mathcal{D}(z_t) + (1/2h)\int_{t-h}^{t}(s-t+h)z^T(s)Rz(s)ds + (1/2)\int_{t-h}^{t} z^T(s)Rz(s)ds, \tag{14}$$

where $P > 0$ and $R > 0$. Taking the time derivative of V along the solution of (7), we have

$$\begin{aligned} dV/dt = {} & 2z^T(t)\bar{A}^T P\left(z(t) + \bar{A}_1 \int_{t-h}^{t} z(s)ds - \bar{A}_2 z(t-h)\right) \\ & + (1/2)z^T(t)Rz(t) - (1/2h)\int_{t-h}^{t} z^T(s)Rz(s)ds + (1/2)z^T(t)Rz(t) \\ & - (1/2)z^T(t-h)Rz(t-h). \end{aligned} \tag{15}$$

By Lemma 2.1, a bound of the term $-\int_{t-h}^{t} z^T(s)Rz(s)ds$ on the right-hand side of (15) can be obtained as

$$-\int_{t-h}^{t} z^T(s)Rz(s)ds \le -\left((1/h)\int_{t-h}^{t} z(s)ds\right)^T (hR)\left((1/h)\int_{t-h}^{t} z(s)ds\right). \tag{16}$$

Substituting (16) into (15) gives

$$\begin{aligned} dV/dt \le\, & z^T(t)\left(P\bar{A}+\bar{A}^T P+R\right)z(t)-2z^T(t)\bar{A}^T P\bar{A}_2 z(t-h) \\ & +2z^T(t)\bar{A}^T P\bar{A}_1\int_{t-h}^{t} z(s)ds-\frac{1}{2}z^T(t-h)Rz(t-h) \\ & -(1/2)\left((1/h)\int_{t-h}^{t} z(s)ds\right)^T R\left((1/h)\int_{t-h}^{t} z(s)ds\right) \\ \equiv\, & Z^T(t)\Sigma Z(t), \end{aligned} \tag{17}$$

where

$$Z(t)=\begin{bmatrix} z(t) \\ z(t-h) \\ (1/h)\int_{t-h}^{t} z(s)ds \end{bmatrix}, \quad \Sigma=\begin{bmatrix} P\bar{A}+\bar{A}^T P+R & -\bar{A}^T P\bar{A}_2 & h\bar{A}^T P\bar{A}_1 \\ \star & -(1/2)R & 0 \\ \star & \star & -(1/2)R \end{bmatrix}.$$

Thus, if the inequality $\Sigma<0$ holds, there exists a positive scalar γ such that

$$dV/dt \le -\gamma \parallel x(t) \parallel^2. \tag{18}$$

In the matrix $\Sigma, P>0$ and $R>0$; the controller parameters A_c, B_c, C_c, which are included in the matrix $\bar{A}$, are unknown and occur in nonlinear fashion. Hence, $\Sigma<0$ cannot be considered a linear matrix inequality problem. In the following, we introduce a change of variables such that the inequality can be solved via convex optimization algorithms (Ref. 17).

First, partition the matrix P and its inverse as

$$P=\begin{bmatrix} S & N \\ N^T & U \end{bmatrix}, \quad P^{-1}=\begin{bmatrix} Y & M \\ M^T & W \end{bmatrix}, \tag{19}$$

where $S, Y \in \Re^{n\times n}$ are positive-definite matrices and M, N are invertible matrices.

Note that the equality $P^{-1}P=I$ gives

$$MN^T=I-YS. \tag{20}$$

Define

$$F_1=\begin{bmatrix} Y & I \\ M^T & 0 \end{bmatrix}, \quad F_2=\begin{bmatrix} I & S \\ 0 & N^T \end{bmatrix}. \tag{21}$$

Then, it follows that

$$PF_1=F_2, \quad F_1^T PF_1=F_1^T F_2=\begin{bmatrix} Y & I \\ I & S \end{bmatrix}>0. \tag{22}$$

Now, postmultiplying and premultiplying the matrix inequality $\Sigma < 0$ by the matrix $\operatorname{diag}\{F_1^T, I, I\}$ and its transpose, respectively, gives

$$\begin{bmatrix} F_2^T\bar{A}F_1 + F_1^T\bar{A}^T F_2 + F_1^T R F_1 & -F_1^T\bar{A}^T P\bar{A}_2 & hF_1^T\bar{A}^T P\bar{A}_1 \\ \star & -R/2 & 0 \\ \star & \star & -R/2 \end{bmatrix} < 0. \tag{23}$$

Here, we define the matrix R as

$$R = \operatorname{diag}\{\varepsilon I, \mathcal{R}\},$$

where $\mathcal{R}$ is a positive-definite matrix and ε is positive scalars to be chosen later.

By utilizing the relations (19)–(22), it can be obtained easily that the inequality (23) is equivalent to

$$\begin{bmatrix} m_{11} & m_{12} & m_{13} & 0 & m_{15} & 0 \\ \star & m_{22} & m_{23} & 0 & m_{25} & 0 \\ \star & \star & -\varepsilon I/2 & 0 & 0 & 0 \\ \star & \star & \star & -\mathcal{R}/2 & 0 & 0 \\ \star & \star & \star & \star & -\varepsilon I/2 & 0 \\ \star & \star & \star & \star & \star & -\mathcal{R}/2 \end{bmatrix} < 0, \tag{24}$$

where

$$m_{11} = AY + YA^T + BC_cM^T + MC_c^T B^T + \varepsilon YY + M\mathcal{R}M^T,$$
$$m_{12} = A + YA^T S + MC_c^T B^T S + YC^T B_c^T N^T + MA_C^T N^T + \varepsilon Y,$$
$$m_{13} = -(SAY + SBC_cM^T + NB_cCY + NA_cM^T)^T A_2,$$
$$m_{15} = h(SAY + SBC_cM^T + NB_cCY + NA_cM^T)^T A_1,$$
$$m_{22} = SA + NB_cC + A^T S + C^T B_c^T N^T + \varepsilon I,$$
$$m_{23} = -(A^T S + C^T B_c^T N^T)A_2,$$
$$m_{25} = h(A^T S + C^T B_c^T N^T)A_1.$$

By defining a new set of variables as follows:

$$X = M\mathcal{R}M^T, \tag{25a}$$

$$\hat{A} = SAY + SB\hat{C} + \hat{B}CY + NA_cM^T, \tag{25b}$$

$$\hat{B} = NB_c, \tag{25c}$$

$$\hat{C} = C_cM^T, \tag{25d}$$

the inequality (24) is simplified to the following inequality:

$$\begin{bmatrix} \Omega_1+\varepsilon YY & \Omega_2 & -\hat{A}^T A_2 & 0 & h\hat{A}T A_1 & 0 \\ \star & \Omega_3 & m_{23} & 0 & m_{25} & 0 \\ \star & \star & -\varepsilon I/2 & 0 & 0 & 0 \\ \star & \star & \star & -\mathcal{R}/2 & 0 & 0 \\ \star & \star & \star & \star & -\varepsilon I/2 & 0 \\ \star & \star & \star & \star & \star & -\mathcal{R}/2 \end{bmatrix} < 0, \qquad (26)$$

where

$$m_{23} = -(A^T S + C^T \hat{B}^T)A_2, \quad m_{25} = h(A^T S + C^T \hat{B}^T)A_1,$$

and $\Omega_1, \Omega_2, \Omega_3$ are defined in (13).

By Fact 2.1 (Schur complement), the inequality (26) is equivalent to the inequality (11). Therefore, by Theorem 9.8.1 of Hale and Lunel (Ref. 1, pp 292–293), with the stable operator $\mathcal{D}(z_t)$ and (18), we conclude that the systems (1a) and (4) are both asymptotically stable. This completes the proof. □

Remark 2.2. The problem of Theorem 2.1 is to determine whether or not the problem is feasible. This is a feasibility problem. The solutions of the problem can be found by solving a generalized eigenvalue problem in $S, Y, \mathcal{R}, X, \hat{A}, \hat{B}, \hat{C}, \varepsilon$, which is a quasiconvex optimization problem. Note that a locally optimal point of a quasiconvex optimization problem with strictly quasiconvex objective is globally optimal (Ref. 15). Various efficient convex optimization algorithms can be used to check whether the matrix inequalities (11) and (12) are feasible. In this paper, in order to solve the matrix inequality, we utilize the Matlab LMI Control Toolbox (Ref. 18), which implements state-of-the-art interior-point algorithms and is significantly faster than classical convex optimization algorithms (Ref. 15).

Remark 2.3. Given any solution of the matrix inequalities (11) and (12) in Theorem 2.1, a corresponding controller of the form (3) will be constructed as follows:

Step 1. Using two solutions $X, \mathcal{R}$, compute the invertible matrix M satisfying the relation $X = M\mathcal{R}M^T$.

Step 2. Using the matrix M, compute the invertible matrix N satisfying (20).

Step 3. Utilizing the matrices M and N obtained above, solve the system of equations (25) for B_c, C_c, A_c in order.

Example 2.1. Consider the following linear differential system of neutral type:

$$\dot{x}(t)=A_0x(t)+A_1x(t-h)+A_2\dot{x}(t-h)+Bu(t), \tag{27a}$$

$$y(t)=C_x(t), \tag{27b}$$

where

$$A_0=\begin{bmatrix}2 & 0\\ 1 & -1\end{bmatrix},\quad A_1=\begin{bmatrix}0.3 & 0.1\\ 0.1 & 0.5\end{bmatrix},\quad A_2=\begin{bmatrix}0.2 & 0\\ 0 & 0.2\end{bmatrix},$$
$$B=\begin{bmatrix}2\\ 0.5\end{bmatrix},\qquad C=[1,0],\qquad h=1,$$

and where the initial condition of the system is as follows:

$$x(t)=\left[e^t,-e^t\right]^T,\qquad \text{for } -1\le t\le 0.$$

Here, we construct a suitable dynamic output feedback controller of the form (3) for the system (27) guaranteeing the asymptotic stability of the closed-loop system. First, let us check the stability of the operator $D(z_t)$ given in (8). By solving the inequality (10), we have

$$\alpha_1=0.3333,\quad \alpha_2=0.3333,\quad \Gamma_1=\begin{bmatrix}0.6354 & 0.0094\\ 0.0094 & 0.6542\end{bmatrix},\quad \Gamma_2=\begin{bmatrix}0.6213 & 0\\ 0 & 0.6213\end{bmatrix},$$

which guarantees the stability of the operator.

Next, by applying Theorem 2.1 to this system and checking the feasibility of the matrix inequalities (11) and (12), we can find that the matrix inequalities are feasible and obtain the solutions of the inequalities,

$$S=\begin{bmatrix}18.1673 & 1.3411\\ 1.3411 & 4.8058\end{bmatrix},\qquad Y=\begin{bmatrix}3.1978 & 0.2072\\ 0.2072 & 0.7007\end{bmatrix},$$

$$\mathcal{R}=\begin{bmatrix}26.2991 & 0\\ 0 & 26.2991\end{bmatrix},\qquad X=\begin{bmatrix}16.5116 & 4.0692\\ 4.0692 & 1.2521\end{bmatrix},$$

$$\hat{A}=\begin{bmatrix}-0.8913 & 0.3856\\ -0.2001 & 0.1666\end{bmatrix},\qquad \hat{B}=\begin{bmatrix}-47.0197\\ -6.9813\end{bmatrix},$$

$$\hat{C}=\begin{bmatrix}-17.1857 & -3.5982\end{bmatrix},\qquad \varepsilon=1.$$

Therefore, in light of Remark 2.3, the invertible matrices M,N are as follows:

$$M=\begin{bmatrix}0.7739 & 0.1699\\ 0.1699 & 0.1369\end{bmatrix},\qquad N=\begin{bmatrix}-91.5031 & 79.1470\\ -3.5571 & -14.9031\end{bmatrix},$$

and the corresponding positive-definite matrix P is

$$P=\begin{bmatrix} 18.1673 & 1.3411 & -91.5031 & 79.1470 \\ 1.3411 & 4.8058 & -3.5571 & -14.9031 \\ -91.5031 & -3.5571 & 473.5732 & -430.7527 \\ 79.1470 & -14.9031 & -430.7527 & 490.7836 \end{bmatrix}.$$

Then, we can get the following stabilizing dynamic output feedback controller for the system (27):

$$A_c=\begin{bmatrix} -12.7534 & 0.0442 \\ -3.9491 & -1.0211 \end{bmatrix}, \quad B_c=\begin{bmatrix} 0.7618 \\ 0.2866 \end{bmatrix}, \quad C_c=[-22.5880 \ 1.7430].$$

The simulation results are in Figs. 1 and 2. From the figures, one can see that the system is indeed well stabilized.

3. Concluding Remarks

We have investigated the problem of the stabilization of a class of neutral delay differential systems. Then, we have designed a dynamic output feedback controller which guarantees the asymptotic stability of the

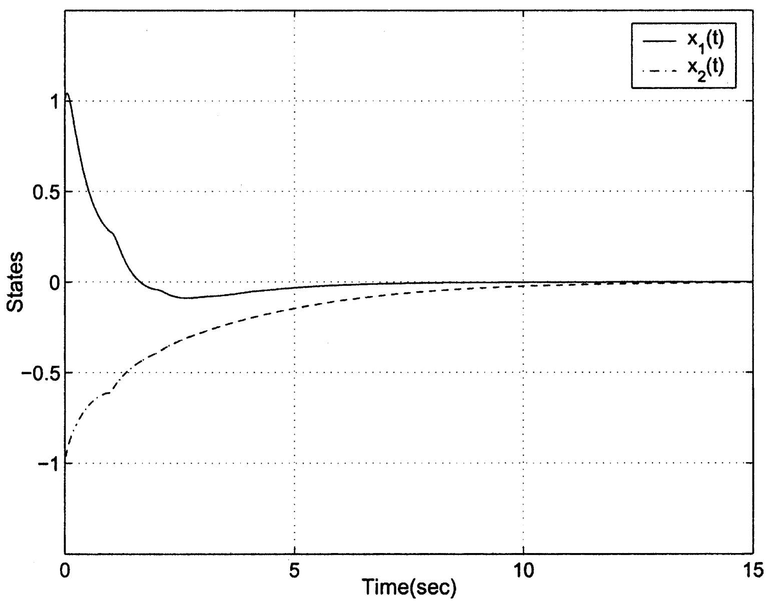

Fig. 1. State responses of the system.

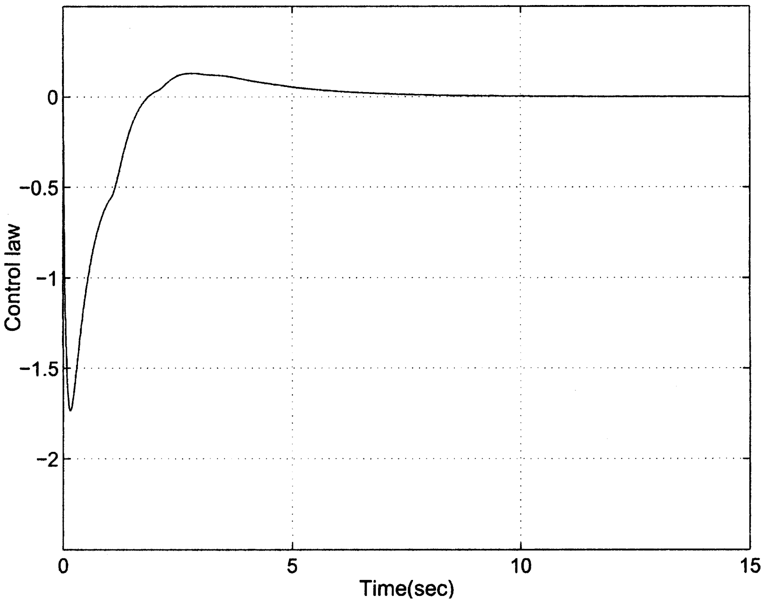

Fig. 2. Control law of the system.

systems and have derived a stabilization criterion in terms of matrix inequalities which can be solved easily by various efficient convex optimization algorithms. Finally, a numerical example is given to illustrate the design procedure.

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