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Decentralized Guaranteed Cost Control for Uncertain Large-Scale Systems Using Delayed Feedback: LMI Optimization Approach[1]

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Abstract. In this paper, we propose a design method of guaranteed cost controllers for uncertain large-scale systems with time delays in subsystem interconnections using delayed feedback. Using the Lyapunov method, a linear matrix inequality (LMI) optimization problem is formulated to design a delayed feedback controller which minimizes the upper bound of a given quadratic cost function. A numerical example is included to illustrate the design procedures.

Key Words. Uncertain large-scale systems, delayed feedback, guaranteed cost control, linear matrix inequalities, Lyapunov method.

1. Introduction

Recently, stabilizing control schemes for large-scale interconnected systems such as electrical power systems, communication networks, social systems, chemical processes, and economic systems have become more interesting. In controlling such systems, decentralized controllers are much preferred because the interconnected systems can be decomposed into several lower-order subsystems such that the design procedures are simplified and the computational burden can be shared by all the subsystem controllers. On the other hand, time delays due to the information transmission between subsystem interconnections exist naturally in large-scale systems and it is well known that their existence is a

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source of instability and poor performance of the systems. Hence, the stabilization problem of uncertain large-scale systems with time delays has received considerable attention by many researchers (see e.g. Refs. 1–8 and references therein). For instance, delay-independent stability conditions have been derived by Ikeda and Siljak (Ref. 1), Bakule (Ref. 2), Wu (Ref. 5), Won and Park (Ref. 6), and Park (Refs. 7, 8); delay-dependent criteria have been considered by De Souza (Ref. 3) and Tasy et al. (Ref. 4). In general, delay-dependent criteria are less conservative than delay-independent ones when the time-delay is small (Ref. 9).

When designing controllers, it is desirable to ensure satisfactory system performance. One possible approach to this problem is the so-called guaranteed cost control which was first introduced by Cheng and Peng (Ref. 10). Since this approach was represented by the LMI framework (Ref. 11), some researchers tackled the problem of the guaranteed cost control for several class of uncertain dynamic systems (Refs. 12–14). More recently, Xie et al. (Ref. 22) investigated the decentralized guaranteed cost control for uncertain large-scale interconnected systems. Mukaidani et al. (Refs. 23–24) extended the work to systems with time delays in the subsystems. However, time delays in the subsystem interconnections was not considered in Refs. 23, 24.

In this paper, we consider the design problem of a decentralized guaranteed cost controller for uncertain large-scale systems with time delays in the subsystem interconnections. A delayed feedback controller has been proposed for the control scheme of the system. The delayed feedback controller with feedback provisions on the current state and past history of the state may improve system performance (Refs. 15, 16). For designing delayed feedback controllers, we use a neutral model transformation (Ref. 18). Using the Lyapunov function method, a convex optimization problem is formulated to construct the delayed feedback controller which stabilizes the resulting closed-loop systems and minimizes the upper bound of the cost function. A delay-dependent stabilization criterion is derived in terms of LMIs which can be solved efficiently by various convex optimization algorithms (Ref. 17).

2. Preliminaries

The following notations are used in the paper. $\mathcal{R}^n$ is the n-dimensional Euclidean space, $\mathcal{R}^{m\times n}$ denotes the set of $m \times n$ real matrices, $*$ denotes the symmetric part, $X > 0$ $[X \geq 0]$ means that X is a real symmetric positive-definitive matrix [positive semidefinite], I denotes the identity matrix with appropriate dimensions. $\lambda_{\min}(X)$ and $\lambda_{\max}(X)$ are the minimum and maximum eigenvalues of X, $\|\cdot\|$ refers to the induced matrix 2-norm, diag$\{\cdots\}$ denotes the block diagonal matrix, $\mathcal{C}_{n,h} = \mathcal{C}([-h, 0], \mathcal{R}^n)$ denotes the Banach space of continuous functions mapping the interval $[-h, 0]$ into $\mathcal{R}^n$, with the topology of uniform convergence.

We need the following well-known facts and lemmas to obtain the main results.

Fact 2.1. Schur Complement. Given the constant symmetric matrices $\Sigma_1, \Sigma_2, \Sigma_3$, with $\Sigma_1 = \Sigma_1^T$ and $0 < \Sigma_2 = \Sigma_2^T$, then

$$\Sigma_1 + \Sigma_3^T \Sigma_2^{-1} \Sigma_3 < 0$$

if and only if

$$\begin{bmatrix} \Sigma_1 & \Sigma_3^T \\ \Sigma_3 & -\Sigma_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Sigma_2 & \Sigma_3 \\ \Sigma_3^T & \Sigma_1 \end{bmatrix} < 0.$$

Fact 2.2. For given matrices D, E, F, with $F^T F \le I$ and scalar $\epsilon > 0$, the following inequality is always satisfied:

$$DFE + E^T F^T D^T \le \varepsilon D D^T + \varepsilon^{-1} E^T E.$$

Lemma 2.1. See Ref. 25. For any $z, y \in \mathcal{R}^{n\times m}$ and any positive-definite matrix $X \in \mathcal{R}^{n\times n}$, the following inequality holds:

$$-2z^T y \le z^T X^{-1} z + y^T X y.$$

Lemma 2.2. See Ref. 19. For any constant matrix $M \in \mathcal{R}^{n\times n}$, $M = M^T > 0$, scalar $\gamma > 0$, and vector function $\omega : [0, \gamma] \to \mathcal{R}^n$ such that the integrations concerned are well defined,

$$\left(\int_0^\gamma \omega(s)ds\right)^T M \left(\int_0^\gamma \omega(s)ds\right) \le \gamma \int_0^\gamma \omega^T(s) M \omega(s) ds.$$

Lemma 2.3. See Ref. 20. Consider an operator $\mathcal{D}(\cdot) : \mathcal{C}_{n,h} \to \mathcal{R}^n$, with $\mathcal{D}(x_t) = x(t) + \hat{B} \int_{t-h}^{t} x(s)ds$, where $x(t) \in \mathcal{R}^n$ and $\hat{B} \in \mathcal{R}^{n\times n}$. For a given scalar δ, with $0 < \delta < 1$, let a positive-definite symmetric matrix M exist such that

$$\begin{bmatrix} -\delta M & h\hat{B}^T M \\ hM\hat{B} & -M \end{bmatrix} < 0.$$

Then, the operator $\mathcal{D}(x_t)$ is stable.

3. Problem Statements

Consider an uncertain large-scale system composed of N interconnected subsystems described by

$$\dot{x}_i(t) = (A_i + \Delta A_i)x_i(t) + (A_{ii} + \Delta A_{ii})x_i(t - h_{ii}) + \sum_{j=1, j\neq i}^{N} (A_{ij}+\Delta A_{ij})x_j(t - h_{ij}) + (B_i+\Delta B_i)u_i(t), \quad i = 1, \ldots, N,$$

$$x_i(s) = \phi_i(s), \quad s \in \left[-\max_{i,j}\{h_{ij}\}, 0\right], \tag{1}$$

where $x_i(t) \in \mathcal{R}^{n_i}$ is the state vector, $u_i(t) \in \mathcal{R}^{m_i}$ is the control input, A_i, A_{ii}, A_{ij}, B_i are known real matrices of appropriate dimensions, ΔA_i, ΔA_{ii}, ΔA_{ij}, ΔB_i are norm-bounded time-varying uncertainties, h_{ii} and h_{ij} are the known constant delays, and $\phi_i(s)$ is a given continuous vector-valued initial function.

The parameter uncertainties are assumed to be in the form

$$\Delta A_i = D_{1i}F_{1i}(t)E_{1i}, \qquad \Delta A_{ii} = D_{dii}F_{dii}(t)E_{dii},$$
$$\Delta A_{ij} = D_{2ij}F_{2ij}(t)E_{2ij}, \quad \Delta B_i = D_{3i}F_{3i}(t)E_{3i},$$

where $D_{1i}, D_{2ij}, D_{3i}, E_{1i}, E_{2ij}, E_{3i}, i, j = 1, \ldots, N, i \neq j$, are known real constant matrices of appropriate dimensions, and where $F_{ki}(t)$, $F_{dii}(t)$, $F_{2ij}(t)$ are unknown matrices, which satisfy

$$F_{ki}^T(t)F_{ki}(t) \leq I, \qquad k = 1, 3, \tag{2a}$$
$$F_{2ij}^T(t)F_{2ij}(t) \leq I, \quad F_{dii}^T(t)F_{dii}(t) \leq I. \tag{2b}$$

It is assumed that the pair $(A_i + A_{ii}, B_i)$ is controllable and that the measurements of the state $x_i(t)$ and the time delay h_{ii} are always available.

In this paper, in order to evaluate the system performance, we define the following integral quadratic cost function for the subsystem i:

$$J_i = \int_0^\infty \left[x_i^T(t)R_{xi}x_i(t) + u_i^T(t)R_{ui}u_i(t)\right] dt, \tag{3a}$$

where $R_{xi} > 0$ and $R_{ui} > 0$ are the given state and control weighting matrices. The total cost of the system (1) is

$$J = \sum_{i=1}^{N} J_i. \tag{3b}$$

Definition 3.1. For the system (1) and cost function (3), if a control law $u_i^*(t)$ and a positive scalar J_i^* exist, such that, for all admissible uncertainties, the resulting closed-loop system is asymptotically stable and the closed-loop value of the cost function satisfies $J_i \leq J_i^*$, then $u^*(t)$ is said to be a guaranteed cost control law for the system (1) and J_i^* is said to be a guaranteed cost of the ith subsystem.

Consider a controller in the form

$$u_i(t) = K_i \left[x_i(t) + \int_{t-h_{ii}}^{t} A_{ii} x_i(s) ds \right], \tag{4}$$

where $K_i \in \mathcal{R}^{m_i \times n_i}$ is a controller gain matrix for the ith subsystem.

Problem 3.1. Consider the system (1)–(3). The goal is to design the matrices K_i so that the controller (4) is a guaranteed cost controller for the system (1)–(3) by using a delay-dependent approach.

4. Controller Design

In the section, we propose a design method of a guaranteed cost controller for the system and use the neutral model transformation method (Ref. 18) as follows:

$$z_i(t) = x_i(t) + \int_{t-h_{ii}}^{t} A_{ii} x_i(s) ds. \tag{5}$$

The guaranteed cost controller (4) is rewritten as

$$u_i(t) = K_i z_i(t). \tag{6}$$

Substituting the controller (6) in the system (1), the resulting closed-loop system is

$$\begin{aligned} \dot{x}_i(t) &= (A_i + \Delta A_i) x_i(t) + (A_{ii} + \Delta A_{ii}) x_i(t - h_{ii}) \\ &\quad + \sum_{j=1, j \neq i}^{N} (A_{ij} + \Delta A_{ij}) x_j(t - h_{ij}) + (B_i + \Delta B_i) K_i z_i(t), \quad i = 1, \ldots, N. \end{aligned} \tag{7}$$

Differentiating $z_i(t)$ with respect to t leads to

$$\begin{aligned} \dot{z}_i(t) &= \dot{x}_i(t) + A_{ii} x_i(t) - A_{ii} x_i(t - h_{ii}) \\ &= [A_i + \Delta A_i(t) + A_{ii} + (B_i + \Delta B_i) K_i] z_i(t) \end{aligned}$$

$$+ \sum_{j \neq i}^{N} (A_{ij} + \Delta A_{ij}) x_j (t - h_{ij})$$
$$- (A_i + A_{ii} + \Delta A_i(t)) \int_{t-h_{ii}}^{t} A_{ii} x_i(s) ds + \Delta A_{ii} x_i(t - h_{ii}). \quad (8)$$

For simplicity, we define

$$D_{2di} = \left(\sum_{j=1, j\neq i}^{N} D_{2ij} D_{2ij}^T \right)^{1/2}, \quad (9)$$

$$\Sigma_i = (A_i + A_{ii}) X_i + X_i (A_i + A_{ii})^T + \sum_{j\neq i}^{N} A_{ij} X_{ij} A_{ij}^T + B_i Y_i + Y_i^T B_i^T$$
$$+ (\varepsilon_{1i} + \varepsilon_{4i}) D_{1i} D_{1i}^T + \varepsilon_{2i} D_{dii} D_{dii}^T + \varepsilon_{3i} D_{3i} D_{3i}^T + \varepsilon_{5i} D_{2di} D_{2di}^T, \quad (10)$$

$$N_{ii} N_{ii}^T = \int_{-h_{ii}}^{0} \phi_i(s) \phi_i^T(s) ds, \quad (11)$$

$$N_{dii} N_{dii}^T = \int_{-h_{ii}}^{0} \int_{s}^{0} \phi_i(u) \phi_i^T(u) du ds, \quad (12)$$

$$N_{dij} N_{dij}^T = \int_{-h_{ij}}^{0} \phi_j(s) \phi_j^T(s) ds, \quad (13)$$

where X_i and X_{ij} are positive-definite matrices, Y_i is a matrix with appropriate dimensions, $\varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}, \varepsilon_{5i}$ are positive scalars. Then we have the following theorem.

Theorem 4.1. Consider the system (1) with the cost function (3). For given delays h_{ii}, consider the following optimization problem:

$$\text{minimize} \left\{ \alpha_i + \text{Trace} \left(M_{ii} + M_{dii} + \sum_{j=1, j\neq i}^{N} (M_{dij}) \right) \right\}, \quad (14)$$

subject to

$$\begin{bmatrix} \Sigma_i & -(A_i + A_{ii})A_{ii}R_{ii} & h_{ii}X_i & X_i & X_i & X_i & \dots & X_i & 0 & X_iE_{1i}^T & Y_i^TE_{3i}^T & Y_i^T \\ * & -R_{ii} & -h_{ii}R_{ii}A_{ii}^T & -R_{ii}A_{ii}^T & -R_{ii}A_{ii}^T & -R_{ii}A_{ii}^T & \dots & -R_{ii}A_{ii}^T & R_{ii}A_{ii}^TE_{1i}^T & 0 & 0 & 0 \\ * & * & -R_{ii} & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ * & * & * & -R_{xi}^{-1} & 0 & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -W_i & 0 & \dots & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & -W_{1i} & \dots & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \ddots & \vdots & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & \dots & -W_{Ni} & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & -\varepsilon_{4i}I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & * & * & -\varepsilon_{1i}I & 0 & 0 \\ * & * & * & * & * & * & * & * & * & * & -\varepsilon_{3i}I & 0 \\ * & * & * & * & * & * & * & * & * & * & * & -R_{ui}^{-1} \end{bmatrix} < 0, \tag{15}$$

$$\begin{bmatrix} -W_i & W_iE_{dii}^T \\ * & -\varepsilon_{2i}I \end{bmatrix} < 0, \tag{16}$$

$$\begin{bmatrix} -W_{ij} & W_{ij}E_{2ij}^T & W_{ij} & \dots & W_{ij} \\ * & -\varepsilon_{2i}I & 0 & \dots & 0 \\ * & * & -X_{i1} & \dots & 0 \\ * & * & * & \ddots & \vdots \\ * & * & * & \dots & -X_{iN} \end{bmatrix} < 0, \quad j = 1, \dots, N, j \neq i, \tag{17}$$

$$\begin{bmatrix} -\alpha_i & z_i^T(0) \\ * & -X_i \end{bmatrix} < 0, \tag{18}$$

$$\begin{bmatrix} -M_{ii} & N_{ii}^T \\ * & -W_i \end{bmatrix} < 0, \tag{19}$$

$$\begin{bmatrix} -M_{dii} & h_{ii}N_{dii}^T \\ * & -h_{ii}R_{ii} \end{bmatrix} < 0, \tag{20}$$

$$\begin{bmatrix} -M_{dij} & N_{dij}^T \\ * & -W_{ij} \end{bmatrix} < 0. \tag{21}$$

Assume that problem (15)–(21) has solutions with positive-definite matrices $X_i, X_{ij}, R_{ii}, M_{ii}, M_{dii}, M_{dij}, W_i, W_{ij}$, matrix Y_i, and positive scalars $\alpha_i, \varepsilon_{1i}, \varepsilon_{2i}, \varepsilon_{3i}, \varepsilon_{4i}, \varepsilon_{5i}$. Then,

$$u_i(t) = Y_i X_i^{-1} z_i(t)$$

is the guaranteed cost controller for the system (1) and minimizes the upper bound of the cost function (3) as

$$J_i^* = \left[\alpha_i + \text{Trace}\left(M_{ii} + M_{dii} + \sum_{j\neq i}^{N}(M_{dij})\right)\right]. \tag{22}$$

Proof. Consider the following Lyapunov function candidate

$$\begin{aligned} V(t) =& \sum_{i=1}^{N} V_i(t) \\ =& \sum_{i=1}^{N} \left\{ z_i^T(t) P_i z_i(t) + \int_{t-h_{ii}}^{t} x_i^T(s) T_i x_i(s) ds \right. \\ & \left. + \int_{t-h_{ii}}^{t} \int_{s}^{t} x_i^T(u) Q_{ii} x_i(u) du ds + \sum_{j\neq i}^{N} \int_{t-h_{ij}}^{t} x_j^T(s) T_{ij} x_j(s) ds \right\}, \end{aligned} \tag{23}$$

where the matrices P_i, T_i, Q_{ii}, T_{ij} are positive definite. Taking the time derivative of $V(t)$ leads to

$$\begin{aligned} \dot{V}(t) = \sum_{i=1}^{N} & \left\{ 2z_i^T(t) P_i (A_i + A_{ii} + \Delta A_i + (B_i + \Delta B_i) K_i) z_i(t) \right. \\ & - 2z_i^T(t) P_i (A_i + A_{ii} + \Delta A_i) \int_{t-h_{ii}}^{t} A_{ii} x_i(s) ds - \int_{t-h_{ii}}^{t} x_i^T(s) Q_{ii} x_i(s) ds \\ & + 2Z_i^T(t) P_i \Delta A_{ii} x_i(t - h_{ii}) + 2z_i^T(t) P_i \sum_{j\neq i}^{N} (A_{ij} + \Delta A_{ij}) x_j(t - h_{ij}) \\ & + x_i^T(t) T_i x_i(t) - x_i^T(t - h_{ii}) T_i x_i(t - h_{ii}) + h_{ii} x_i^T(t) Q_{ii} x_i(t) \\ & \left. + \sum_{j\neq i}^{N} \left(x_j^T(t) T_{ij} x_j(t) - x_j^T(t - h_{ij}) T_{ij} x_j(t - h_{ij}) \right) \right\}. \end{aligned} \tag{24}$$

Define

$$\hat{V}(t) = \dot{V}(t) + \sum_{i=1}^{N} \left[x_i^T(t) R_{xi} x_i(t) + u_i^T(t) R_{ui} u(t) \right]. \tag{25}$$

Note that, if $\hat{V}(t)$ is negative, then $\dot{V}(t)$ is negative.

Using Fact 2.2 and Lemma 2.1, we obtain

$$\begin{aligned}
&\sum_{i=1}^{N} 2z_i^T(t)P_i \sum_{j\neq i}^{N} A_{ij}x_j(t) \\
&\leq \sum_{i=1}^{N} z_i^T(t)P_i\left(\sum_{j\neq i}^{N} A_{ij}X_{ij}A_{ij}^T\right)P_i z_i(t) \\
&\quad + \sum_{i=1}^{N}\sum_{j\neq i}^{N} x_j^T(t-h_{ij})X_{ij}^{-1}x_j(t-h_{ij}),
\end{aligned} \tag{26}$$

$$\begin{aligned}
&\sum_{i=1}^{N} 2z_i^T(t)P_iD_{1i}F_{1i}(t)E_{1i}z_i(t) \\
&\leq \sum_{i=1}^{N} \varepsilon_{1i}z_i^T(t)P_iD_{1i}D_{1i}^TP_iz_i(t) + \sum_{i=1}^{N}\varepsilon_{1i}^{-1}z_i^T(t)E_{1i}^TE_{1i}z_i(t),
\end{aligned} \tag{27}$$

$$\begin{aligned}
&\sum_{i=1}^{N} 2z_i^T(t)P_iD_{dii}F_{dii}(t)E_{dii}x_i(t-h_{ii}) \\
&\leq \sum_{i=1}^{N} \varepsilon_{2i}z_i^T(t)P_iD_{dii}D_{dii}^TP_iz_i(t) \\
&\quad + \sum_{i=1}^{N}\varepsilon_{2i}^{-1}\left[x_i^T(t-h_{ii})E_{dii}^TE_{dii}x_i(t-h_{ii})\right],
\end{aligned} \tag{28}$$

$$\begin{aligned}
&\sum_{i=1}^{N} 2z_i^T(t)P_iD_{3i}F_{3i}(t)E_{3i}K_iz_i(t) \\
&\leq \sum_{i=1}^{N} \varepsilon_{3i}z_i^T(t)P_iD_{3i}D_{3i}^TP_iz_i(t) + \sum_{i=1}^{N}\varepsilon_{3i}^{-1}z_i^T(t)K_i^TE_{3i}^TE_{3i}K_iz_i(t),
\end{aligned} \tag{29}$$

$$\begin{aligned}
&\sum_{i=1}^{N} 2z_i^T(t)P_iD_{1i}F_{1i}(t)E_{1i}\int_{t-h_{ii}}^{t} A_{ii}x_i(s)ds \\
&\leq \sum_{i=1}^{N} \varepsilon_{4i}z_i^T(t)P_iD_{1i}D_{1i}^TP_iz_i(t) \\
&\quad + \sum_{i=1}^{N}\left[\varepsilon_{4i}^{-1}\left(\int_{t-h_{ii}}^{t} x_i(s)ds\right)^T A_{ii}^TE_{1i}^TE_{1i}A_{ii}\left(\int_{t-h_{ii}}^{t} x_i(s)ds\right)\right],
\end{aligned} \tag{30}$$

$$\sum_{i=1}^{N} 2z_i^T(t)P_i \sum_{j\neq i}^{N} D_{2ij}F_{2ij}(t)E_{2ij}x_j(t-h_{ij})$$
$$\leq \sum_{i=1}^{N} \varepsilon_{5i} z_i^T(t)P_i D_{2di} D_{2di}^T P_i z_i(t)$$
$$+\sum_{i=1}^{N} \varepsilon_{5i}^{-1} \sum_{j\neq i}^{N} x_j^T(t-h_{ij})E_{2ij}^T E_{2ij}x_j(t-h_{ij}). \tag{31}$$

Here, note that

$$\sum_{i=1}^{N}\sum_{j\neq i}^{N} x_j^T(t)T_{ij}x_j(t) = \sum_{i=1}^{N} x_i^T(t)\left(\sum_{j\neq i}^{N} T_{ji}\right)x_i(t) \tag{32}$$

and define

$$\hat{T}_i = h_{ii}Q_{ii} + R_{xi} + T_i + \sum_{j\neq i}^{N} T_{ji}. \tag{33}$$

Then,

$$x_i^T(t)\hat{T}_i x_i(t) = \left[z_i(t) - \int_{t-h_{ii}}^{t} A_{ii}x_i(s)ds\right]^T \hat{T}_i \left[z_i(t) - \int_{t-h_{ii}}^{t} A_{ii}x_i(s)ds\right]$$
$$= z_i^T(t)\hat{T}_i z_i(t) - 2z_i^T(t)\hat{T}_i \int_{t-h_{ii}}^{t} A_{ii}x_i(s)ds$$
$$+\left[\int_{t-h_{ii}}^{t} A_{ii}x_i(s)ds\right]^T \hat{T}_i \left[\int_{t-h_{ii}}^{t} A_{ii}x_i(s)ds\right]. \tag{34}$$

Substituting (26)–(32) and (34) into (24) and applying Lemma 2.2 to the term $-\int_{t-h_{ii}}^{t} x_i^T(s)Q_{ii}x_i(s)ds$, we see that $\hat{V}(t)$ has a new upper bound as follows:

$$\hat{V}(t) \leq \sum_{i=1}^{N} \begin{bmatrix} z_i(t) \\ \int_{t-h_{ii}}^{t} x_i(s)ds \end{bmatrix}^T G_i \begin{bmatrix} z_i(t) \\ \int_{t-h_{ii}}^{t} x_i(s)ds \end{bmatrix} ds$$
$$+\sum_{i=1}^{N} \left[x_i^T(t-h_{ii})\big(-T_i + \varepsilon_{2i}^{-1}E_{dii}^T E_{dii}\big)x_i(t-h_{ii})\right]$$
$$+\sum_{i=1}^{N}\sum_{j\neq i}^{N} \left[x_j^T(t-h_{ij})\big(-T_{ij} + \varepsilon_{5i}^{-1}E_{2ii}^T E_{2ij} + X_{ij}^{-1}\big)x_j(t-h_{ij})\right], \tag{35}$$

where

$$G_i = \begin{bmatrix} \Sigma_{1i} + \hat{T}_i & G_{i12} \\ * & G_{i22} \end{bmatrix}, \tag{36}$$

$$G_{i12} = -P_i(A_i + A_{ii})A_{ii} - \hat{T}_i A_{ii}, \tag{37}$$

$$G_{i22} = -h_{ii}^{-1} Q_{ii} + A_{ii}^T \hat{T}_i A_{ii} + \varepsilon_{4i}^{-1} A_{ii}^T E_{1i}^T E_{1i} A_{ii}, \tag{38}$$

$$\begin{aligned} \Sigma_{1i} = {} & P_i(A_i + A_{ii}) + (A_i + A_{ii})^T P_i + P_i \left(\sum_{j \neq i}^{N} A_{ij} X_{ij}^T A_{ij} \right) P_i \\ & + P_i B_i K_i + K_i^T B_i^T P_i + (\varepsilon_{1i} + \varepsilon_{4i}) P_i D_{1i} D_{1i}^T P_i \\ & + \varepsilon_{2i} P_i D_{dii} D_{dii}^T P_i + \varepsilon_{3i} P_i D_{3i} D_{3i}^T P_i \\ & + \varepsilon_{5i} P_i D_{2di} D_{2di}^T P_i + \varepsilon_{1i}^{-1} E_{1i}^T E_{1i} + \varepsilon_{3i}^{-1} K_i^T E_{3i}^T E_{3i} K_i + K_i^T R_{ui} K_i. \end{aligned} \tag{39}$$

From (35), if the following inequalities hold:

$$G_i < 0, \tag{40}$$

$$-T_i + \varepsilon_{2i}^{-1} E_{dii}^T E_{dii} < 0, \tag{41}$$

$$\sum_{j \neq i}^{N} \left[-T_{ij} + \varepsilon_{5i}^{-1} E_{2ii}^T E_{2ij} + X_{ij}^{-1} \right] < 0, \tag{42}$$

for $i = 1, \ldots, N$, then there are positive scalars λ_i satisfying

$$\dot{V}_i(t) < -\lambda_i \|z_i(t)\|^2. \tag{43}$$

From (36) and (38), the inequalities $G_i < 0$ imply $G_{i22} < 0$, which guarantee that

$$-h_{ii}^{-1} Q_{ii} + A_{ii}^T \hat{T}_i A_{ii} < 0. \tag{44}$$

Also, from relation (33), the inequality (44) implies

$$-h_{ii}^{-1} Q_{ii} + h_{ii} A_{ii}^T Q_{ii} A_{ii} < 0. \tag{45}$$

If the above inequality (45) holds, then we can prove that there is a scalar $0 < \delta_i < 1$ such that

$$\begin{bmatrix} -\delta_i Q_{ii} & h_{ii} A_{ii}^T Q_{ii} \\ * & -Q_{ii} \end{bmatrix} < 0, \tag{46}$$

by Fact 2.1 and matrix theory. Therefore, if $G_i < 0$ holds, then

$$z_i(t) = x_i(t) + \int_{t-h_{ii}}^{t} A_{ii} x_i(s) ds$$

is a stable operator by Lemma 2.3 According to Theorem 9.8.1 in Ref. 18, we conclude that the system (7) is asymptotically stable.

Again, using Fact 2.1, $G_i < 0$ is equivalent to

$$\begin{bmatrix} \Sigma_{1i} & -P_i(A_i + A_{ii})A_{ii} & h_{ii}I & I & I & I & \dots & I \\ * & \begin{bmatrix} -h_{ii}^{-1}Q_{ii} \\ +\varepsilon_{4i}^{-1}A_{ii}^T E_{1i}^T E_{1i} A_{ii} \end{bmatrix} & -h_{ii}A_{ii}^T & -A_{ii}^T & -A_{ii}^T & -A_{ii}^T & \dots & -A_{ii}^T \\ * & * & -h_{ii}Q_{ii}^{-1} & 0 & 0 & 0 & \dots & 0 \\ * & * & * & -R_{xi}^{-1} & 0 & 0 & \dots & 0 \\ * & * & * & * & -T_i^{-1} & 0 & \dots & 0 \\ * & * & * & * & * & -T_{1i}^{-1} & \dots & 0 \\ * & * & * & * & * & * & \ddots & 0 \\ * & * & * & * & * & * & \dots & -T_{Ni}^{-1} \end{bmatrix} < 0. \tag{47}$$

Let

$$X_i = P_i^{-1}, \quad W_i = T_i^{-1}, \quad W_{ij} = T_{ij}^{-1}, \quad R_{ii} = h_{ii}Q_{ii}^{-1}, \quad Y_i = K_i X_i. \tag{48}$$

Postmultiplying and premultiplying the matrix inequalities (47), (41), (42) by the matrices diag$\{X_i, R_{ii}, I, I, I, I, \dots, I\}$, W_i, W_{ij}, the inequalities (47), (41), (42) are equivalent to the inequalities (15), (16), (17) by using Fact 2.1. Therefore, the system (1) under the controller (3) is asymptotically stable if (15), (16), (17) hold.

Now, we state the minimization of the upper bound of the cost function (3). If the inequalities (15), (16), (17) hold, then from (25) we have

$$\sum_{i=1}^{N} \dot{V}_i(t) < -\sum_{i=1}^{N} \left[x_i^T(t)R_{xi}x_i(t) + u_i^T(t)R_{ui}u_i(t)\right] < 0. \tag{49}$$

Integrating both sides of (49) from 0 to t_f gives

$$\sum_{i=1}^{N} \Bigg\{ z_i^T(t_f)P_i z_i(t_f) + \int_{t_f-h_{ii}}^{t_f} x_i^T(s)T_i x_i(s)ds + \int_{t_f-h_{ii}}^{t_f}\int_{s}^{t_f} x_i^T(u)Q_{ii}x_i(u)duds$$
$$+ \sum_{j\neq i}^{N} \int_{t_f-h_{ij}}^{t_f} x_j^T(s)T_{ij}x_j(s)ds - z_i^T(0)P_i z_i(0) - \int_{-h_{ii}}^{0} \phi_i^T(s)T_i\phi_i(s)ds$$

$$
\left. - \int_{h_{ii}}^{0} \int_{s}^{0} \phi_i^T(u) Q_{ii} \phi_i(u) du ds - \sum_{j \neq i}^{N} \int_{-h_{ij}}^{0} \phi_j^T(s) T_{ij} \phi_j(s) ds \right\}
$$

$$
< - \sum_{i=1}^{N} \int_{0}^{t_f} \left[x_i^T(t) R_{xi} x_i(t) + u_i^T(t) R_{ui} u_i(t) \right] ds. \tag{50}
$$

Since we have established already the asymptotic stability of the closed-loop system (7), when $t_f \longrightarrow \infty$,

$$
\begin{aligned}
& z_i^T(t_f) P_i z_i(t_f) \longrightarrow 0, \\
& \int_{t_f - h_{ii}}^{t_f} x_i^T(s) T_i x_i(s) ds \longrightarrow 0, \\
& \int_{t_F - h_{ii}}^{t_f} \int_{s}^{t_f} x_i^T(u) Q_{ii} x_i(u) du ds \longrightarrow 0, \\
& \sum_{j \neq i}^{N} \int_{t_f - h_{ij}}^{t_f} x_j^T(s) T_{ij} x_j(s) ds \longrightarrow 0 \, .
\end{aligned}
$$

Therefore, we obtain the upper bound of the cost function (3) as

$$
\begin{aligned}
J_i \leq {} & z_i^T(0) P_i z_i(0) + \int_{-h_{ii}}^{0} \phi_i^T(s) T_i \phi_i(s) ds \\
& + \int_{-h_{ii}}^{0} \int_{s}^{0} \phi_i^T(u) Q_{ii} \phi_i(u) du ds + \sum_{j \neq i}^{N} \int_{-h_{ij}}^{0} \phi_j^T(s) T_{ij} \phi_j(s) ds.
\end{aligned} \tag{51}
$$

If the LMIs (18), (19), (20), (21) in Theorem 4.1 hold, then the following inequality holds:

$$
\begin{aligned}
& \alpha_i + \mathrm{Trace}\left(M_{ii} + M_{dii} + \sum_{j \neq i}^{N} M_{dij} \right) \\
& > z_i^T(0) X_i^{-1} z_i(0) + \mathrm{Trace}\left\{ N_{ii}^T W_i^{-1} N_{ii} + N_{dii}^T \left(h_{ii} \mathcal{R}_{ii}^{-1} \right) N_{dii} \right. \\
& \quad \left. + \sum_{j \neq i}^{N} N_{dij}^T W_{ij}^{-1} N_{dij} \right\},
\end{aligned} \tag{52}
$$

as can be seen by applying Fact 2.1 to the LMIs (18), (19), (20), (21) and adding each term. Since $P_i = X_i^{-1}$ and

$$\int_{-h_{ii}}^{0} \phi_i^T(s)T_i\phi_i(s)ds = \text{Trace}\big(N_{ii}N_{ii}^T W_i^{-1}\big) = \text{Trace}\big(N_{ii}^T W_i^{-1} N_{ii}\big), \tag{53}$$

$$\int_{-h_{ii}}^{0}\int_{s}^{0} \phi_i^T(u)Q_{ii}\phi_i(u)duds = \text{Trace}\big(N_{dii}N_{dii}^T Q_{ii}\big) = \text{Trace}\big(N_{dii}^T Q_{ii} N_{dii}\big) = \text{Trace}\big(N_{dii}^T h_{ii}\mathcal{R}_{ii}^{-1} N_{dii}\big), \tag{54}$$

$$\sum_{j\neq i}^{N}\int_{-h_{ij}}^{0} \phi_j^T(s)T_{ij}\phi_j(s)ds = \sum_{j\neq i}^{N}\int_{-h_{ij}}^{0} \phi_j^T(s)T_{ij}\phi_j(s)ds = \sum_{j\neq i}^{N}\text{Trace}\big(N_{dij}N_{dij}^T T_{ij}\big) = \sum_{j\neq i}^{N}\text{Trace}\big(N_{dij}^T W_{ij}^{-1} N_{dij}\big), \tag{55}$$

we obtain

$$z_i^T(0)X_i^{-1}z_i(0) + \text{Trace}\left\{N_{ii}^T W_i^{-1}N_{ii} + N_{dii}^T(h_{ii}\mathcal{R}_{ii}^{-1})N_{dii} + \sum_{j\neq i}^{N} N_{dij}^T W_{ij}^{-1}N_{dij}\right\}$$
$$= z_i^T(0)P_i z_i(0) + \int_{-h_{ii}}^{0}\phi_i^T(s)T_i\phi_i(s)ds + \int_{-h_{ii}}^{0}\int_{s}^{0}\phi_i^T(u)Q_{ii}\phi_i(u)duds$$
$$+\sum_{j\neq i}^{N}\int_{-h_{ij}}^{0}\phi_j^T(s)T_{ij}\phi_j(s)ds. \tag{56}$$

From (51) and (56), it is obvious that the upper bound of the cost function (3) for each subsystem is J_i^* given in (22). Therefore, the controller

$$u_i(t) = Y_i X_i^{-1} z_i(t)$$

constructed from Theorem 4.1 is the guaranteed cost controller minimizing the upper bound value of the cost function (3). This completes our proof. □

Remark 4.1. Since conditions (15)–(21) in Theorem 4.1 are LMIs with respect to the solution variables, various efficient convex algorithms can be used to ascertain the LMI solutions. In this paper, we utilize the Matlab LMI Control

Toolbox (Ref. 21) which implements interior-point algorithms. These algorithms are significantly faster than classical convex optimization algorithms (Ref. 17).

Remark 4.2. Recently, a new delay-independent stability criterion for uncertain large-scale system with delays has been presented by Mukaidani (Ref. 24). The system considered in this work (Ref. 24) does not have the delays in the subsystem interconnection. However, our result is delay-dependent and our work considers the delays in the subsystem interconnections. Note that it is well-known that the delay-dependent criterion is less conservative than the delay-independent one when the delay is small. Another advantage of the method proposed in this paper is that it can be applied to systems in which the pair (A_i, B_i) is uncontrollable, since our work use the assumption that $(A_i + A_{ii}, B_i)$ is controllable.

5. Numerical Example

Consider the following uncertain large-scale systems with $N = 3$:

$$\begin{aligned}
\dot{x}_1(t) =& \left\{\begin{bmatrix} 0 & 0 \\ -4 & -3 \end{bmatrix} + \Delta A_1(t)\right\} x_1(t) + \left\{\begin{bmatrix} -0.45 & 0.45 \\ 0 & 0.4.5 \end{bmatrix} + \Delta A_{11}(t)\right\} x_1(t-0.9) \\
&+ \left\{\begin{bmatrix} -0.5 & 0.1 \\ 0.3 & 0 \end{bmatrix} + \Delta A_{12}(t)\right\} x_2(t-1) \\
&+ \left\{\begin{bmatrix} -0.4 & 0.2 \\ 0.1 & 0.3 \end{bmatrix} + \Delta A_{13}(t)\right\} x_3(t-1) + \left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \Delta B_1(t)\right\} u_1(t), \qquad (57a)
\end{aligned}$$

$$\phi_1(s) = \begin{bmatrix} e^{s+1} \\ 0 \end{bmatrix}, \quad s \in [-1, 0], \qquad (57b)$$

$$\begin{aligned}
\dot{x}_2(t) =& \left\{\begin{bmatrix} -4 & 0.1 \\ 0 & -2.5 \end{bmatrix} + \Delta A_2(t)\right\} x_2(t) \\
&+ \left\{\begin{bmatrix} -0.675 & 0.675 \\ 0 & 0.45 \end{bmatrix} + \Delta A_{22}(t)\right\} x_2(t-0.8) \\
&+ \left\{\begin{bmatrix} 0.3 & -0.5 \\ 0.2 & 0.1 \end{bmatrix} + \Delta A_{21}(t)\right\} x_1(t-1) \\
&+ \left\{\begin{bmatrix} 0.2 & 0.5 \\ 0.1 & 0.2 \end{bmatrix} + \Delta A_{23}(t)\right\} x_3(t-1) + \left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix} + \Delta B_2(t)\right\} u_2(t), \qquad (57c)
\end{aligned}$$

$$\phi_2(s) = \begin{bmatrix} 0.5e^{0.5s} \\ -e^{0.5s} \end{bmatrix}, \quad s \in [-1, 0], \qquad (57d)$$

$$\dot{x}_3(t) = \left\{ \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix} + \Delta A_3(t) \right\} x_3(t)$$
$$+ \left\{ \begin{bmatrix} -0.45 & 0 \\ -0.45 & -0.45 \end{bmatrix} + \Delta A_{33}(t) \right\} x_3(t - 0.7)$$
$$+ \left\{ \begin{bmatrix} 0.3 & 0.1 \\ 0.1 & 0.3 \end{bmatrix} + \Delta A_{31}(t) \right\} x_1(t - 1)$$
$$+ \left\{ \begin{bmatrix} 0.2 & 0.4 \\ 0.1 & 0.3 \end{bmatrix} + \Delta A_{32}(t) \right\} x_2(t - 1) + \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \Delta B_3(t) \right\} u_3(t), \qquad (57e)$$
$$\phi_3(s) = \begin{bmatrix} 0.5e^s \\ -0.5e^s \end{bmatrix}, \quad s \in [-1, 0]. \qquad (57f)$$

where

$$\Delta A_1(t) = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.2 \end{bmatrix} \sin(2t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_{12}(t) = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.2 \end{bmatrix} \cos(0.5t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_{12}(t) = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \cos(t)[0 \quad 1],$$

$$\Delta A_{13}(t) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \cos(t)[0 \quad 1],$$

$$\Delta A_2(t) = \begin{bmatrix} 0 & 0 \\ 0.1 & 0.1 \end{bmatrix} \sin(2t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_{22}(t) = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.2 \end{bmatrix} \sin(0.5t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_{21}(t) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \sin(3t)[1 \quad 1],$$

$$\Delta A_{23}(t) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \sin(3t)[1 \quad 0],$$

$$\Delta A_3(t) = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.1 \end{bmatrix} \cos(2t) \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_{33}(t) = \begin{bmatrix} 0 & 0 \\ 0.2 & 0.2 \end{bmatrix} \sin(0.5t) \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix},$$

$$\Delta A_{31}(t) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \sin(3t)[1 \quad 1],$$

$$\Delta A_{32}(t) = \begin{bmatrix} 0.2 \\ 0 \end{bmatrix} \sin(3t)[1 \quad 0],$$

$$\Delta B_1(t) = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} \cos(2t),$$

$$\Delta B_2(t) = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix} \cos(3t),$$

$$\Delta B_3(t) = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix} \cos(3t).$$

The weighting matrices are chosen as

$$R_{x1} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \quad R_{u1} = 0.2, \quad R_{x2} = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$R_{u2} = 0.1, \quad R_{x3} = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad R_{u3} = 0.2.$$

By applying Theorem 4.1 to the above system, we found the solutions of the LMIs to be

$$X_1 = \begin{bmatrix} 0.0641 & -0.0295 \\ -0.0295 & 0.2664 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} 0.1591 & 0.2007 \\ 0.2007 & 0.4528 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 0.0952 & 0.0588 \\ 0.0588 & 0.1377 \end{bmatrix}, \qquad X_{12} = \begin{bmatrix} 0.5587 & 2.5955 \\ 2.5955 & 13.1555 \end{bmatrix},$$

$$X_{13} = \begin{bmatrix} 0.3209 & 0.5540 \\ 0.5540 & 1.1855 \end{bmatrix}, \qquad X_{21} = \begin{bmatrix} 15.8485 & 8.1079 \\ 8.1079 & 4.8051 \end{bmatrix},$$

$$X_{23} = \begin{bmatrix} 60.4800 & -24.2279 \\ -24.2279 & 9.9646 \end{bmatrix}, \qquad X_{31} = \begin{bmatrix} 11.14137 & 11.1799 \\ 11.1799 & 12.3407 \end{bmatrix},$$

$$X_{32} = \begin{bmatrix} 21.6904 & -19.2685 \\ -19.2685 & 19.0277 \end{bmatrix}, \qquad Y_1 = [0.0001 \quad -4.5133],$$

$$Y_2 = [0.0021 \quad -4.1071], \qquad Y_3 = [0.0021 \quad -4.1071],$$

$$R_{11} = \begin{bmatrix} 0.3169 & -0.0549 \\ 0.0549 & 0.1751 \end{bmatrix}, \qquad R_{22} = \begin{bmatrix} 0.2050 & 0.2661 \\ 0.2661 & 0.3908 \end{bmatrix},$$

$$R_{33} = \begin{bmatrix} 0.0774 & -0.0726 \\ -0.0726 & 0.2118 \end{bmatrix}, \qquad \varepsilon_{11} = 3.8551, \quad \varepsilon_{21} = 4.8979,$$

$$\varepsilon_{31} = 46.2535, \quad \varepsilon_{41} = 2.9860, \quad \varepsilon_{51} = 0.1996, \quad \varepsilon_{12} = 14.3724,$$

$$\varepsilon_{22} = 8.5665, \quad \varepsilon_{32} = 7.0292, \quad \varepsilon_{42} = 6.0140, \quad \varepsilon_{52} = 2.5612,$$

$$\varepsilon_{13} = 0.0039, \quad \varepsilon_{23} = 0.0062, \quad \varepsilon_{33} = 1.0021, \quad \varepsilon_{43} = 0.0863,$$

$$\varepsilon_{53} = 1.3864,$$

$$W_1 = \begin{bmatrix} 4.7303 & -0.1241 \\ -0.1241 & 4.3675 \end{bmatrix}, \qquad W_2 = \begin{bmatrix} 6.6552 & 0.6488 \\ 0.6488 & 7.4779 \end{bmatrix},$$

$$W_3 = \begin{bmatrix} 0.8579 & -0.0701 \\ -0.0701 & 0.9228 \end{bmatrix}, \qquad W_{12} = \begin{bmatrix} 0.0545 & 0.1029 \\ 0.1029 & 0.3833 \end{bmatrix},$$

$$W_{13} = \begin{bmatrix} 0.0441 & 0.0596 \\ 0.0596 & 0.2095 \end{bmatrix}, \qquad W_{21} = \begin{bmatrix} 1.1252 & 0.1058 \\ 0.1058 & 0.3575 \end{bmatrix},$$

$$W_{23} = \begin{bmatrix} 1.3271 & -0.4271 \\ -0.1271 & 0.3261 \end{bmatrix}, \qquad W_{31} = \begin{bmatrix} 0.7455 & 0.2239 \\ 0.2239 & 0.6369 \end{bmatrix},$$

$$W_{32} = \begin{bmatrix} 0.5978 & -0.1349 \\ -0.1349 & 0.6563 \end{bmatrix}, \qquad W_{d11} = \begin{bmatrix} 5.5922 & 0 \\ 0 & 0.4692 \end{bmatrix},$$

$$M_{d22} = \begin{bmatrix} 22.1174 & 0 \\ 0 & 0.4692 \end{bmatrix}, \qquad M_{d33} = \begin{bmatrix} 2.4302 & 0 \\ 0 & 0.4692 \end{bmatrix},$$

$$M_{d12} = \begin{bmatrix} 15.7579 & 0 \\ 0 & 0.4692 \end{bmatrix}, \qquad M_{d13} = \begin{bmatrix} 28.5932 & 0 \\ 0 & 0.4692 \end{bmatrix},$$

$$M_{d21} = \begin{bmatrix} 3.1601 & 0 \\ 0 & 0.4692 \end{bmatrix}, \qquad M_{d23} = \begin{bmatrix} 1.6298 & 0 \\ 0 & 0.4692 \end{bmatrix},$$

$$M_{d31} = \begin{bmatrix} 15.0269 & 0 \\ 0 & 0.4692 \end{bmatrix}, \qquad M_{d32} = \begin{bmatrix} 1.3184 & 0 \\ 0 & 0.4692 \end{bmatrix},$$

$$M_{11} = \begin{bmatrix} 0.9287 & 0 \\ 0 & 0.4692 \end{bmatrix}, \qquad M_{22} = \begin{bmatrix} 0.5233 & 0 \\ 0 & 0.4692 \end{bmatrix},$$

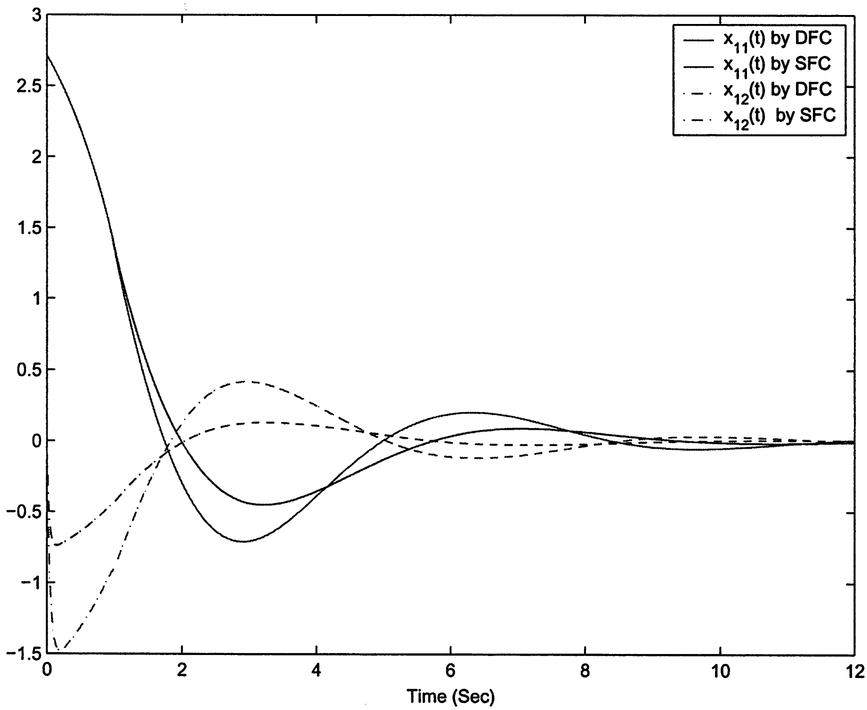

Fig. 1. State responses in Subsystem 1.

$$M_{33} = \begin{bmatrix} 1.0526 & 0 \\ 0 & 0.4692 \end{bmatrix},$$
$$\alpha_1 = 65.2342, \quad \alpha_2 = 6.3032, \quad \alpha_3 = 28.0545.$$

Thus, the stabilizing controllers are

$$u_1(t) = K_1 z_1(t) = Y_1 X_1^{-1} z_1(t)$$
$$= [-7.6225 \quad -16.5473] \left(x_1(t) + \int_{t-0.9}^{t} A_{11} x_1(s) ds \right), \tag{58a}$$
$$u_2(t) = K_2 z_2(t) = Y_2 X_2^{-1} z_2(t)$$
$$= [26.0129 \quad -20.6013] \left(x_2(t) + \int_{t-0.8}^{t} A_{22} x_2(s) ds \right), \tag{58b}$$
$$u_3(t) = K_3 z_3(t) = Y_3 X_3^{-1} z_3(t)$$
$$= [-23.4345 \quad -10.7930] \left(x_3(t) + \int_{t-0.7}^{t} A_{33} x_3(s) ds \right), \tag{58c}$$

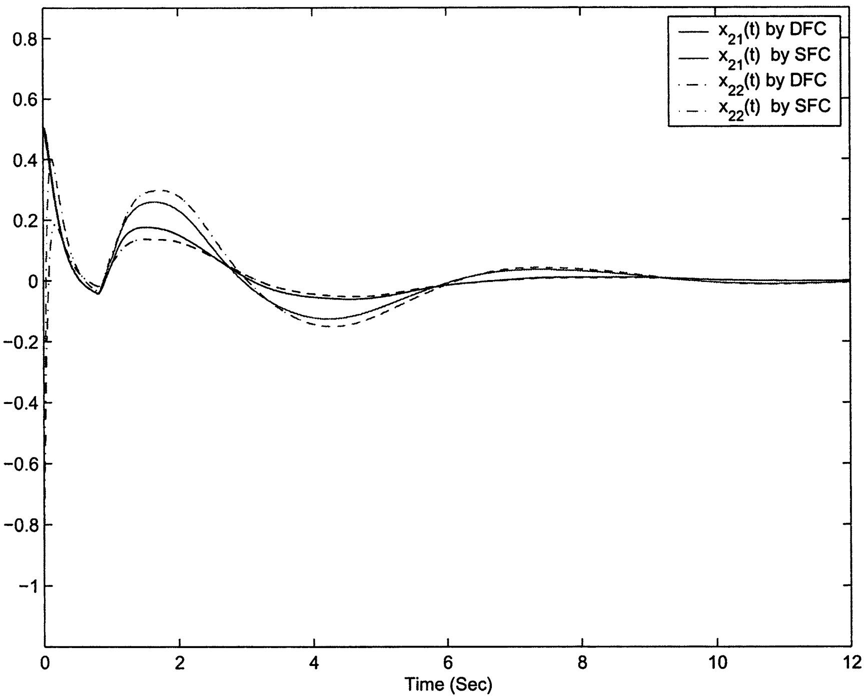

Fig. 2. State responses in Subsystem 2.

and the guaranteed costs for each subsystem are obtained as follows:

$$J_1^* = \alpha_1 + \text{Trace}(M_{11} + M_{d11} + M_{d12} + M_{d13}) = 117.9849,$$
$$J_2^* = \alpha_2 + \text{Trace}(M_{22} + M_{d22} + M_{d21} + M_{d23}) = 35.6107,$$
$$J_3^* = \alpha_3 + \text{Trace}(M_{33} + M_{d33} + M_{d31} + M_{d32}) = 39.7595.$$

In the numerical simulation, two types of controller are implemented. One is the delayed feedback controller (DFC) (58); the other is the standard memoryless feedback controller (SFC),

$$u_i(t) = K_i x_i(t).$$

The simulation results for the system (57) are illustrated in Figures 1–4. From the figures, we can see that the delayed feedback controllers (58) yield better performance than those of the memoryless feedback controllers, which do not have integral term in (58). Finally, note that the system (57) is unstable when the

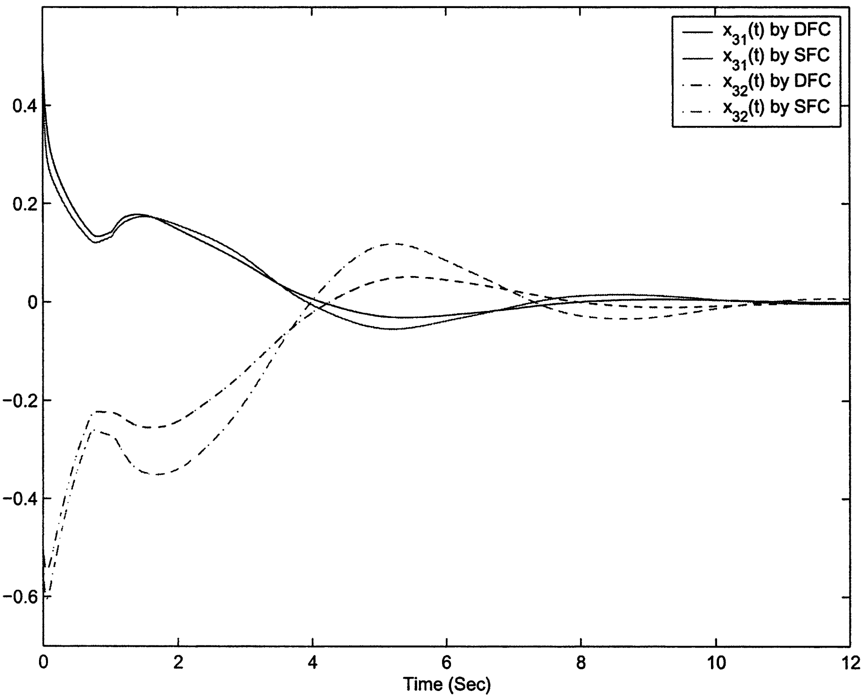

Fig. 3. State responses in Subsystem 3.

pure delay controller,

$$u_i(t) = K_i \int_{t-h_{ii}}^{t} A_{ii} x_i(s) ds,$$

is implemented.

6. Conclusions

In this paper, a decentralized delayed feedback controller design method was proposed for the guaranteed cost stabilization of uncertain large-scale interconnected systems with time delays. Based on the Lyapunov function method, a stabilization criterion has been derived in terms of LMIs. Using a numerical example, we showed that the obtained controllers stabilize the system and guarantee an adequate level of performance in spite of the uncertainties.

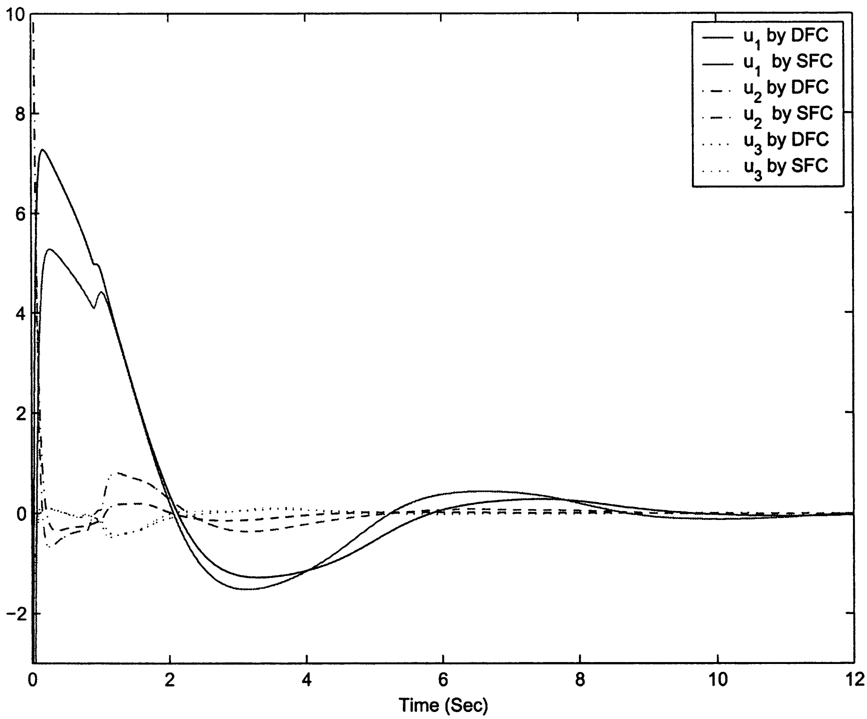

Fig. 4. Control inputs.

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